Vertical Mixing Due to Penetrative Convection

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ABSTRACT

A method for estimating environmental temperature and moisture increases due to penetrative convection is discussed. The estimates depend on the upward flux across the earth surface, the large-scale horizontal convergence, and the variation of entrainment of outside air by the convective element with height. The effect of thermals originating at the surface layer as well as those at upper levels are incorporated. The method could be generalized to include the exchange of other atmospheric properties such as momentum and atmospheric pollutants. An application of the method to determine the heating of the environment by dry convection is described.

1. Introduction

During days of strong solar heating, the thermal stratification of the lower atmosphere is characterized by a superadiabatic layer adjacent to the ground as shown schematically in Fig. 1. The stratification is most unstable near the ground, becoming less unstable upward, and finally stable at upper levels. Under this condition of stratification, upward heat transfer arises mainly from the motion of discrete, buoyant masses of hot air, the so-called thermals or penetrative convective elements. Each thermal breaks away from the surface layer and ascends up to the level where its temperature equals that of the environment. The maximum level which may be attained by an undiluted thermal is the equilibrium level Z_E. During its ascent, the thermal mixes with its surroundings and sheds off some of its excess heat content. The upward heat flux is directed towards lower values of the environmental potential temperature in the lower unstable layers; in the upper levels, the heat flux is directed towards higher values of the potential temperature, i.e., countergradient. It is quite clear that, besides transporting heat, the ascending thermals could transport other properties such as momentum, moisture and atmospheric pollutants.

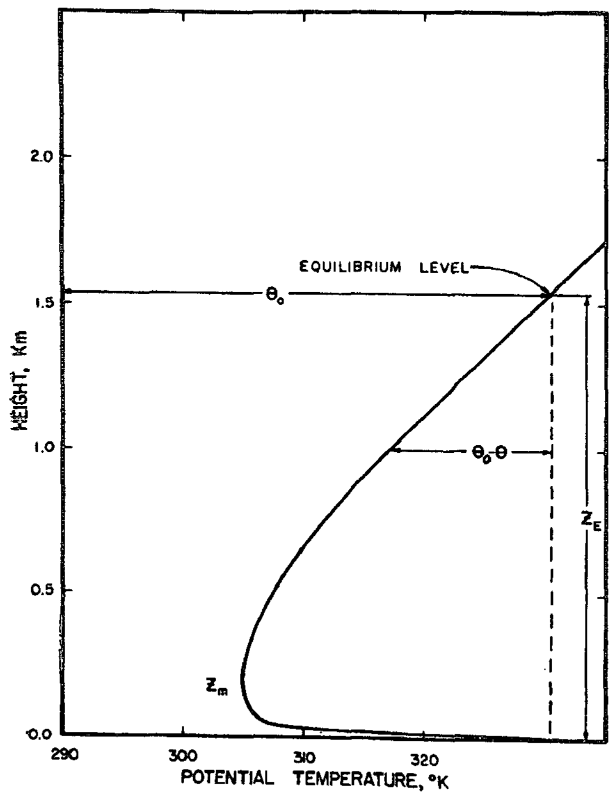

FIG. 1. Schematic diagram showing the potential temperature distribution under conditions of strong solar heating.

This paper describes a method for determining environmental temperature and other property changes associated with ascending thermals. The formulation follows, in general, a philosophy which has been proposed for parameterizing cumulus convection in hurricane modeling (Kuo, 1965).

2. Basic considerations

The vertical redistribution of atmospheric properties as envisaged in the preceding section may be analyzed in terms of three factors. The first factor is the difference between the value of the property inside the original undiluted thermal and the corresponding value of the property in the environment. As an example, for potential temperature, the difference at a height Z is simply the ambient potential temperature at the height Z_0 of origin of the thermal minus the ambient potential temperature at Z, i.e., $\theta(Z_0)-\theta(Z)$. Thus, for any given temperature sounding it is a function of both Z_0 and Z. For a fixed origin Z_0, it has a maximum value at the level Z_m

of minimum potential temperature and vanishes at the level of origin Z_0 and the equilibrium level Z_E. For a fixed Z and for the sounding shown in Fig. 1, it has a maximum value for $Z_0=0$. It is reasonable to expect that heating of the environment will occur whenever the difference is positive; the rate of heating is directly proportional to this difference.

The second factor is the amount of the property which is available for redistribution in the environment by thermals. For a given portion of the environment the availability of the property depends on the flux of the property into the volume and also on the intensity of the sources and sinks inside the volume. Hence, it is clear that if the flux is large, large increases of the property will occur in the environment as a result of penetrative convection. In general, the available amount of the property can be computed as a function of the large-scale synoptic condition and the characteristics of the earth surface. Thus, on clear sunny days and over terrain with low albedo, the available sensible heat flux is large. Large available flux of any property will also occur whenever there is a strong large-scale convergence of the horizontal flux of the property within or below the convectively unstable layer.

The third factor is the variation with height of the intensity of mixing between the thermal and its environment. This factor controls how the total available flux of the property gets redistributed or partitioned with height in the environment. The intensity of mixing may vary with height due to corresponding variations in the intensity of the environmental turbulence. Thus, over rough surfaces during strong wind conditions, the thermals would tend to break up rapidly at relatively low levels. Hence, most of the property is deposited at these levels and very little reaches the upper levels. The intensity of mixing also depends on the size of the thermal. Large thermals are able to attain greater heights than small ones. Therefore, they are able to deposit relatively greater amounts of the property at higher levels than small thermals.

3. Dry thermals

We now describe how the three aforementioned factors are incorporated in deriving a method for estimating the rate of heating due to penetrative convection. Consider the simple case of heating due to dry thermals originating from the ground under conditions shown in Fig. 1. Thermals with potential temperature θ_0 rise from the surface, gradually shedding off heat to the environment; they come to rest at levels where their potential temperature is equal to that of the environment. The maximum height which may be attained by thermals is the equilibrium level Z_E, where θ_0 is equal to the potential temperature of the environment. The layer bounded by the level of origin of the thermal and its equilibrium level is defined as the convective layer. Actually, the thermals may overshoot the level of equilibrium but this is not considered in our formulation. We will try to estimate the heating of a column due to the collective effect of such thermals if the total upward heat flux at $Z=0$ is given as F_0. Let us assume that the resulting temperature change in the environment is given by the equation

$$\left(\frac{\partial\theta}{\partial t}\right)_c = K(\theta_0-\theta), \quad 0\le Z\le Z_E, \tag{1}$$

where the subscript c indicates temperature change due to convection and K is a proportionality constant. This equation simply indicates that the environmental heating is directly proportional to the difference between the ambient temperature θ and the undiluted temperature θ_0 of the thermal. It is clear from Eq. (1) that heating ceases as soon as the difference vanishes and this occurs when the lapse rate becomes adiabatic.

The next step is to determine the constant of proportionality K. For this purpose, we will assume that the heat flux F_0 from the bottom is entirely used up in heating the layer from $Z=0$ to $Z=Z_E$. Using the thermodynamic energy equation (assuming no heat sources or sinks)

$$\frac{\partial\theta}{\partial t} = -\mathbf{V}\cdot\nabla\theta,$$

and a simplified equation of continuity

$$\nabla\cdot\rho_s\mathbf{V}=0,$$

one can obtain the thermodynamic energy equation in flux form. Thus,

$$\frac{\partial}{\partial t}(\rho_s\theta) = -\nabla\cdot\rho_s\mathbf{V}\theta,$$

where ∇ is the 3-dimensional gradient operator, $\mathbf{V}$ the 3-dimensional velocity vector and ρ_s the density of the standard atmsophere.

Integrating the above equation over the column (assuming unit horizontal cross section) and considering only the heating effect of convection due to the heat flux F_0 through the bottom, one obtains

$$\int_0^{Z_E}\frac{\partial}{\partial t}(\rho_s\theta)_c dz = F_0.$$

Multiplying Eq. (1) by ρ_s and integrating over the same volume, one obtains

$$\int_0^{Z_E}\frac{\partial}{\partial t}(\rho_s\theta)_c dz = K\int_0^{Z_E}\rho_s(\theta_0-\theta)dz,$$

which can be combined with the preceding equation to give

$$K=\frac{F_0}{\int_0^{Z_E}\rho_s(\theta_0-\theta)dz}.$$

This expression for K is then substituted in Eq. (1) to yield an equation describing the heating effect of thermals, i.e.,

$$\left(\frac{\partial\theta}{\partial t}\right)_c=\frac{F_0(\theta_0-\theta)}{\int_0^{Z_E}\rho_s(\theta_0-\theta)dz}. \qquad (2)$$

It is important to note that this equation is free from arbitrary parameters which are related to mixing processes.

Eq. (2) has been tested using observations of potential temperature (Fig. 2) and heat flux (Fig. 3) by Telford and Warner (1964). With the aid of both of these observations, we extrapolated the surface values of the potential temperature θ_0 and the heat flux F_0. These values, together with the observed potential temperature $\theta(Z)$, were substituted in Eq. (2) in order to compute the theoretical local temperature tendency. The result of this computation is shown in Fig. 4 as the dashed curve which is labeled "Theory," $M(Z)=1$. For comparison, we computed the height derivative of the observed heat flux (smoother) shown in Fig. 3 to obtain an "observed" local temperature tendency. It is seen that there is rough agreement between theory and observations; both show larger temperature changes at lower levels as compared to those at upper levels. However, the theory underestimates the values at lower levels, while overestimating them at upper levels. It appears, therefore, that Eq. (2) does not provide enough mixing between thermals and the environment at lower levels. An attempt to remedy this weakness has been made by modifying Eq. (1) so that it becomes

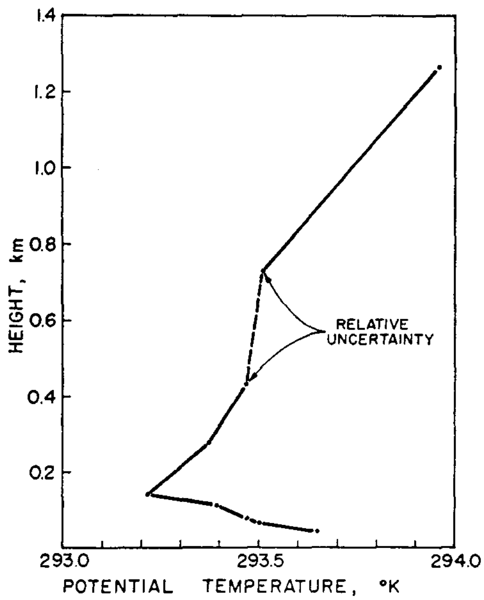

FIG. 2. The average potential temperature from several runs on 12 June 1961, plotted as a function of height. The uncertainty between the two points marked of is the order of 0.1K due to a range change in the thermometer used (after Telford and Warner, 1964).

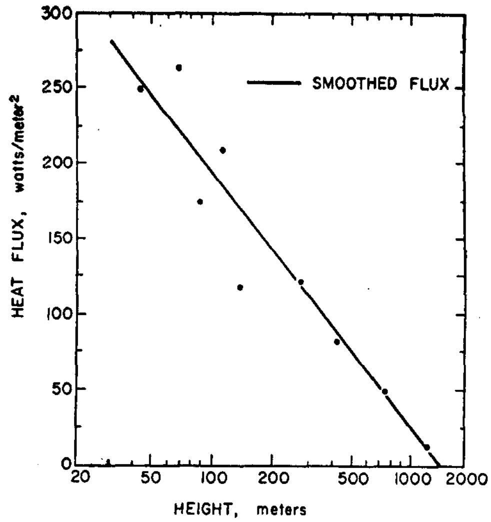

FIG. 3. The variation of heat flux with height for 12 June 1961, where dots represent actual data (after Telford and Warner, 1964).

$$\left(\frac{\partial\theta}{\partial t}\right)_c=KM(Z)(\theta_0-\theta),$$

where $M(Z)$ is an arbitrary parameter which is proportional to the intensity of mixing. Using this equation and following the same procedure used in deriving Eq. (2), one obtains

$$\left(\frac{\partial\theta}{\partial t}\right)_c=\frac{F_0M(Z)(\theta_0-\theta)}{\int_0^{Z_E}\rho_sM(Z)(\theta_0-\theta)dz}. \qquad (3)$$

Thus, we have derived an equation which incorporates the three factors which were enumerated in Section 2. These are the differences between the undiluted and ambient values of the property $(\theta_0-\theta)$, the available flux (F_0) and the variation of the mixing with height, $M(Z)$.

Using this equation and the same F_0 used previously, we recomputed $(\partial\theta/\partial t)_c$ on the assumption that

$$M(Z)=1-\frac{Z}{Z_E}.$$

This particular expression for M was chosen so that greater mixing would occur at lower levels. The results are also plotted in Fig. 4 as a solid curve labeled with the appropriate $M(Z)$. It may be seen that better

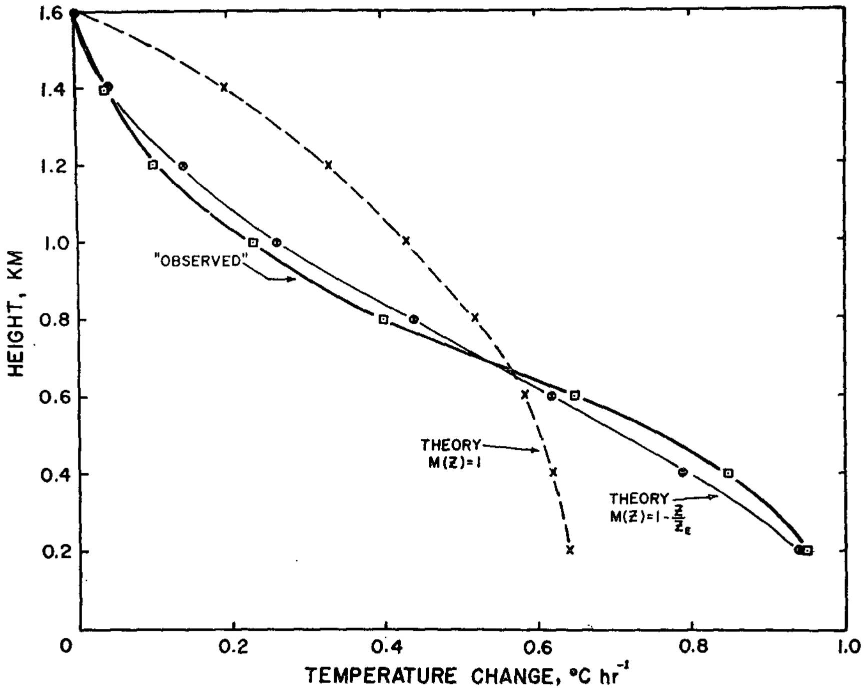

Fig. 4. Comparison between the theoretical and "observed" temperature change (°C hr^{-1}).

agreement between theory and observation has been achieved.

The usefulness of Eq. (3) depends on our ability to specify correctly the parameter $M(Z)$. This parameter is expected to be related to the turbulence characteristics of the atmosphere which, in turn, are related to the large-scale flow pattern and the characteristics of the underlying terrain. There are presently no studies which could shed light on this relationship and one could only make an educated guess as to what the relationship is. It is fortunate, however, that the computed $(\partial\theta/\partial t)_c$ is influenced to a greater degree by $(\theta_0-\theta)$ than by M. And even a crude guess for M could give reasonable results.

4. Moist penetrative convection

The formulation described for dry convection in the preceding section is in terms of the potential temperature, a quantity which is conserved in dry adiabatic process. When convection is accompanied by condensation, it is convenient to use the equivalent potential temperature, a property which is analogous to potential temperature. Using this quantity, and following the same procedure used in deriving Eq. (3), one can show easily that the appropriate governing equation for the equivalent potential temperature is

$$\left(\frac{\partial\theta_e}{\partial t}\right)_c = \frac{F_{e0}M(Z)(\theta_{e0}-\theta_e)}{\displaystyle\int_0^{Z_E} \rho_s M(Z)(\theta_{e0}-\theta_e)dz}, \tag{4}$$

where the subscript e indicates equivalent temperature and F_{e0} is the upward flux of equivalent temperature at the ground. This equation provides a means for obtaining the local increase in the equivalent potential temperature when the atmosphere is conditionally unstable. Again, the upper limit of the integration with respect to Z is the level at which the undiluted temperature of the rising thermal is equal to that of the environment.

Since the equivalent potential temperature is a function of both the temperature and the moisture content, a separate equation for determining the mixing ratio must be formulated so that one can calculate the potential temperature. The form of the tendency equations for both the potential and the equivalent potential temperatures suggests the following equation for the mixing ratio:

$$\left(\frac{\partial q}{\partial t}\right)_c = \frac{F_{q0}M(Z)(q_L-q)}{\displaystyle\int_0^{Z_E} \rho_s M(Z)(q_L-q)dz}, \tag{5}$$

where q is the ambient mixing ratio and q_L the mixing ratio of the undiluted ascending thermal. In the layer between the surface and the condensation level, q_L is equal to the surface mixing ratio; above this level, it decreases with height and equals the moist adiabatic value q_m, given by the saturated adiabat through the condensation level. The variation of q_m with height may be obtained from the thermodynamics of moist air together with temperature, pressure and mixing ratio of the surface air.

The quantity F_{qa} is that portion of F_{q0}, the total upward moisture flux across the surface (due to evaporation), which is available for increasing the ambient water vapor through convective processes. The difference, $F_{q0}-F_{qa}$, is, of course, the portion which is precipitated in liquid or solid form. We will assume on the basis of intuition that the available mositure is given by

$$F_{qa}=\frac{F_{q0}}{Z_E-Z_{CL}}\int_{Z_{CL}}^{Z_E}\left[1-\left(\frac{q}{q_m}\right)^2\right]dz,\quad Z_E-Z_{CL}\geqslant D,$$

$$F_{qa}=F_{q0},\quad Z_E-Z_{CL}<D,$$

where Z_{CL} is the condensation level for the surface air. Here, D is an empirical cloud depth which separates a precipitating cloud from a nonprecipitating cloud, an approximate value being 2 km. This equation implies that if $q=q_m$ in the interval $Z_{CL}\leqslant Z\leqslant Z_E$, and $Z_E-Z_{CL}\geqslant D$, all the moisture which is carried upward by convection is precipitated and no increase in the ambient mixing ratio occurs. If the air is absolutely dry ($q=0$), then all the moisture is used to increase the ambient mixing ratio. If $Z_E-Z_{CL}<D$, no precipitation occurs, and all the flux is again used to moisten the environment. In conjunction with the customary equations describing the thermodynamics of moist air, Eqs. (4)–(6) provide enough relationships for computing the local tendencies of the temperature and moisture fields.

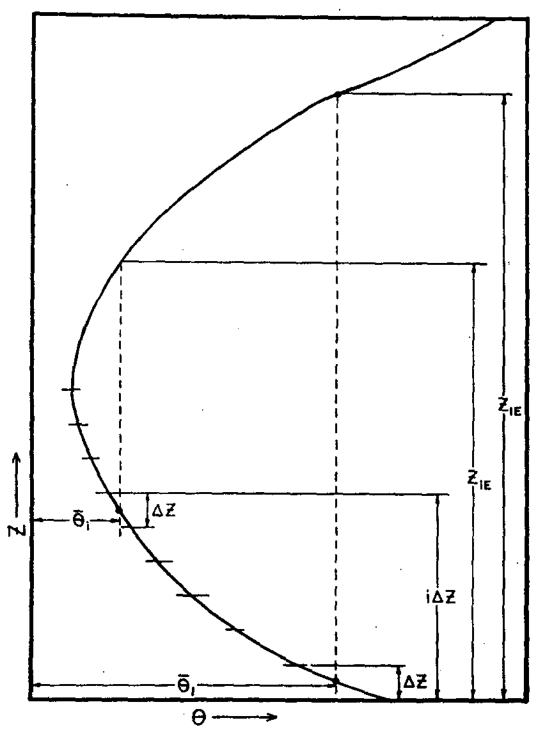

FIG. 5. Schematic diagram showing division of height into incremental layers.

In some cases it may be more desirable to use θ and q instead of θ_e and q as variables. In such cases the equation for equivalent potential temperature (4) should be replaced by the following equation for potential temperature:

$$\frac{\partial\theta}{\partial t}=\frac{F_0M(Z)(\theta_{th}-\theta)}{\displaystyle\int_0^{Z_E}\rho_sM(Z)(\theta_{th}-\theta)dz}+\frac{\dfrac{L}{C_p}(F_{q0}-F_{qa})(\theta_{th}-\theta)\delta}{\displaystyle\int_{Z_{CL}}^{Z_E}\rho_sM(Z)(\theta_{th}-\theta)dz}$$

$$\delta=\begin{cases}1, & Z\geqslant Z_{CL}\\ 0, & Z<Z_{CL}\end{cases}$$

Here L is the latent heat of condensation and C_p the specific heat at constant pressure. The quantity θ_{th} is the potential temperature of the undiluted thermal and is equal to the surface potential temperature in the layer below the condensation level. Above this level, the temperature of the thermal is given by the moist adiabat corresponding to the temperature, pressure and mixing ratio at the level of origin of the thermal.

5. Convection due to convergence

Observations show that there is a strong positive correlation between large-scale horizontal convergence in the lower troposphere and cumulus convection. The convection which is induced by convergence brings about strong upward transports of heat and moisture. In this case, the thermals which are responsible for the transports originate at levels above the earth's surface. Thus, their starting undiluted values of potential temperature and mixing ratios are, in general, less than those of thermals originating at the ground. Therefore, the corresponding convective layers are thinner. The equations which have been derived in the previous section would not apply directly. But they can be modified to cover this more general case. The necessary modification will be illustrated for the case of heating due to dry convection. We will assume that the local tendency of temperature at any given level is the integrated contribution of heating by thermals which originate at different levels below this given level. The computation of the tendency may be done conveniently by dividing the column into thin horizontal layers of thickness ΔZ as shown in Fig. 5. Let us consider the first layer above the ground whose bottom is at $Z=0$ and whose top is at $Z=\Delta Z$. The upward flux of heat across the top of this first layer due to horizontal convergence (indicated by H) is

$$F_1=-\overline{(\nabla_H\cdot\rho_s\mathbf{V}_H\theta)}_1\Delta Z,$$

where the bar denotes the average value for the layer $0\leqslant Z\leqslant\Delta Z$. Then, applying the results in Section 2, the heating at the level Z due to thermals originating in this layer is

$$\left(\frac{\partial\theta}{\partial t}\right)_{c1}=\frac{F_1M(\bar{\theta}_1-\theta)}{\int_{\Delta Z}^{Z_{1E}}\rho_s M(\bar{\theta}_1-\theta)dz},$$

where $\bar{\theta}_1$ represents the average potential temperature for the layer while Z_{1E} is the equilibrium level proper to the undiluted potential temperature $\bar{\theta}_1$. By induction, the contribution to the local potential temperature change of thermals originating in the second layer, $\Delta Z\leq Z\leq 2\Delta Z$, is

$$\left(\frac{\partial\theta}{\partial t}\right)_{c2}=\frac{F_2M(\bar{\theta}_2-\theta)}{\int_{2\Delta Z}^{Z_{2E}}\rho_s M(\bar{\theta}_2-\theta)dz},$$

where

$$F_2=-\overline{(\nabla_H\cdot\rho_s \mathbf{V}_H\theta)}_2\Delta Z,$$

and Z_{2E} is the equilibrium level appropriate for $\bar{\theta}_2$. The contribution due to the ith layer is then

$$\left(\frac{\partial\theta}{\partial t}\right)_{ci}=\frac{F_iM(\bar{\theta}_i-\theta)}{\int_{i\Delta Z}^{Z_{iE}}\rho_s M(\bar{\theta}_i-\theta)dz}.$$

It is understood, of course, that the contribution is to be computed only if there is convergence in the layer and if the atmosphere above the layer is unstable.

The total tendency at any level Z is the sum of the contributions from all layers below Z where convective elements originate plus the contribution of thermals originating from the ground. Thus,

$$\left(\frac{\partial\theta}{\partial t}\right)_c=\sum_{i=0}^{n}\left(\frac{\partial\theta}{\partial t}\right)_{ci},$$

where n is the number of such layers and $(\partial\theta/\partial t)_{c0}$ represents the contribution of thermals originating from the ground as given by Eq. (3). The corresponding formulae governing the local tendencies of equivalent potential temperature and moisture for the case of moist convection will not be presented. These formulae can be derived in a straightforward manner similar to the procedure used in deriving the above formula.

6. Concluding remarks

The present paper has described a method for estimating the ambient temperature and water vapor increases due to redistribution by penetrative convection. The principle used in formulating the method is sufficiently general so that it may be applied to the redistribution of other properties such as, for example, momentum and atmospheric pollutants. For an application of the method to any other property, it is simply necessary to know the value of the property for the undiluted thermal and the available flux of the property. Both of these quantities should be modified for any non-conservative effect in a manner similar to that used in the treatment of water vapor. The quantities are then substituted in the appropriate equations which have been derived. The main weakness of the present method is in the specification of the empirical function $M(Z)$ in terms of the large-scale synoptic situation and the physical characteristics of the earth surface. It is hoped that future studies will enable us to specify this function satisfactorily.

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