Comments on "Estimating the Vertical Ozone Distribution"

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This paper (Herman and Yarger, 1969) represents an important advance in indirect sensing in that it bridges successfully the gap between the inversion of linear problems typified by a matrix equation $\mathbf{y}=\mathbf{A}\mathbf{x}$, with $\mathbf{A}$ a fixed matrix; and nonlinear problems, typified by the equation $\mathbf{y}=\mathbf{A}(\mathbf{x})\mathbf{x}$, $\mathbf{A}$ now being a function of $\mathbf{x}$. The iterative technique used by Herman and Yarger amounts to a linearization at each step according to the scheme $\mathbf{y}=\mathbf{A}(\mathbf{x}_{m-1})\mathbf{x}_m$, the previously determined solution $\mathbf{x}_{m-1}$ being used to derive a value of $\mathbf{A}$ for the next iteration. Since many other indirect sensing problems —for instance, the inference of temperature structure from infrared radiance measurements (Yamamoto, 1961; Wark 1961)—have been forced into linear form at the expense of an exact mathematical modelling of the physical situation, the technique described by Herman and Yarger has obvious advantages, especially when highly accurate measurements are available and the overall error is dominated by the "linearization error."

There are, however, some aspects to the paper which may give rise to misunderstanding. Certainly, many readers will infer that the good fit of Herman and Yarger's solutions in the lower part of the ozone layer is a direct result of their having successfully accounted for multiple scattering [in contrast to results like those of Twomey (1961), who failed to infer the lower part of the ozone distribution]. In fact, however, the good fit of their solutions in this region depends on the choice of a wavelength (3250 Å) which penetrates appreciably the entire ozone layer and gives rise therefore to back-scattered radiation which contains appreciable energy contributions from all parts of the ozone layer. Such wavelengths were excluded in Twomey (1961) on the grounds that they reach the troposphere and the surface and are there reflected; the reflection function of the surface plus the non-Rayleigh atmospheric components must be known *to high accuracy* before such partially absorbed wavelengths can be utilized in practice. Whether this formidable proviso can be met in practical situations is another question.

A second, less important, point is the introduction of multiple Lagrangian multipliers by means of a vector $\boldsymbol{\gamma}$ rather than a scalar. Fundamentally, the Lagrangian multiplier in this procedure is a function of the prescribed error norm $\sum_j \epsilon_j^2$, within which the solution vector is allowed to vary and thereby maximize some *selected* quadratic measure of smoothness. When larger deviations can meaningfully be permitted at certain levels, this can easily be achieved by appropriate weighting in the measure of smoothness, i.e., in the elements of the matrix $\mathbf{H}$. Computationally, this is identical with the procedure adopted by Herman and Yarger but it places the weighting where it really belongs; i.e., in the quadratic measure of smoothness, which is *chosen*, rather than in the error bounds, which are dictated by observational and other errors. In this context it might be noted that there is no reason why the number of unknowns N cannot exceed the number of observations M. The unknowns are no longer independent, since a degree of correlation has been forced among them or, to put it another way, the matrix $(\mathbf{A}^*\mathbf{A}+\gamma\mathbf{H})$ will not be singular when $\mathbf{A}^*\mathbf{A}$ is. The writer has in several problems used $N>M$ without introducing any instabilities or computational difficulties.

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