Reply

R. C. SRIVASTAVA AND D. ATLAS

Dept. of the Geophysical Sciences, The University of Chicago

14 November 1969 and 19 December 1969

We are grateful to Dr. Das for his critical reading of our paper (hereafter referred to as SA). In his comments (hereafter referred to as D), Dr. Das has raised a number of points. Here we shall confine ourselves to 1) the main criticism in D, i.e., the derivation of Eq. (28) of SA (all equation numbers will refer to SA), and 2) a remark concerning the interpretation of the infinite concentration at the turn-around level in the case of a discrete distribution.

The main criticism in D concerns the differentiation of (27) to obtain (28). For reference, let us restate (23) and (24):

$$N\Delta V|W-V|r^2=N_0\Delta V_0|W_0-V_0|r_0^2, \quad \text{(23SA)}$$

$$\frac{N}{N_0}=\left|\frac{W_0-V_0}{W-V}\right|\left(\frac{r_0}{r}\right)^2\frac{dV_0}{dV}. \quad \text{(24SA)}$$

Here N is the concentration density (number per unit volume per unit terminal velocity interval) of precipitation particles, V the particle terminal velocity (positive downward), W the vertical velocity of air (positive upward), and r the radius of the circle over which the particles are distributed. The subscript zero refers to some particular height, say the initial level, and the unsubscripted variables refer to any other level z. The unsubscripted and subscripted variables are related as follows: Particles of fallspeeds V_0 and $V_0+\Delta V_0$ at the initial level grow to particles of fallspeeds V and $V+\Delta V$, respectively, on reaching the height z, and a particle of fallspeed V_0 situated at a distance r_0 from the cloud axis is situated at a distance r on reaching the height z. This distance will vary with the particle fallspeed, but in the limit of $\Delta V_0\rightarrow 0$ this variation does not enter in (23) or (24). Eqs. (23) and (24) express the equality of the rates at which particles cross the initial level and the level z per unit time, and the combination is thus an expression of particle conservation. Note that the time of growth from V_0 to V and from $V_0+\Delta V_0$ to $V+\Delta V$ will, in general, be different. However, this is immaterial in the formulation of the conservation equation (23).

From the foregoing, the manner in which the derivative dV_0/dV in (24) is to be obtained should be clear. It is merely necessary to express V as a function of V_0 and z, and differentiate the resulting equation at constant z. However, in the examples considered in SA, V was not explicitly available as such a function. Rather, we had V as a function of V_0 and t [Eq. (25)], and z as a function of V_0 and t [Eq. (26)]. Therefore, the required functional relationship between V, V_0 and z was obtained by eliminating t between (25) and (26). The resulting equation [(27)] was then differentiated at constant z to obtain the derivative in (28).

We hope that the above will convince the reader of the correctness of Eqs. (28) and (29), and the subsequent results based on these and similarly derived equations. We believe that the criticism in D is due to a too literal interpretation of the phrase "same group of particles" occurring in SA just after Eq. (24). Perhaps the choice of this phrase was unfortunate.

We would now like to make a general remark concerning the interpretation of the infinite concentration at the turn-around level when a discrete distribution is considered. If the concentration (number per unit volume) is represented by n, then we have Eq. (22), i.e.,

$$n|W-V|r^2=n_0|W_0-V_0|r_0^2, \quad \text{(22SA)}$$

which shows that $n\rightarrow\infty$ as $W\rightarrow V$. However, this does not violate any physical law. The occurrence of this infinite n does not necessarily mean that the total number of particles in the system is infinite, or that an infinite time will be required to establish a steady state.

For the cylindrically symmetric model considered in SA, this may be seen as follows. Consider a cylindrical region of radius r between any two levels z_1 and z_2, where r is the radius of the circle over which the particles are distributed and is therefore a function of z. The total number of particles in this cylindrical region denoted by X is

$$X=\pi\int_{z_1}^{z_2} r^2 n dz. \qquad (1)$$

On substituting for n from (22), we have

$$X=\pi r_0^2(W_0-V_0)n_0\int_{z_1}^{z_2}\frac{dz}{W-V}. \qquad (2)$$

Now we need to express $W-V$ as a function of z in order to carry out the integration. However, it is much simpler, and more general, to change the variable of integration from z to the time t through

$$dz/dt=W-V. \qquad (3)$$

Then

$$X=\pi r_0^2(W_0-V_0)n_0\int_{t_1}^{t_2} dt, \qquad (4)$$

where t_1 and t_2 are the times at which a given particle is at z_1 and z_2, respectively. In Eqs. (1) and (2), z_2 may be taken as the turn-around level or even equal to z_1, the integration being then understood to be from z_1 to the turn-around level and back to z_1. Eq. (4) shows that the total number of particles in the region between any two levels z_1 and z_2 is equal to the number of particles entering the initial level during a time interval (t_2-t_1) equal to the time it takes a particle to travel from z_1 to z_2. This is merely an expression of the conservation of particles, hardly requiring the demonstration given above. Since the time interval (t_2-t_1) will be finite in any physically realistic model, the particle number in any volume will also be finite even though the turn-around level may be included in the volume. These comments regarding the infinite concentration at the turn-around level in the case of a discrete distribution should be compared with the "physical feeling" expressed in D that "the analysis of particle motion given by Donaldson and Wexler (1968), as well as the trajectory picture indicated above points strongly to the fact that for each particle that reaches the TA level there is one that leaves it. Thus, in the steady state, there appears to be no accumulation of particles at the TA level."