The Transfer of Solar Irradiance Through Inhomogeneous Turbid Atmospheres Evaluated by Eddington's Approximation

E. P. SHETTLE AND J. A. WEINMAN

Dept. of Meteorology, University of Wisconsin, Madison

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ABSTRACT

Eddington's approximation was employed to compute the irradiances passing through atmospheres consisting of several different, albeit internally homogeneous, layers, as a function of solar zenith angle, albedo for single scattering, the asymmetry factor of the phase function, and the albedo of the underlying surface. Results computed for a single layer atmosphere were found to agree with more exact computations within a few percent.

Irradiances *within* several vertically inhomogeneous three-layer model atmospheres were computed. Effects caused by the vertically inhomogeneous structure are considered. It is noted, for example, that the irradiance within an atmosphere can be greater than that incident upon the atmosphere because radiation may be partially trapped within the atmosphere. The Eddington approximation affords a means to rapidly compute irradiances within realistic inhomogeneous atmospheres with an accuracy of several percent.

1. Introduction

Most computations of the transfer of light in turbid atmospheres have been concerned with the transmission and reflection properties of vertically homogeneous media, i.e., properties which may be measured at either boundary of the atmosphere. Relatively less attention seems to have been devoted to the computation of irradiances *within* vertically inhomogeneous turbid atmospheres. While Preisendorfer (1965) has considered a general solution to this problem, a simpler solution to this more restricted problem may be obtained by means of Eddington's approximation to the equation of transfer.

Eddington's approximation provides some qualitative insight into the mechanism whereby radiation is transferred within turbid atmospheres of moderate or great optical thicknesses. It will be shown that irradiances computed by means of Eddington's approximation are accurate to within a few percent and they may be rapidly evaluated. Irradiances within several inhomogeneous model turbid atmospheres are computed.

2. Analysis

The equation of transfer defining the diffuse radiance $I(\tau,\mu,\phi)$ is

$$\mu\frac{dI}{d\tau}(\tau,\mu,\phi)=-I(\tau,\mu,\phi)+\frac{a}{4\pi}\int_0^{2\pi}\int_{+1}^{+1}P(\mu,\phi;\mu',\phi')I(\tau,\mu',\phi')d\mu'd\phi'+\tfrac{1}{4}aF_0P(\mu,\phi;\mu_0,\phi_0)e^{-\tau/\mu_0}, \quad (1)$$

where the following notation is used:

- μ $\cos\theta$
- θ zenith angle
- ϕ azimuth angle
- $k(z)$ extinction coefficient
- $\tau=\int_0^z k(z')dz'$ optical depth
- $a(\tau)$ albedo for single scattering
- $P(\mu,\phi;\mu',\phi')$ phase function defining the light incident at μ',ϕ' which is scattered in the direction μ,ϕ.
- πF_0 solar irradiance perpendicular to the direction of incidence

Eddington's approximation assumes that the radiance can be given by

$$I(\tau,\mu)=I_0(\tau)+I_1(\tau)\mu. \quad (2)$$

If the phase function is expanded as a series of associated Legendre functions, in the manner of Chandrasekhar (1960), and we assume that $I(\tau,\mu)$ is given by (2), all terms of order greater than one will vanish when (1) is integrated over μ and ϕ. The phase function may therefore be approximated by

$$P(\Theta)=1+\omega_1(\tau)\cos\Theta, \quad (3)$$

where

$$\cos\Theta=\mu\mu'+(1-\mu^2)^{\frac{1}{2}}(1-\mu'^2)^{\frac{1}{2}}\cos(\phi-\phi'),$$

i.e., Θ is the angle between incident and scattered radiances. The integral in (1) thus becomes

$$\int_0^{2\pi}\int_{-1}^{1} P(\mu,\phi;\mu',\phi')I(\mu',\phi')d\mu'd\phi' = 4\pi(I_0+\mu g I_1), \quad (4)$$

where

$$g=\frac{\omega_1}{3}=\frac{1}{2}\int_{-1}^{+1} P(\Theta)\cos\Theta d(\cos\Theta).$$

Using (4) in (1) after integrating over ϕ and dividing by 2π, we have

$$\mu\frac{d(I_0+\mu I_1)}{d\tau} = -(I_0+\mu I_1)+a(I_0+g\mu I_1) + \tfrac{1}{4}aF_0e^{-\tau/\mu_0}(1+3g\mu_0\mu). \quad (5)$$

We obtain a pair of equations for I_0 and I_1 by integrating (5) and by integrating μ times (5) both over μ, i.e.,

$$\frac{dI_1}{d\tau} = -3[1-a(\tau)]I_0+\tfrac{3}{4}a(\tau)F_0e^{-\tau/\mu_0}, \quad (6)$$

$$\frac{dI_0}{d\tau} = -[1-a(\tau)g(\tau)]I_1+\tfrac{3}{4}a(\tau)g(\tau)\mu_0F_0e^{-\tau/\mu_0}. \quad (7)$$

Diffuse irradiances may be computed from I_0 and I_1 from

$$F(\tau)=2\pi\int_0^{\pm1}(I_0+\mu I_1)\mu d\mu = \pi[I_0(\tau)\pm\tfrac{2}{3}I_1(\tau)], \quad (8)$$

where $\mu>0$ corresponds to $F\downarrow(\tau)$ and $\mu<0$ corresponds to $F\uparrow(\tau)$.

The boundary conditions defining I_0 and I_1 are:

1) The downward directed diffuse irradiance at the top of the atmosphere is zero, viz.,

$$F\downarrow(0)=0=2\pi\int_0^1 \mu(I_0+\mu I_1)d\mu = \pi[I_0(0)+\tfrac{2}{3}I_1(0)]. \quad (9)$$

2) The upward directed irradiance at the bottom of the atmosphere is equal to the product of the downward directed irradiances and the albedo of the ground, i.e., the ground diffusely reflects all incident irradiance; thus,

$$F\uparrow(\tau^*)=\pi[I_0(\tau^*)-\tfrac{2}{3}I_1(\tau^*)] = A\pi[I_0(\tau^*)+\tfrac{2}{3}I_1(\tau^*)+\mu_0F_0e^{-\tau^*/\mu_0}], \quad (10)$$

where τ^* is the optical thickness of the entire atmosphere and A is the albedo of the ground.

The effective albedo is the ratio of the upward directed diffuse irradiance to the incident downward directed irradiance; thus,

$$A=F\uparrow(0)/(\pi\mu_0F_0). \quad (11)$$

In general it is not possible to find analytical solutions to (6) and (7). However, these equations can be solved in the special case of an atmosphere composed of homogeneous layers where

$$\frac{\partial a_i}{\partial\tau}=\frac{\partial g_i}{\partial\tau}=0.$$

We will concern ourselves with an atmosphere which is represented by layers each of which is homogeneous, and we will assume that each layer is characterized by different values of g_i and a_i.

a. Non-conservative case

For the homogeneous non-conservative case ($a\neq1$), the following solutions to (6) and (7) are appropriate within each layer; thus, for the ith layer ($i=1,2,\cdots N$), we have

$$I_0(\tau)=I_0^i(\tau)=C_1^ie^{-k\ \tau}+C_2^ie^{+k\ \tau}-\alpha_ie^{-\tau/\mu_0}, \quad \tau_{i-1}<\tau<\tau_i, \quad (12a)$$

$$I_1(\tau)=I_1^i(\tau)=P_i(C_1^ie^{-k\ \tau}-C_2^ie^{+k_i\tau})-\beta_ie^{-\tau/\mu_0}, \quad \tau_{i-1}<\tau<\tau_i, \quad (12b)$$

where:

$$k_i=[3(1-a_i)(1-a_ig_i)]^{\frac{1}{2}},$$
$$p_i=[3(1-a_i)/(1-a_ig_i)]^{\frac{1}{2}},$$
$$\alpha_i=3a_iF_0\mu_0^2[1+g_i(1-a_i)]/4(1-k_i^2\mu_0^2),$$
$$\beta_i=3a_iF_0\mu_0[1+3g_i(1-a_i)\mu_0^2]/4(1-k_i^2\mu_0^2),$$

and where $\tau_0=0$, and $\tau_N=\tau^*$ is the total optical depth of the atmosphere. In order to determine the C_1^i and C_2^i we must formulate $2N$ equations. Eqs. (9) and (10) provide the two boundary conditions

$$(1+2p_1/3)C_1^1+(1-2p_1/3)C_2^1=\alpha_1+2\beta_1/3, \quad (13)$$

$$[1-A-2(1+A)p_N/3]e^{-k_N\tau^*}C_1^N + [1-A+2(1+A)p_N/3]e^{+k_N\tau^*}C_2^N = [(1-A)\alpha_N-2(1+A)\beta_N/3+A\mu_0F_0]e^{-\tau^*/\mu_0}. \quad (14)$$

The remaining $(2N-2)$ equations are determined by requiring that $I_0(\tau)$ and $I_1(\tau)$ are continuous, i.e.,

$$I_0^i(\tau_i)=I_0^{i+1}(\tau_i), \quad i=1,2,\cdots,(N-1), \quad (15a)$$

$$I_1^i(\tau_i)=I_1^{i+1}(\tau_i), \quad i=1,2,\cdots,(N-1). \quad (15b)$$

b. Conservative case

For inhomogeneous turbid atmospheres with no absorption $[a(\tau)=1]$, the solution becomes simpler. Eq. (6) may be integrated to find $I_1(\tau)$ and this result can be used to find $I_0(\tau)$ from (7). The solution to the equation of transfer for this case is

$$I_0(\tau)=B_1-\tfrac{3}{4}\mu_0^2F_0e^{-\tau/\mu_0}-B_2T(\tau), \quad (16a)$$

$$I_1(\tau)=B_2-\tfrac{3}{4}\mu_0F_0e^{-\tau/\mu_0}, \quad (16b)$$

where

$$B_2=\frac{3\mu_0F_0(1-A)[2+3\mu_0+(2-3\mu_0)e^{-\tau^*/\mu_0}]}{4[4+3(1-A)T(\tau^*)]}, \quad (17a)$$

$$B_1=(3\mu_0^2/4+\mu_0/2)F_0-2B_2/3, \quad (17b)$$

and

$$T(\tau)=\int_0^\tau [1-g(\tau')]d\tau' \quad (18)$$

is the effective optical depth. Note that this solution for a conservative atmosphere is valid for an arbitrary $g(\tau)$ and is not restricted to a layered atmosphere.

The irradiances within various model atmospheres may be computed from (8) and (12) or (16) as a function of depth and incident sun angle for various underlying surfaces.

3. Comparison with previous work

It is desirable to compare solutions obtained from Eddington's approximation with more rigorously derived solutions to the transfer equation in order to establish the limits of accuracy that may be expected from irradiances computed by means of Eddington's approximation. Because such comparisons are only feasible for homogeneous turbid media, we compare results of the present analysis for $N=1$ with results obtained by other investigators. No layer index subscripts or superscripts i are specified for these examples.

a. Non-conservative case

1) For solar irradiances incident perpendicularly to the atmosphere ($\mu_0=1$) and zero ground reflectivity ($A=0$), (13) and (14) reduce to a form equivalent to the corresponding equations derived by Pitts (1954) [his Eqs. (4.1.13) and (4.1.14)].

2) Irvine (1968) computed the albedo of a non-conservative uniform atmosphere as a function of solar zenith angle. He assumed that the atmosphere was characterized by an optical thickness $\tau^*=16$, an albedo for single scattering $a=0.95$, and a phase function

$$P(\Theta)=bP_{HG}(g_1')+(1-b)P_{HG}(g_2'), \quad (19)$$

where

$$P_{HG}(g')=(1-g'^2)/(1+g'^2-2g'^2\cos\Theta)^{\frac{3}{2}},$$

$$g=\frac{1}{2}\int_{-1}^{+1}P(\Theta)\cos\Theta d(\cos\Theta)=bg_1'+(1-b)g_2'.$$

Fig. 1 presents a comparison of this computation with the albedo computed by means of Eddington's approximation for an atmosphere characterized by $a=0.95$, $\tau^*=16$, and $g=0.786$, which is the asymmetry factor of the phase function used by Irvine. Making use of (8), (11), (12), (13) and (14), we find that the albedo is

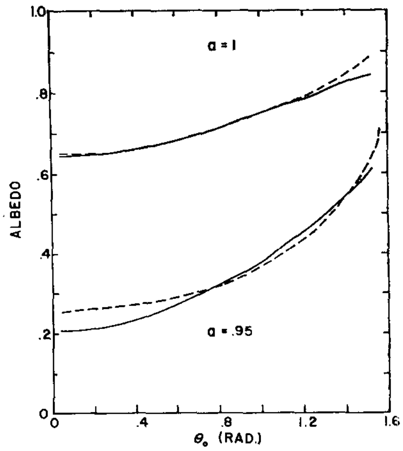

FIG. 1. Albedo as a function of the incident sun angle μ_0: comparison between Eddington's approximation (solid lines) and Irvine's (1968) results (dashed lines) for $\tau^*=16$ and $g=0.786$.

$$A=\frac{2(C_1+C_2)}{\mu_0F_0}-\frac{3a\mu_0(1+g-ag)}{2(1-k^2\mu_0^2)}. \quad (20)$$

In addition to the Irvine's exact calculations referred to above, he also presented an expression for the albedo using Eddington's approximation [his Eq. (16)]. That equation and the results he bases on it appear to be valid only for $\mu_0=1$.

b. Conservative case

For a conservative single layer atmosphere, the radiances $I_0(\tau)$ and $I_1(\tau)$ are given by (16). The effective optical thickness from (18) is now

$$T=(1-g)\tau^*. \quad (21)$$

The irradiance from (8) may be used to compute the albedo (11), i.e.,

$$A=1-(4\beta_2/3\mu_0F_0). \quad (22)$$

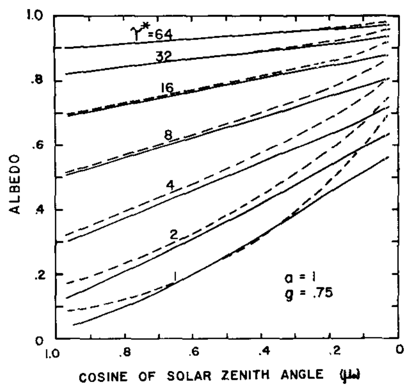

FIG. 2. Albedo as a function of the total optical thickness τ^* and the incident sun angle: comparison between Eddington's approximation (solid lines) and the exact results of van de Hulst and Grossman (1968) (dashed lines).

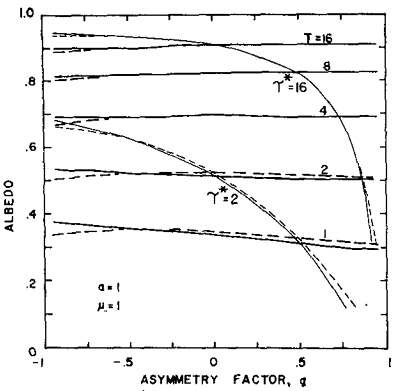

FIG. 3. Albedo as a function of the asymmetry factor g and the effective optical thickness $T=(1-g)\tau^*$: comparison between Eddington's approximation (solid lines) and the exact results of van de Hulst and Grossman (1968) (dashed lines).

Solutions to this problem may be compared with results obtained more accurately by other investigators.

1) Fig. 1 illustrates Irvine's exact computation of the albedo as a function of solar zenith angle for a turbid atmosphere characterized by $a=1$, $\tau^*=16$, $b=0.9724$, $g'_1=0.824$ and $g_2'=-0.55$ [for Eq. (19)]. The albedo obtained by means of Eddington's approximation for an atmosphere characterized by $a=1$, $\tau^*=16$ and $g=0.786$ [for Eq. (3)] is also shown for comparison.

Fig. 2 provides a comparison between the albedo as a function of solar zenith angle computed by Eddington's approximation and that computed rigorously by van de Hulst and Grossman (1968) for a turbid atmosphere characterized by $a=1$, $b=1$, and $g'=g=0.75$ [for Eqs. (19) and (3)].

2) Fig. 3 presents the comparison between the albedo as a function of asymmetry factor g as computed by Eddington's approximation and the results computed by van de Hulst and Grossman. The problem is assumed to be characterized by $a=1$, $\mu_0=1$ and $b=1$ in all of these computations.

TABLE 1. Irradiances emerging from a Rayleigh atmosphere over a Lambertian reflecting surface ($\mu_0=1$).

	$F\downarrow(\tau^*)$		$F\uparrow(0)$	
A	Eddington approximation	Kahle (1968)	Eddington approximation	Kahle (1968)
$\tau^*=1$				
0	0.92	0.91	1.06	1.05
0.25	1.17	1.2	1.40	1.4
0.8	2.00	2.1	2.50	2.5
1.0	2.48	2.7	π	π
$\tau^*=5$				
0	0.81	0.81	2.32	2.3
0.25	1.00	1.05	2.37	2.4
0.8	2.22	2.4	2.69	2.7
1.0	3.90	4.0	π	π

TABLE 2. Three-layer model atmospheres.

	Wavelength λ		
	0.76 μ*	0.4 μ (blue)	0.7 μ (red)
Clear layer ($z>4$ km)			
τ_1	0.492	0.291	0.069
g_1	0.553	0.180	0.511
a_1	0.124	1.000	1.000
Cloud layer ($3<z<4$ km)			
$(\tau_2-\tau_1)$	20.000	20.000	20.000
g_2	0.848	0.848	0.848
a_2	0.997	1.000	1.000
Aerosol laden air ($0<z<3$ km)			
$(\tau_3-\tau_2)$	0.370	0.346	0.169
g_3	0.708	0.507	0.701
a_3	0.434	1.000	1.000
Surface albedo ($z=0$)			
Ocean	0.02	0.09	0.02
Snow	0.75	0.78	0.75

* Oxygen A band.

One important feature evident in this figure, pointed out by van de Hulst and Grossman, is that the albedo depends mainly on the effective optical depth, Eq. (21).

3) The effect of the underlying surface albedo on the irradiance transmitted and reflected by a turbid atmosphere was evaluated from a model turbid atmosphere characterized by $g=0$. Kahle (1968) computed similar irradiances emerging from Rayleigh atmospheres above Lambertian surfaces characterized by various albedos.

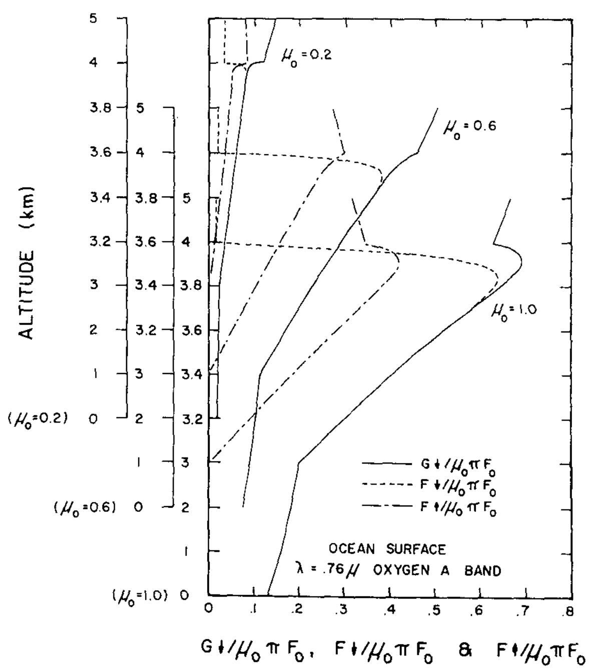

FIG. 4. Normalized irradiances within a non-conservative atmosphere, above the ocean as a function of height and incident sun angle. Note that the ordinate scale is nonlinear.

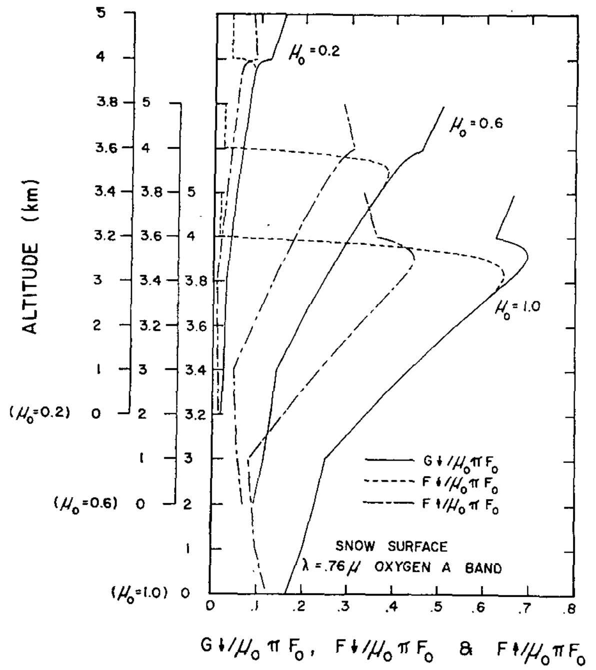

FIG. 5. Normalized irradiances within a non-conservative atmosphere, above snow as a function of height and incident sun angle. Note that the ordinate is nonlinear.

Table 1 summarizes the comparison of Kahle's solutions obtained from her Figs. 4 and 5 with those derived from Eddington's approximation.

4. Vertically inhomogeneous atmospheres

Since the real atmosphere is not homogeneous, the methods discussed above are of limited applicability. The Eddington approximation, however, may be generalized to apply to an inhomogeneous atmosphere. We will determine the irradiance within somewhat more realistic models of the earth's atmosphere summarized in Table 2.

We assume that these atmospheres consist of three internally homogeneous, albeit different, layers and that a cloud layer exists at an altitude between 3 and 4 km. A typical optical thickness for such a cloud is $\tau^*=20$ with only a slight variation depending on wavelength, which we ignore (Feigelson, 1964). The optical thickness of the atmosphere, above and below the cloud deck, due to both Rayleigh scattering from the air molecules and Mie scattering from aerosols, was obtained from Elterman (1968).

The asymmetry factor for turbid atmospheric layers is given by

$$g_i=(g_{\mathrm{Ray}}\Delta\tau_{\mathrm{Ray}}+g_{\mathrm{Mie}}\Delta\tau_{\mathrm{Mie}})/(\Delta\tau_{\mathrm{Ray}}+\Delta\tau_{\mathrm{Mie}}),\quad i=1,3, \tag{23}$$

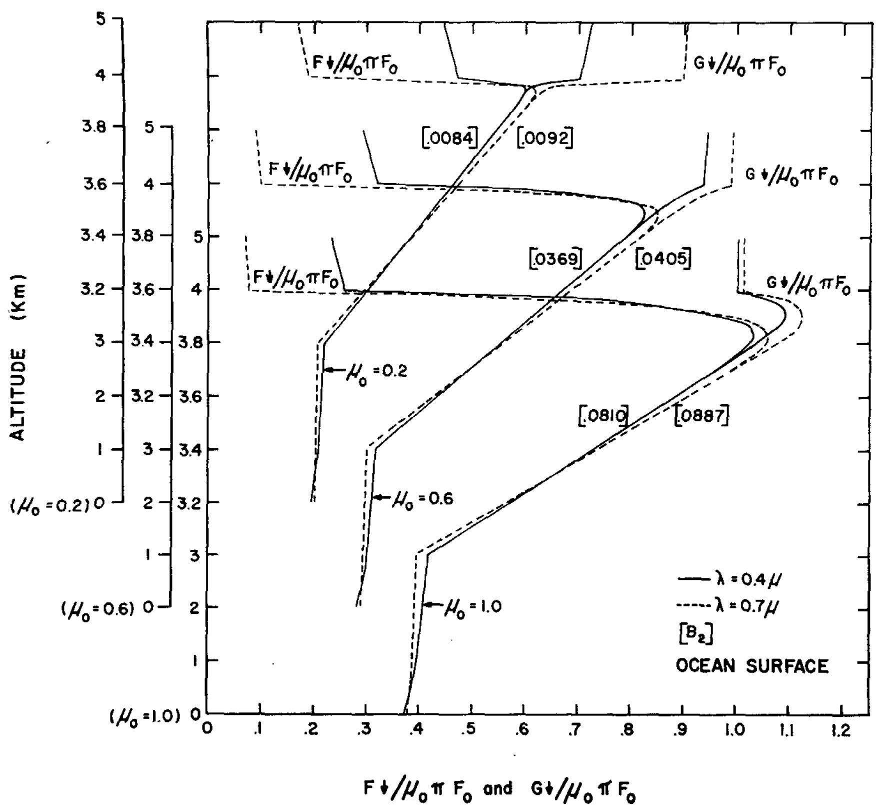

FIG. 6. Normalized irradiances above the ocean as a function of height and incident sun angle. Upward directed irradiances can be obtained from Eq. (27) by employing values of $[B_2]$ with the values of $G\downarrow(\tau)$ shown above. Note that the ordinate scale is nonlinear.

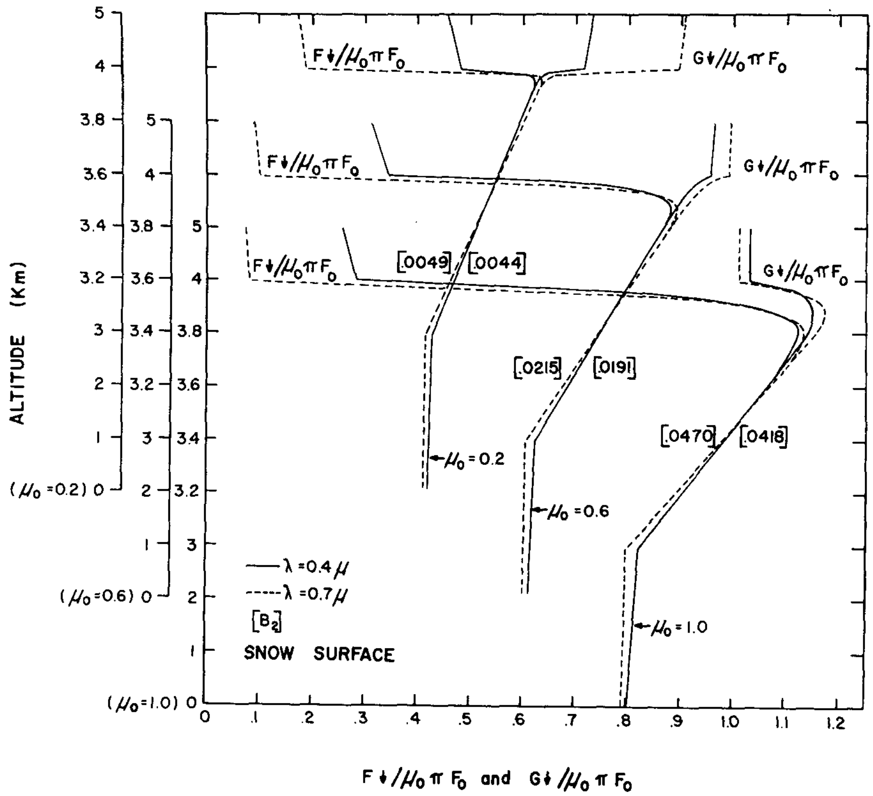

FIG. 7. Normalized irradiances above snow as a function of height and incident sun angle. Upward directed irradiances can be obtained from Eq. (27) by employing values of $[B_2]$ with the values of $G\downarrow(\tau)$ shown above. Note that the ordinate scale is nonlinear.

which for the cloud layer becomes $g_2 \sim g_{\mathrm{cloud}}$. The asymmetry factors are $g_{\mathrm{Ray}}=0$, $g_{\mathrm{Mie}}=0.75$ and $g_{\mathrm{cloud}}=0.848$ (see Irvine, 1968).

The ground surfaces were assumed to be Lambertian reflectors. The ground surfaces considered were ocean and snow because albedos of such surfaces represent extreme values for natural surfaces. The dependence of the ground albedo on the wavelength which is shown in Table 2 was obtained from the thesis of Bartman (1967).

a. Non-conservative case

Consider the optical characteristics of a cloudy atmosphere at a wavelength $\lambda=0.76\ \mu$ where absorption takes place in the oxygen A band. In accord with Wark and Mercer (1965), we assume that one air mass absorbs half of the radiation normally incident upon it, corresponding to $\tau=0.69$. It is assumed that the optical thickness of each layer is attributable to oxygen absorption in proportion to the pressure differential ΔP between the upper and lower boundaries of the layer, i.e.,

$$(\tau_i-\tau_{i-1}) = \begin{cases} \Delta\tau_{\mathrm{Ray}}+\Delta\tau_{\mathrm{Mie}}+0.69(\Delta P/1000), & i=1,3, \\ \tau_{\mathrm{Cloud}}+0.69(\Delta P/1000), & i=2. \end{cases} \tag{24}$$

The albedo for single scattering therefore becomes

$$a_i = \begin{cases} (\Delta\tau_{\mathrm{Ray}}+\Delta\tau_{\mathrm{Mie}})/(\tau_i-\tau_{i-1}), & i=1,3, \\ \tau_{\mathrm{Cloud}}/(\tau_i-\tau_{i-1}), & i=2. \end{cases} \tag{25}$$

We assumed that the atmosphere was composed of three layers each characterized by different values of g_i and a_i found from (23) and (25). Thus, the solution given by (12a) and (12b) with $N=3$ is the appropriate one to use. Figs. 4 and 5 illustrate the downward and upward directed diffuse irradiance, given by (8), and the total global downward directed irradiance, given by

$$G\downarrow(\tau)=F\downarrow(\tau)+\mu_0\pi F_0 e^{-\tau/\mu_0}. \tag{26}$$

b. Conservative case

We consider an inhomogeneous cloudy atmosphere with no absorption $[a(\tau)=1]$. In this case the appropriate solution to (6) and (7) is given by (16a) and (16b).

The downward directed diffuse irradiance and the total global downward directed irradiance are both shown in Figs. 6 and 7. The upward directed irradiance is not shown, because, for a conservative atmosphere, it differs from the total global downward irradiance by a constant only, i.e.,

$$F\uparrow(\tau)=G\downarrow(\tau)-\frac{4\pi}{3}B_2. \qquad (27)$$

This constant, $(4\pi B_2)/3$, is the net irradiance.

5. Discussion and conclusions

We have compared the irradiances emerging from homogeneous atmospheres as determined by the Eddington approximation with results obtained by other investigators who employed more accurate solutions (see Figs. 1–3 and Table 1). The Eddington approximation is found to give fairly accurate results for atmospheres of at least moderate thicknesses.

While the rigorous solutions of the radiative transfer equation yield more accurate results, they frequently take large amounts of computer time, and they only give the radiation emerging from the upper and lower boundaries of the atmosphere. The Eddington approximation provides a means for rapidly calculating the irradiances within as well as those emerging from the boundaries of turbid atmospheres. For example, the time required for computing the irradiances and absorbed radiation for each layer of a six layer non-conservative atmosphere took between 5 and 10 *milliseconds* using a UNIVAC 1108. Calculations for a conservative atmosphere are considerably faster, since it is not necessary to solve a large system of equation for the constants C_1^i and C_2^i, $i=1, 2, \cdots, N$. This speed might render the Eddington approximation a convenient method for including solar radiative heating effects in numerical models of the atmospheric circulation.

Fig. 3 shows that the albedo provides a measure of the effective optical thickness. Computations by Irvine (1963, 1965) showed that $(1-g)$ is relatively insensitive to the size of the particles comprising a cloud. Furthermore, Dobbins and Jizmigian (1966) have shown that the extinction coefficient is a function of the volume occupied by the drops and the volume/surface ratio of the drop ensemble, independent of the shape of the ensemble's size distribution. Albedo measurements may therefore yield information on the drop density and a weighted mean radius of drops comprising a cloud.

It is interesting to note in a thick atmosphere, for nearly normally incident solar irradiance, that the downward directed irradiance will initially increase as one descends from the top of the atmosphere. In some cases involving conservative atmospheres, the downward global irradiance in the interior of the atmosphere is actually greater than the irradiance incident on the top of the atmosphere (see e.g., Figs. 6 and 7). Kahle (1968) found a similar condition for the diffuse irradiance transmitted by a Rayleigh atmosphere above a highly reflecting surface (see Table 1). Our results differ from Kahle's result because we find that this is true *within* moderately thick atmospheres above less reflecting ground surfaces, not just those with $A>0.9$. The explanation of this "paradox" is that the downward irradiance passing through a given level in the atmosphere can be partially trapped between the highly reflective combination of the atmosphere and ground surface below that level and the diffusely reflecting atmosphere above the level. Light can therefore be multiply reflected through a given level several times before escaping to space or being absorbed by the ground or atmosphere. See the Appendix for more detailed discussion of this point.

The present analysis revealed another interesting characteristic of the irradiances within an inhomogeneous atmosphere; namely, that the downward irradiance may have several maxima, if the forward scattering increases with depth.

The downward directed global irradiance within non-conservative atmospheres, shown in Figs. 4 and 5, is never greater than the amount incident on the top of the atmosphere. This is because the oxygen above the cloud strongly attentuates the light. However, for near normal incidence, the downward directed diffuse irradiance near the top of the cloud is greater than the downward directed global irradiance incident upon the top of the cloud.

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APPENDIX

Comparison of Internal and Incident Downward Irradiances

It may seem paradoxical that the global downward irradiance within a turbid medium may exceed the incident downward irradiance, i.e., $G\downarrow(\tau)>G\downarrow(0)$. The following exposition illustrates why this is feasible.

We can regard the atmosphere as being divided into two layers, one above and one below a given level (at a depth τ). Each layer is assumed to be partially reflecting. A layer of reflectivity r represents the upper portion of the atmosphere, and a layer of reflectivity r' the lower portion of the atmosphere and/or the ground. The reflectivity of such layers depends on the angular distribution of the incident radiances, and this distribution depends on the different orders of reflections. The subscript k denotes the kth reflection from a given layer.

The upper layer transmits a fraction $(1-r_0)$ of the irradiance $G\downarrow(0)$ initially incident upon it from above. A fraction r_1' of this transmitted light is reflected upward by the lower reflecting layer. The part of the upward directed irradiance which has been reflected once from the lower layer is therefore $r_1'(1-r_0)G\downarrow(0)$. Of this radiation, a fraction r_1 is reflected back downward by the upper layer, and so on. Thus, the total downward directed irradiance between the two layers is

$$\left.\begin{aligned} G\!\downarrow\!(\tau) &= G\!\downarrow\!(0)[(1-r_0) \\ &\quad + r_1r_1'(1-r_0) + r_2r_2'r_1r_1'(1-r_0) \\ &\quad + r_3r_3'r_2r_2'r_1r_1'(1-r_0) + \cdots] \\ G\!\downarrow\!(\tau) &= G\!\downarrow\!(0)(1-r_0)[1+\sum_{n=1}^{\infty}(\prod_{k=1}^{n} r_kr_k')] \end{aligned}\right\}. \quad \text{(A1)}$$

Let us choose

$$r = \min(r_1, r_2, r_3, \cdots),$$
$$r' = \min(r_1', r_2', r_3', \cdots).$$

Then

$$G\!\downarrow\!(\tau) \geqslant G\!\downarrow\!(0)(1-r_0)[1+\sum_{n=1}^{\infty}(rr')^n]$$
$$= G\!\downarrow\!(0)\frac{(1-r_0)}{(1-rr')}, \quad \text{(A2)}$$

and thus

$$G\!\downarrow\!(\tau) > G\!\downarrow\!(0), \quad \text{if } rr' > r_0. \quad \text{(A3)}$$

The albedo of a turbid medium is a minimum for normally incident irradiances and it increases as the angle of incidence θ_0 approaches $\pi/2$. We always have $r>r_0$ for nearly normal incident solar radiance, as long as the upward directed irradiance is diffuse. Under this circumstance a significant fraction of the upward directed radiances may be incident at angles nearer to grazing than the incident solar radiance. All that is needed for (A3) to be satisfied is that r', the reflectivity of the lower layer, be sufficiently large. The lower portion of the atmosphere must therefore be thick or the ground albedo be high. This effect is illustrated by the fact that $G\!\downarrow\!(\tau)>G\!\downarrow\!(0)$ over a greater depth of the atmosphere above the snow surface (Fig. 7) which has a higher reflectivity than it is over the ocean with a lower reflectivity (see Fig. 6). We also see that when the sun is lower in the sky ($\mu_0=0.6$ or 0.2) so that $r<r_0$, then $G\!\downarrow\!(\tau)<G\!\downarrow\!(0)$ for all τ.

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