# The Dynamics of Updraft Vaults in Hailstorms as Inferred from the Entraining Jet Model

J. D. Marwitz, J. R. Middleton, A. H. Auer, Jr., and D. L. Veal

*Natural Resources Research Institute, University of Wyoming, Laramie*

27 May 1969 and 29 June 1970

## ABSTRACT

The one-dimensional steady-state entraining jet model of cumulonimbus by Squires and Turner was applied to the updraft vaults of 11 hailstorms which occurred in Nebraska and Alberta during July 1968. The initial conditions of cloud base temperature, height, updraft speed and radius were directly observed from an aircraft flying in the updrafts at cloud base. Environmental soundings of temperature and mixing ratio were obtained for each storm from nearby radiosonde observations.

The storm tops as estimated from the radar were within ±1.4 km of the model tops. The radii of the echo free vaults were within ±1 km of the model radii in the lower half of the storm. The significance of the other parameters predicted by the model are discussed.

## 1. Introduction

The entraining jet model of cumulonimbus by Squires and Turner (1962) considers many of the processes which act within the updraft regions of hailstorms. This model is based on a steady-state, fully turbulent, condensing plume which entrains environmental air according to the simple law that the inflow velocity at any height is proportional to the upward velocity of the plume. With this model, the velocity and radius of the updraft region as well as other properties follow from the dynamics. The set of four differential equations includes the conservation of air mass, momentum, energy and water mass. The processes considered by the set of equations are condensation, buoyancy, entrainment, water load and linear conversion from liquid to ice between $-15$ and $-40$C. The input parameters (initial conditions) required are cloud base height, temperature, updraft velocity and radius. Experience indicates that one cannot reliably predict hailstorm cloud base heights to ±500 ft on the High Plains using only radiosonde observations. Generally, observed cloud base temperatures agree quite closely with radiosonde observations, if one knows the cloud base height. However, mixing ratios required to produce the observed cloud base heights often are greater than any shown by radiosonde data. Therefore, the required cloud base observations were obtained by flying an instrumented aircraft at cloud base in the same manner as outlined by Auer and Sand (1966) and Auer and Marwitz (1968). In addition, the model requires the environmental temperature and mixing ratio above cloud base as boundary conditions. These were obtained from nearby representative radiosondes.

The radar observations of echo free vaults (EFV's) in hailstorms presented by Browning (1965), Browning and Donaldson (1963) and Chisholm (1968) have been shown to be updraft vaults by Marwitz *et al.* (1969). This was accomplished by flying an instrumented aircraft in the organized cloud base updraft fields (Auer *et al.*, 1970) and noting that these updraft fields were at the base of the EFV's.

Even though these updraft vaults are continuously undergoing evolutionary changes, it still appears that one can assume that they are a quasi-steady-state phenomenon during a 10–20 min period. This assumption is supported by observations of constant cloud base updrafts plus radar observations of the integrity of the EFV's during a 10–20 min period. Based on model

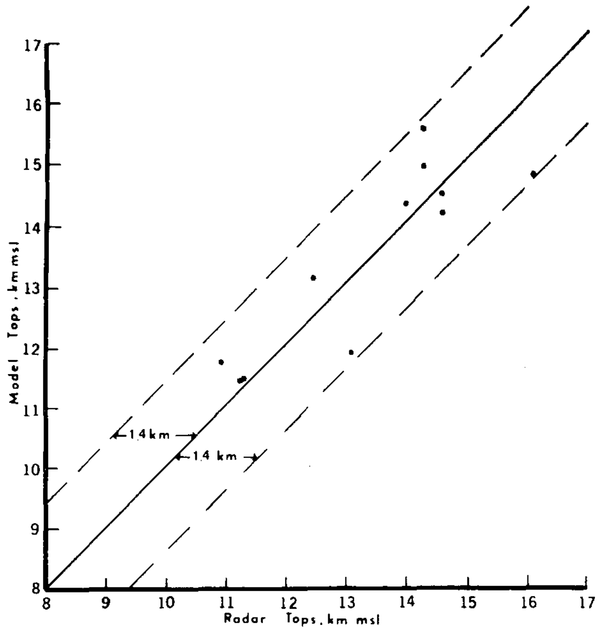

FIG. 1. Radar tops vs model tops.

results presented herein, the travel time from cloud base to cloud top is on the order of 10 min. This result lends additional credence to the steady-state assumption.

## 2. Methods

The reflectivity factors within the EFV are less than $10^2$ mm$^6$ m$^{-3}$. Hailstones with a 5-mm radius and a concentration of 0.01 m$^{-3}$ or a 10-mm radius and a concentration of 0.0001 m$^{-3}$ will produce these reflectivity factors. With these possible sizes and concentrations, the assumption of insignificant dynamic effect on the updraft vault by the hailstones present appears to be justified.

The model as presented by Squires and Turner assumes that the virtual temperature at cloud base is the same in the updrafts as in the environment. Our observations from aircraft of temperature and dew point generally indicate that the sub-cloud air is not particularly well mixed, i.e., there is not a uniform vertical and horizontal distribution of potential temperature and specific humidity. Additional analysis is being done on this point, but for the moment, it appears more reasonable to assume that the temperatures at cloud base are the same as the temperatures in the environment at cloud base level. Hence, there is a positive virtual temperature anomaly (i.e., buoyancy exists) at the cloud base of cumulonimbi. This observation is reasonable when one combines it with radiosonde results. Radiosondes often indicate a dry adiabatic lapse rate below cloud base and a strong lapse rate of mixing ratio over the High Plains. One would thus expect to observe buoyant updrafts as cloud base.

The model assumes that the jet is fully turbulent and, hence, that entrained air is instantly and uniformly mixed across the updraft vault. This is not reasonable since it further implies that the horizontal velocity of the turbulent eddies is greater than the vertical velocities. Sensitivity tests were conducted with the model assuming several entrainment constants $\alpha$ with less than 1 km effect upon the model tops for hailstorms which penetrate a strong stabilizing tropopause. Hence, the value $\alpha=0.1$ for the entrainment constant was adopted.

The input parameters were obtained for five hailstorms in Nebraska and six in Alberta during July 1968 and are given in Table 1. The differential equations of the model were integrated by the Runga-Kutta method for the first 300 m and then by the Adams-Moulton predictor-corrector method for the remaining height (Diesel *et al.*, 1966) on the Philco 2000 digital computer at the University of Wyoming.

TABLE 1. Squires-Turner model: Cloud base data.

| Date | Time (MST) | Mass flux (gm sec$^{-1}$) | Cloud base | | | Sounding used | Tops | | Model maximum updraft (m sec$^{-1}$) | Hail size |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Updrafts (m sec$^{-1}$) | Temperature (°C) | Height (km) | | Radar (km) | Model (km) | | |
| *Nebraska and South Dakota* | | | | | | | | | | |
| 3 July 68 | 1600 | $6.6\times10^{10}$ | 4.5 | 5.0 | 3.38 | RCA 3/12Z | 13.2 | 12.0 | 18.0 | Pea |
| 9 July 68 | 1700 | $17.8\times10^{10}$ | 4.0 | 12.0 | 2.92 | LBF 10/00Z | 14.2 | 14.5 | 27.0 | Grape |
| 10 July 68 | 1430 | $87.5\times10^{10}$ | 4.0 | 7.0 | 3.68 | LBF 11/00Z | 14.5 | 15.1 | 29.1 | Pea |
| 10 July 68 | 1750 | $19.5\times10^{10}$ | 5.0 | 9.5 | 3.52 | LBF 11/00Z | 14.8 | 14.7 | 27.0 | Grape |
| 10 July 68 | 1900 | $14.8\times10^{10}$ | 3.5 | 9.5 | 3.52 | LBF 11/00Z | 16.3 | 14.9 | 28.2 | Grape |
| *Alberta* | | | | | | | | | | |
| 17 July 68 | 1530 | $23.7\times10^{10}$ | 4.5 | 5.0 | 2.44 | QF 18/00Z | 11.4 | 11.6 | 18.6 | Grape |
| 17 July 68 | 1700 | $30.6\times10^{10}$ | 4.5 | 5.0 | 2.44 | QF 18/00Z | 11.4 | 11.6 | 19.3 | Walnut |
| 18 July 68 | 1420 | $14.8\times10^{10}$ | 4.5 | 8.0 | 2.28 | QF 19/00Z | 11.1 | 11.7 | 20.2 | Grape |
| 25 July 68 | 1930 | $56.9\times10^{10}$ | 6.0 | 9.7 | 2.44 | QF 26/00Z | 12.7 | 13.2 | 27.4 | Walnut |
| 28 July 68 | 1830 | $6.2\times10^{10}$ | 4.0 | 14.0 | 2.59 | QF 29/00Z | 14.8 | 14.3 | 26.4 | 6 cm dia |
| 28 July 68 | 2030 | $56.6\times10^{10}$ | 7.0 | 14.0 | 2.59 | QF 29/03Z | 14.5 | 15.7 | 38.2 | Golfball |

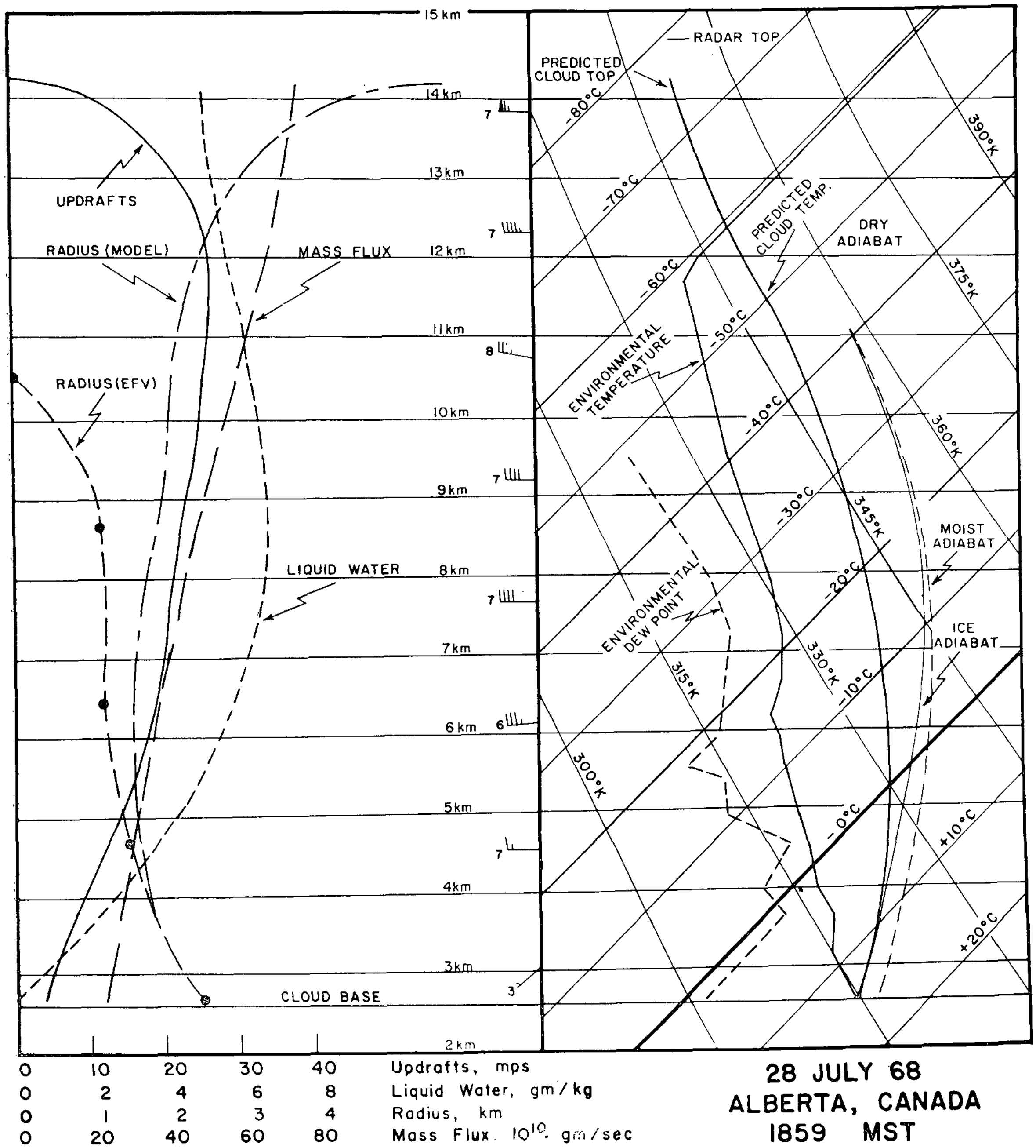

Fig. 2. Environmental sounding and model results for thunderstorm which occurred 1859 MST on 28 July 1968. Dots near radius line are measured radar radii of echo free vaults (EFV's).

The parameters predicted by the Squires-Turner model apply only to the updraft vault. The updraft vault occupies less than 25% (generally <10%) of the volume of the radar echo from a typical severe thunderstorm. Conditions existing outside of the updraft vault are not predicted by the model.

## 3. Results

The cloud tops predicted by the model were compared with the highest radar tops observed within 0 to 15 min of the cloud base observations. The model tops were assumed to be the heights at which the predicted vertical velocities decreased to zero. The radar tops were observed in Nebraska by an M-33 track radar and in Alberta by a high resolution radar (Zadwadski and Ballantyne, 1968). These radar tops were not corrected for beam width based on the observation by Jones and Marwitz (1966) that the visual tops approximately equal the radar tops for mature hailstorms on the High Plains. It may be seen that the radar tops agreed with the model tops to ±1.4 km (Fig. 1 and Table 1).

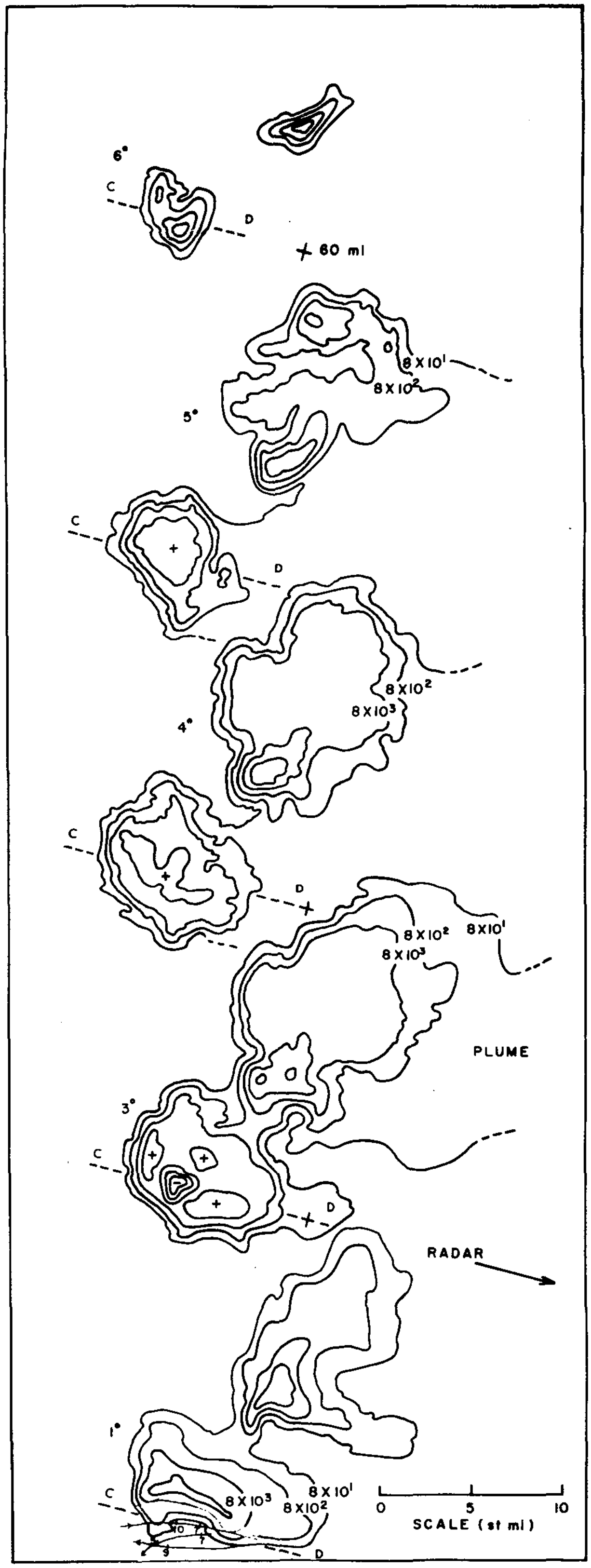

FIG. 3. PPI sections for 1859 CST 28 July 1968 for elevation angles of 1°, 3°, 4°, 5° and 6°. Contours are values of $Z_e$ (mm$^6$ m$^{-3}$). Plus signs imply a peak in $Z_e$, while minus signs imply a decrease. Line 0 is a fiducial line for relating various PPI sections. Space-corrected aircraft tracks are presented on the 1° PPI section. Aircraft determined updrafts (100×ft min$^{-1}$) are noted on the tracks. North is at top.

The model results for 1859 MST 28 July 1968 from Alberta are presented in Fig. 2 as an example of the results obtained from the model. The radar data for this storm from Marwitz *et al.* (1969) are reproduced as Fig. 3. The radii of the updraft vaults predicted by the model were compared with the radii of the EFV's observed by the high resolution radar. It was determined that the radii of the EFV's were equal to the radii predicted by the model to within ±1 km for the lower half of this thunderstorm.

Inasmuch as the top of the hailstorm penetrates an extremely stable environment (stratosphere), it is recognized that storm tops are not a particularly strong verification of the model. The confidence one places on the results of the model must rest on the fact that the initial conditions were direct observations and the degree to which the numerical simulation represents the significant processes which occur within real thunderstorms. The model predicted maximum updrafts of 20–40 m sec$^{-1}$ and cloud water maxima of 4–9 gm kg$^{-1}$. The predicted temperature anomalies between the updraft region and environment were 4–10C at 500 mb.

*Acknowledgments.* This research was made possible by National Science Foundation Grant GA-1527 and Research Council of Alberta Contract PO-8305-A to the University of Wyoming.

## REFERENCES

Auer, Jr., A. H., and W. Sand, 1966: Updraft measurements beneath the base of cumulus and cumulonimbus clouds. *J. Appl. Meteor.*, **5**, 461–466.

——, and J. D. Marwitz, 1968: Estimates of air and moisture flux into hailstorms on the High Plains. *J. Appl. Meteor.*, **7**, 196–198.

——, D. L. Veal and J. D. Marwitz, 1970: The identification of organized cloud base updrafts. *J. Rech. Atmos.*, 4, 1–6.

Browning, K. A., 1965: A family outbreak of severe local storms—A comprehensive study of the storms in Oklahoma on 26 May 1963, Part I. AFCRL-65-695(1), Spec. Rept. 32, 346 pp.

——, and R. J. Donaldson, 1963: Airflow and structure of a tornadic storm. *J. Atmos. Sci.*, **20**, 533–545.

Chisholm, A. J., 1968: Observations by 10 cm radar of Alberta hailstorm in a sheared environment. *Proc. 13th Weather Radar Conf.*, Montreal, 82–87.

Diesel, J. W., J. M. Taylor, C. O. Culver, J. H. Davis and W. P. Cramer, 1966: Modularized six-degree-of-freedom (MOD6DF) computer program. Vol. 1, prepared for Systems Engineering Group (PTD), Wright Patterson AFB, Ohio, under Contract AF33 (615)-3204, Litton Systems, Inc., Woodland Hills, Calif.

Jones, G. W., and J. D. Marwitz, 1966: Visual and associated radar tops of thunderstorms in northeastern Colorado. *Proc. 12th Conf. Radar Meteorology*, Norman, Okla., 366–370.

Marwitz, J. D., A. J. Chisholm and A. H. Auer, Jr., 1969: The kinematics of severe thunderstorms sheared in the direction of motion. *Preprints of Papers Sixth Conf. Severe Local Storms*, 8–10 April, Chicago, 6–12.

Squires, P., and J. S. Turner, 1962: An entraining jet model for cumulonimbus updrafts. *Tellus*, 14, 422–434.

Zadwadski, I. I., and E. Ballantyne, 1968: HARPI, 1967. The development and use of a Height-Azimuth-Range Position Indicator. Sci. Rept. MW-56, McGill University, Stormy Weather Group.