On Boundary Layer Dynamics and the ITCZ¹

J. R. HOLTON AND J. M. WALLACE

Dept. of Atmospheric Sciences, University of Washington, Seattle

AND J. A. YOUNG

Dept. of Meteorology, University of Wisconsin, Madison 30 July 1970 and 7 December 1970

1. Introduction

The intertropical convergence zone (ITCZ) is a narrow east-west band of vigorous cumulonimbus convection and heavy precipitation which forms along the equatorward boundary of the trade wind regimes. Within this region precipitation exceeds evaporation by a factor of 2 or more, the excess being provided by the moist, converging low-level flow.

The spatial and temporal variability of the ITCZ are well documented in the recent literature (Bjerknes et al., 1969; Hubert et al., 1969; Kornfield and Hasler, 1969). The ITCZ is particularly persistent and sharply defined over the North Pacific and Atlantic, between 5 and 10N. It occasionally appears over the southeast Pacific at 5-10S. The influence of large land masses apparently inhibits the development of a well-defined ITCZ at other longitudes. There are seasonal and year-to-year variations in position and intensity. For example, in the western Atlantic the zone shifts from just north of the equator in winter and spring to around 8-10N in summer. The western Pacific ITCZ is well defined during some years and virtually absent during others.

The ITCZ is rarely, if ever, found at the equator, except in the vicinity of large land masses. The conspicuously cloud-free "equatorial dry zone" has been viewed by Bjerknes et al. (1969) and others as a consequence of cool, upwelling ocean water associated with the equatorial current system. This hypothesis has been examined by Pike (1970) who obtained a low-latitude steady convergence zone by numerical integration of a coupled atmosphere-ocean model.

While the coincidence between equatorial upwelling and clear skies is quite remarkable over the eastern oceans, the equatorial dry zone occurs in the absence of a well-defined minimum of sea surface temperature over the western oceans. A mechanism to explain the development of an ITCZ in the absence of an equatorial minimum in the sea surface temperature has been proposed by Charney (1966). In this model the ITCZ is viewed as a zone of cumulus convection controlled by the frictionally induced meridional convergence of moisture in the trades. This model is thus a line symmetric analogue of the CISK models for tropical cyclone

development. The growth rate in Charney's stability analysis increases with the Coriolis torque due to the meridional circulation set up by the Ekman layer convergence, and with the lapse rate of equivalent potential temperature; the former increases with latitude, while Charney assumed the latter to be a maximum at the equator. The effect of these two influences acting jointly is to produce a maximum growth rate, and hence ITCZ development, several degrees away from the equator.

When viewed at a particular instant in time, rather than in the usual climatological sense, the ITCZ exhibits a somewhat more complicated structure. Inspection of individual satellite photographs shows that it can only rarely be identified as a long, unbroken band of heavy cloudiness. Rather, it is usually made of a number of "cloud clusters," separated by large expanses of relatively clear skies. Chang (1970) and Reed (1970) have presented strong evidence to the effect that cloud clusters are, in turn, manifestations of synopticscale westward propagating wave disturbances. They are marked by heavy precipitation (>2 cm day-1), which requires strong low-level convergence ($\sim 10^{-5}$ \sec^{-1}), strong ascent at middle levels (~ 2 cm \sec^{-1}), and strong upper level divergence. Fujita et al. (1969) and Williams (1970) have reported that divergent motions of this size are typical in the vicinity of cloud clusters. They have also shown that the clusters represent concentrations of cyclonic vorticity at low levels and anticyclonic vorticity at 200 mb. At low levels the convergence is mainly in the meridional wind component. The clear areas between clusters are marked by weak subsidence together with vertical distributions of vorticity and divergence opposite in sign to those with the clusters.

Thus, in a climatological sense, the ITCZ may be considered to be the locus of the cloud clusters associated with a train of westward propagating wave disturbances. The low-level convergence and cyclonic shear within the zone in the long-term mean is an average of periods of strong convergence and cyclonic vorticity, coincident with the passage of cloud clusters, together with periods of weak divergence and anticyclonic vorticity which mark the passage of clear areas. The concentration of precipitation, divergence and vorticity which we call the ITCZ is not simply a steady atmospheric state, but should be viewed as a conse-

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quence of the dynamical properties of large-scale equatorial wave disturbances.

The purpose of this note is to propose a mechanism which can account for the strong latitudinal preference of traveling equatorial wave disturbances, and to qualitatively begin to answer the following questions:

1) Why is the ITCZ often well defined, even in the absence of a distinct sea-surface temperature maximum?

2) How might we account for seasonal and yearly variations in the intensity and latitude of this band? Finally, we wish to comment upon the problem of relating the low-level vertical motion to the vorticity of the horizontal flow field at low latitudes.

2. Theoretical background

Yamasaki (1969) and Hayashi (1970) have recently investigated the possibility that the CISK mechanism is the energy source for planetary waves in the tropics. Those studies each have certain weaknesses. Yamasaki treated only two-dimensional quasi-geostrophic flows and Hayashi did not properly take into account the role of boundary layer friction. However, their analyses do provide at least some theoretical evidence that latent heat release is the driving mechanism for the observed equatorial wave disturbances. Further evidence is provided by the diagnostic model of Holton (1970) in which it is shown that a specified heating distribution with cloud cluster scale can force an atmospheric response very similar to that of the observed waves.

An essential feature of the CISK mechanism is the existence of low-level convergence to provide the necessary moisture to maintain the large-scale convection pattern. For mid-latitude quasi-geostrophic waves, Charney and Eliassen (1949) have shown that the convergence in the Ekman layer is proportional to the vorticity at the top of the boundary layer. However, such a simple relationship does not hold near the equator. In fact, it is shown below that for a propagating wave on the equatorial β plane, the Ekman layer solution is singular along a *critical latitude* at which the angular frequency of the wave is equal to the Coriolis frequency. As a consequence of this singularity the convergence in the boundary layer tends to be concentrated at the critical latitude.

To illustrate this singular behavior it is sufficient to consider the linearized equations for a forced barotropic disturbance in a resting Boussinesq fluid on the equatorial β plane. If we nondimensionalize the equations using a length scale $L \equiv (c/\beta)^{\frac{1}{2}}$, a time scale $\tau \equiv (c\beta)^{-\frac{1}{2}}$, and a depth scale H, where c is the horizontal velocity scale and $\beta \equiv 2\Omega/a$ (with Ω the angular velocity of the earth and a the radius of the earth), we obtain

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + yv + D\frac{\partial^2 u}{\partial z^2},\tag{1}$$

$$\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} - yu + D\frac{\partial^2 v}{\partial z^2},\tag{2}$$

$$\frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w) = -\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}.$$
 (3)

Here $D=KH^{-2}(c\beta)^{-\frac{1}{2}}$ is the nondimensional vertical eddy viscosity, assumed constant, ρ_0 is a standard density, and u, v, w and p are the nondimensional perturbation meridional velocity, zonal velocity, vertical velocity and pressure, respectively.

Since for motions of interest here $D \ll 1$, the system (1)-(3) may be analyzed by conventional boundary layer techniques. We assume that solutions exist of the form

$$\begin{cases}
 u \\
 v \\
 w \\
 p
\end{cases} = \operatorname{Re} \left\{ \begin{cases}
 u^{I}(y) + u^{B}(y,\xi) \\
 v^{I}(y) + v^{B}(y,\xi) \\
 D^{\frac{1}{2}} \llbracket w^{I}(y,z) + w^{B}(y,\xi) \rrbracket \\
 p(y)
\end{cases} \right\} e^{i(\omega t + kx)}, \quad (4)$$

where $\xi=zD^{-\frac{1}{2}}$ is a stretched coordinate. (Here the superscript I indicates an interior variable, and the superscript B a boundary layer contribution which must decay exponentially away from the boundary.) Substituting (4) into (1)-(3) and taking the limit $D \to 0$ while holding z constant, we obtain the interior equations

$$i\omega u^I = -ikp + yv^I, \tag{5}$$

$$i\omega v^{I} = -\frac{\partial p}{\partial y} - yu^{I},\tag{6}$$

$$+iku^{I} + \frac{\partial v^{I}}{\partial y} = 0. (7)$$

We next substitute (4) into (1)–(3) and subtract out the interior variables by applying (5)–(7). The result is the boundary layer system

$$i\omega u^B = yv^B + \frac{\partial^2 u^B}{\partial \xi^2},\tag{8}$$

$$i\omega v^B = -yu^B + \frac{\partial^2 v^B}{\partial \xi^2},\tag{9}$$

$$\frac{\partial w^B}{\partial x} = -\left(\frac{\partial u^B}{\partial x} + \frac{\partial v^B}{\partial y}\right). \tag{10}$$

To solve (8)–(10) we must apply appropriate boundary conditions. We require that u, v, w=0 at $\xi=0$ and $u \to u^I$, $v \to v^I$, $w \to w^I$ as $\xi \to \infty$. Thus,

$$u^{B} = -u^{I}, \quad v^{B} = -v^{I}, \quad w^{B} = -w^{I}, \quad \text{at} \quad \xi = 0 \\ u^{B} \to 0, \quad v^{B} \to 0, \quad w^{B} \to 0, \quad \text{at} \quad \xi \to \infty$$
 \}. (11)

The solution of (8)-(9) subject to the boundary condi-

tions (11) is

$$v^{B}(\xi) = A(y)e^{-r_{1}\xi} + B(y)e^{-r_{2}\xi},$$
 (12)

$$u^{B}(\xi) = iA(y)e^{-r_{1}\xi} - iB(y)e^{-r_{2}\xi},$$
 (13)

where (assuming $\omega > 0$)

$$r_{1} = \begin{cases} (1+i) [(\omega+y)/2]^{\frac{1}{2}}, & y > -\omega \\ (1-i) [(\omega+y)/2]^{\frac{1}{2}}, & y < -\omega \end{cases},$$

$$r_{2} = \begin{cases} (1+i) [(\omega+y)/2]^{\frac{1}{2}}, & y < \omega \\ (1-i) [(\omega+y)/2]^{\frac{1}{2}}, & y > \omega \end{cases},$$

$$A(y) = -\frac{1}{2} (v^{I} - iu^{I}) \\ B(y) = -\frac{1}{2} (v^{I} + iu^{I}) \end{cases}.$$

and

In terms of p, we have

$$A(y) = -\frac{i}{2} \left(kp + \frac{\partial p}{\partial y} \right) / (y + \omega)$$

$$B(y) = -\frac{i}{2} \left(kp - \frac{\partial p}{\partial y} \right) / (y - \omega)$$

In an equatorial wave forced by latent heating the pressure perturbation p appearing in the coefficients A(y) and B(y) would itself be determined in part by the convergence of mass and moisture in the boundary layer. However, to illustrate the basic nature of the boundary layer dynamics, it is sufficient to simply specify a reasonable latitudinal distribution of pressure. For convenience, we assume here that the pressure field corresponds to one of the free equatorial β plane modes discussed by Rosenthal (1965) and Matsuno (1966). According to Matsuno, the pressure perturbations for these free modes can be expressed as

$$p = +\frac{1}{2}(\omega_n - k)\psi_{n+1} - n(\omega_n + k)\psi_{n-1}, \tag{14}$$

where $\psi_n = e^{-\frac{i}{2}y^2} H_n(y)$, with $H_n(y)$ the Hermite polynomial of order n.

In (14), ω_n must satisfy

$$\omega_n^2 - k^2 + \frac{k}{\omega_n} = 2n + 1, \quad n = 0, 1, 2 \cdot \cdot \cdot.$$
 (15)

The vertical structure of the solutions (12) and (13) exhibits interesting behavior, particularly near the "resonance points" where $y = \pm \omega$. Unlike ordinary resonances, the solutions do not possess infinite amplitudes at these points. Instead, it may be verified that the coefficients A(y) and B(y) are everywhere finite and smoothly varying, in accord with the character of the flow above the boundary layer.²

However, since the depth of the boundary layer is seen to be proportional to $|y-\omega|^{-\frac{1}{2}}$ as $y \to \omega$, the vertically integrated mass transport in the boundary layer possesses a singularity of the order $|y-\omega|^{\frac{1}{2}}$ at $y=\omega$. (Similar arguments hold for $y=-\omega$.) This property of boundary layers in contained, rotating viscous fluids has been previously noted (Greenspan, 1968). Its direct importance in the earth's atmosphere seems uncertain, since other factors may restrict the boundary layer thickness.

Another important feature of the boundary layer solution is the phase shift which occurs at the critical latitudes where $y=\pm\omega$. This phase shift is capable of producing large meridional variations in the velocity components, and hence large values of horizontal convergence and vorticity.

The origin of this phase shift can be understood by consideration of (12) and (13). The first mode of each solution varies as $e^{-r_1(y)\xi}$, and corresponds to the upward diffusion of momentum away from the surface at all latitudes. It may be thought of as a vertically damped viscous wave of the sort found above an oscillating plane in Stokes' problem of the second kind (Schlichting, 1960) which has been modified by Coriolis forces.

The second mode behaves in a similar way at low latitudes where Coriolis forces are small. However, at higher latitudes $|y| \gtrsim |\omega|$ the Coriolis forces produce a downward phase propagation which accounts for a 90° phase shift for this mode across the critical latitude. In the low-frequency limit $|\omega| \ll |y|$ the downward propagation matches the upward propagation in such a way that the classical Ekman structure of a vertically damped standing wave results.

In Figs. 1-3, some characteristics of the solutions (12)-(13) are illustrated for pressure perturbations corresponding to n=0 and n=1 in (14)-(15). According to Holton (1970), the n=1 mode most closely resembles the maximum response mode for a moving ITCZ heat

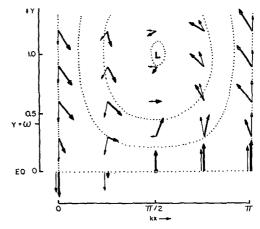


Fig. 1. Flow patterns for n=0 mode. Thick arrows represent boundary-layer flow at $\xi=0.6$ exaggerated three times relative to flow above boundary layer (thin arrows). Dotted lines represent disturbance geopotential field.

² Charney (private communication) has recently obtained a solution with similar properties by assuming that the flow above the boundary layer is in geostrophic balance, in contrast to the boundary conditions used in Eq. (10).

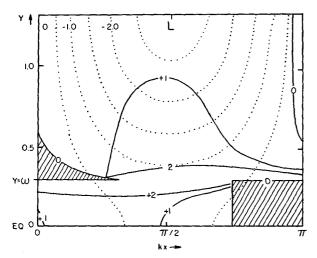


Fig. 2. Vertical motion at top of boundary layer for n=1 mode. Shading denotes area of downward motion, with intensity (in nondimensional units of 10) indicated by solid lines. Dotted lines are disturbance geopotential.

source. On the other hand, the n=0 mode possesses strong cross-equatorial flow and is thus further from geostrophic balance.³

Fig. 1 illustrates the flow pattern which occurs at low latitudes for the n=0 mode. We see that deviations from the classical Ekman boundary flow are responsible for a circulation center lying between those associated with the actual and geostrophic flows above the boundary layer. Similar deviations for the n=1 mode (not shown here) were noted only near the equator.

Integrating the continuity equation (10) from $\xi=0$ to $\xi\to\infty$ and applying (11), we obtain the vertical velocity at the top of the boundary layer, i.e.,

$$w^I(y,0) = -\bigg[ik\int_0^\infty u^B d\xi + \int_0^\infty \frac{\partial v^B}{\partial y} d\xi\bigg].$$

Fig. 2 shows the pattern of upward motion produced by mass convergence in the boundary layer for the n=1 mode. This motion possesses a maximum along the latitude where the wave disturbance frequency equals the Coriolis parameter. In addition, this latitude also separates the "mid-latitude" regime to the north (where the upward motion occurs along the trough) from the "equatorial" regime (where the upward motion is not centered in the trough). Similar characteristics were found for the n=0 case, not shown here.

Fig. 3 shows that the proportionality between vertical motion and vorticity at any level is quite good well away from the equator. However, near and equatorward of the critical latitude these relationships are quite complicated, both with regard to amplitude and phase. For example, the infinite vertical motion found at the critical latitude is far equatorward of the local vorticity maxima. At the equator, finite vertical motion occurs where the geostrophic vorticity ζ_g is infinite and the actual flow vorticity is zero. Investigation of the n=0 case shows that the relationships at low latitudes are similarly complicated, e.g., at the equator the vertical motion is zero, although both ζ_g and ζ^I are not.

In conclusion, it should be emphasized that this idealized model treats the disturbance as a pure mode of single frequency ω . In reality, atmospheric waves are transient disturbances whose energy is distributed over a continuous frequency spectrum. The boundary layer convergence should thus be distributed over a finite

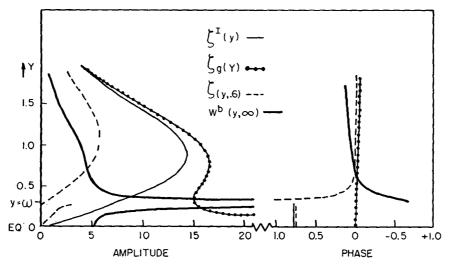


Fig. 3. Vorticity and boundary-layer vertical motion relationships for n=1 mode. Vertical motion amplitude is expressed in nondimensional units of 2.5. Phase is in radians.

 $^{^3}$ It should be emphasized that these pure equatorial β -plane mode solutions are used here merely for simplicity of illustration. The singular behavior at $y{=}\pm\omega$ will occur for any lateral distribution of the pressure wave. In particular, it is not required that the motion be symmetric or antisymmetric about the equator as is implied by the pure modes discussed here.

band of latitudes rather than restricted to a single latitude. Presumably, then, the maximum convergence would occur at the latitude where the Coriolis frequency corresponded to the frequency of maximum power in the wave spectrum.

3. Discussion

From the foregoing it is apparent that the boundary layer convergence associated with a spectrum of equatorial waves whose Doppler-shifted frequencies (i.e., the frequency relative to the mean zonal wind) lie between ω_1 and ω_2 will mainly be confined to the band of latitude in which the value of the Coriolis parameter lies between ω_1 and ω_2 . If we associate boundary layer convergence with convection, it follows that the latitudinal distribution of rainfall produced by the passage of equatorial waves is to some extent determined by the frequency spectrum of the waves.

For equatorial waves which are forced by latent heat release, the above argument can be applied in reverse. If conditions favorable for convection (e.g., warm ocean temperatures, availability of moisture) are confined to a band of latitude in which the Coriolis parameter lies between f_1 and f_2 , those waves whose Doppler-shifted frequencies lie between f_1 and f_2 will tend to be favored. In other words, the frequency distribution of equatorial waves driven by latent heat release is to some extent determined by the latitudinal distribution of ocean temperature and moisture in the zonal mean.

Thus, we conclude that there is a relationship between the frequency distribution of equatorial wave disturbances and the latitudinal distribution of precipitation in the waves. At this point we can make no statement as to which is cause and which is effect. In order to understand how this type of interaction is likely to result in a convergence zone in the long-term mean, we must consider refinements to the linear theory discussed in the previous section.

The critical latitude represents a region of large velocity gradients and hence nonlinear effects. It is quite likely that the Reynolds stresses associated with the moving wave could force a mean circulation pattern near the critical latitude, in much the same manner that steady currents are produced by oscillating tidal flows in variable estuaries (Abbott, 1970).

However, in the tropical atmosphere, the dominant nonlinearity would seem to be associated with the release of latent heat, where temperature changes accompanying rising motion are quite different from those associated with sinking motion. As a result, it seems likely that the low-level convergence associated with the waves will be more strongly concentrated near the "critical latitude" than the corresponding divergence. This means that in the long-term mean there would be convergence at the latitude of maximum wave activity and divergence at other latitudes. Because of the earth's rotation, this steady-state circulation may tend to pro-

duce a concentration of vorticity in the convergence

4. Observational evidence

Observational verification of the above mechanism would require, as a starting point, a large sample of simultaneous measurements of Doppler-shifted wave frequency and latitude of the ITCZ, preferably from a number of geographical regions, and during several seasons and years. The spectral data currently available are not adequate for this purpose. For the present, we can only indicate that the range of "critical latitudes" inferred from the frequency spectrum of tropical wave disturbances is not inconsistent with the observed position of the ITCZ.

As evidence of this we note that spectral peaks in the range 0.20–0.25 cycle per day (cpd) have been observed at a large number of tropical stations and during a number of different time periods (Rosenthal, 1960; Yanai et al., 1968; Wallace and Chang, 1969; Chang et al., 1970). In at least some of these cases the mean zonal wind speed in the boundary layer was sufficiently small compared to the wave speed so that the Doppler-shifted frequency was approximately the same as the ground-based frequency. The range of "critical latitudes" corresponding to this frequency band is 6°–7°, which is close to the most frequent ITCZ position.

The observed lack of stationarity in time series of tropical data may be linked to variations in the position and intensity of the ITCZ. There have been extended periods when the frequency spectrum of tropical disturbances in the western Pacific has failed to show any distinct peak in the 0.20-0.25 cpd frequency range. At such times most of the variance is contributed by waves with much lower frequencies, and the Doppler-shifted spectrum resembles red noise. If the above arguments are correct, we should not expect to see a sharply defined ITCZ in such situations. It is then perhaps significant that the satellite maps of mean cloud amounts presented by Bjerknes et al. do not show a well-defined ITCZ in the western Pacific during the period January 1963-July 1964, a time marked by predominantly lowfrequency waves in this region (Wallace and Chang, 1969, Fig. 9). The same maps show a well-defined ITCZ during 1967, a period marked by a strong spectral peak in the 0.20-0.25 cpd range.4

5. Concluding remarks

It should be emphasized that the proposed relationships between ITCZ position and the frequency spectrum of equatorial wave disturbances represents but one possible link in the complex chain of interactive mechanisms which control tropical climate. A complete explanation of the ITCZ and its spatial and temporal vari-

⁴ A detailed study of this period is now in progress at the University of Washington.

ations must also take into account proper modeling of the CISK process, air-sea interaction, and topographical influences. In a similar manner, a complete explanation of the frequency spectrum of tropical wave disturbances must also take into account many factors not considered here. The primary value of the simple interactive scheme proposed above is to provide a conceptual framework which may be helpful in interpreting observational data and numerical models of the tropical general circulation.

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