The Relationship Between Visibility and Liquid Water Content in Fog

RALPH G. ELDRIDGE

6 Preston Rd., Lexington, Mass.

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ABSTRACT

An analytical technique employing a fog drop-size distribution model is used to demonstrate the importance of the width of measured drop-size distributions when evolving a visibility-liquid water content relationship. Consistency is achieved between two apparently different visibility-liquid water content curves generated from two sets of fog drop-size distributions with this analysis.

1. Introduction

Although the relationship between liquid water content and visibility in fog was not the central theme of Platt's (1970) paper, he comments that there is an inconsistency when the results of Eldridge (1966) and Houghton and Radford (1938) are compared. Therefore, it appears propitious to comment on the difference and discuss the major cause of the apparent inconsistency. The purpose of this paper is to discuss one essential difference in the drop-size distributions of Houghton and Radford, and Eldridge, i.e., the width of the fog drop-size distributions. The difference in the size range of the droplets in the distributions can be used to reduce the discrepancy between the two visibility-liquid water content curves.

2. Background

Although measured drop-size distributions show similar major features, every distribution has variations in size and number which mitigate against the formulation of an exact relationship between liquid water content and visibility. At best, this relationship will be approximate.

Papers by aufm Kampe and Weikmann (1952), Atlas and Bartnoff (1953), Fritz (1954), and Eldridge (1957, 1961) have inferred the necessity of considering the spread of the fog drop-size distributions used when forming a relationship between liquid water content and visibility. The larger droplets, those droplets much greater than several microns radius, make the major contribution to the liquid water content, whereas the smaller droplets are the most effective in reducing visibility. These researchers generally agreed that more *in situ* simultaneous measurements of visibility, liquid water content, and appropriate drop-size distributions are necessary to establish a valid relationship between these parameters. Lacking these measurements, criteria for this relationship can only be defined by using a model drop-size distribution to assess the contribution of various size droplets to the liquid water content and to the attenuation of visible light.

3. Drop-size distribution data

The inconsistency between the visibility-liquid water content relationships as derived from the data of Houghton and Radford (1938) and Eldridge (1966) is a consequence of significantly different limits of the drop-size distributions. Houghton and Radford collected fog droplets on coated slides which resulted in distributions ranging from about 3 to about 70 μ radius. The collection efficiency of this technique is very low for droplets of a few microns radius. The distribution will result in reasonable liquid water contents (and are reasonably good for computing far infrared transmittances), but will severely underestimate the transmission of visible radiation.

The fog drop-size distributions published by Eldridge (1966) were inferred from measured spectral transmissions through fog with the aid of Mie scattering theory. The droplets range in size from 0.3–8 μ radius. These droplets are the most effective for reducing the visibility, but will only account for a portion of the liquid water content.

The stable fog model illustrated in Fig. 1 is a composite of the stable fog drop-size distributions of Eldridge (1966) for aerosol droplets $<10\,\mu$ radius, and that of Houghton and Radford for droplets $>10\,\mu$. This model has the same general shape as the distributions presented by Neiburger and Chien (1960). The other curve, labeled haze, is a continental or Junge haze model (1955) for an environmental relative humidity of 100%, and results in a visibility of 1 km. The haze model is included for reference.

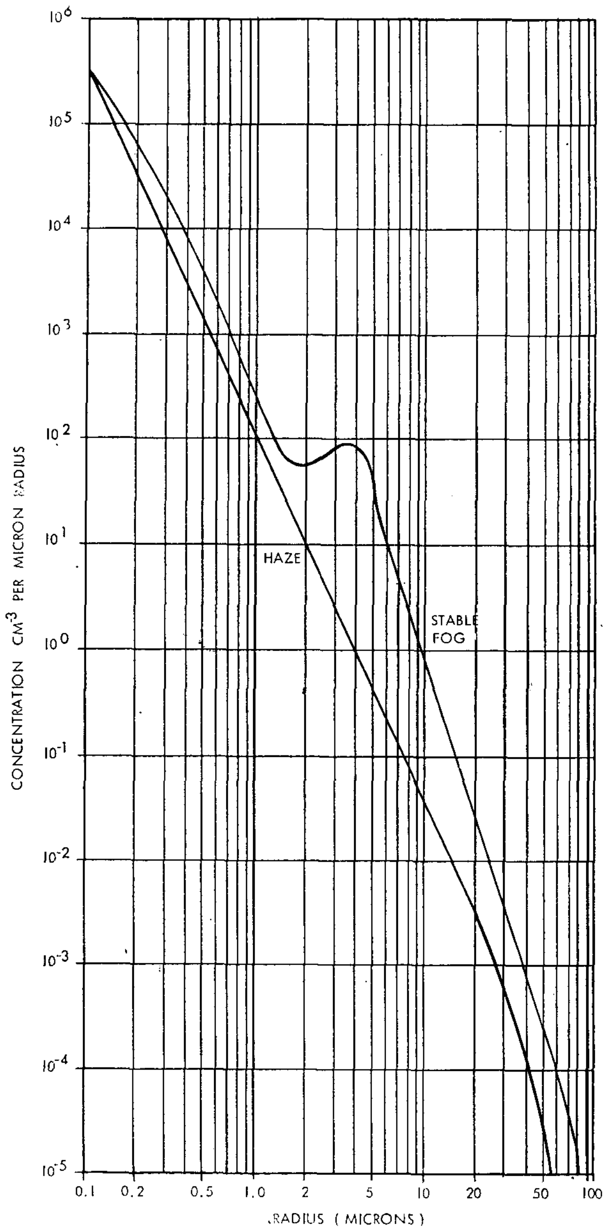

FIG. 1. Model aerosol droplet-size distributions for a stable fog and a continential type haze.

4. Determination of visibility and liquid water content

The stable fog drop-size distribution shown in Fig. 1 is used to compute the attenuation coefficient σ_λ, where

$$\sigma_\lambda=\pi N\Sigma_i(N_i/N)K_{\lambda i}r_i^2. \quad (1)$$

The quantity N is the total number of aerosols in the distribution, while N_i is the number of aerosols of size r_i, which is the mean radius of the aerosol in class i. The scattering cross section $K_{\lambda i}$ is a consequence of Mie scattering principles using the appropriate complex index of refraction for each wavelength.

Visibility is related to the attenuation coefficient via Koschmieder's (1924) formula when the wavelength is $0.55\,\mu$, the wavelength of maximum visual response of the normal human eye.

The liquid water content is determined by summing the volume of all the droplets in the fog distribution.

5. Results

The cumulative attenuation coefficient and liquid water content as a function of particle radius for the stable fog aerosol drop-size distribution model is presented in Fig. 2. If it is assumed that the distributions

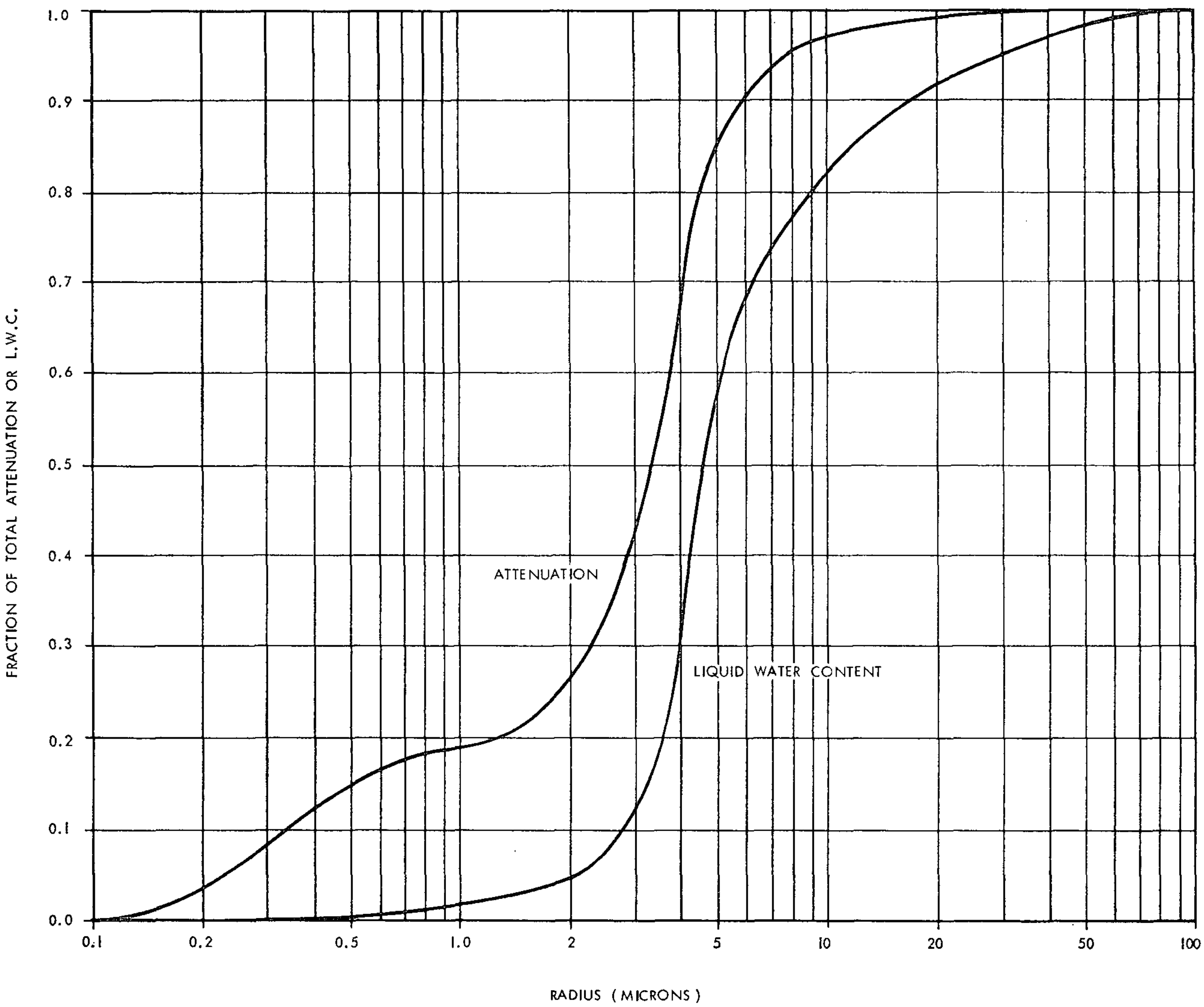

FIG. 2. Cummulative attenuation and liquid water content as a function of aerosol droplet radius for the stable fog model.

of Houghton and Radford (1938) and Eldridge (1966) may be represented over the size range from 0.1 to 100 μ radius by the model distribution illustrated in Fig. 1, then Fig. 2 can be used to relate these distributions to the proper visibility and to account for the liquid water content. The fraction of the attenuation omitted by Eldridge's distribution is 0.09 for aerosols $<0.3\,\mu$ radius, and 0.05 for droplets $>8\,\mu$. This distribution accounts for 86% of the attenuation, or overestimates the visibility by 14%. In the same manner, this distribution underestimates the liquid water content by 33%.

The Houghton and Radford droplet distribution underestimates the visibility by 42% because it omits all the aerosol droplets $<3\,\mu$ radius. In contrast, the Houghton and Radford distribution accounts for at least 88% of the liquid water content.

In summary, Eldridge's distribution underestimates the visibility and the liquid water content by 14 and 33%, respectively. Houghton and Radford's distribution underestimates the visibility by 42% and the liquid water content by 12%.

The two visibility-liquid water content curves of Platt's (Fig. 2, 1970) are presented in Fig. 3 as labeled dashed curves. When Eldridge's curve is moved down 14% (decreased visibility) and to the right by 33% (increased water content), the solid curve just above the labeled curve results. In the same manner, after moving Houghton and Radford's curve down 42% and to the right 12%, we obtain the solid curve below their labeled curve.

This analysis indicates the importance of giving proper consideration to the drop-size interval of selected data when making a visibility-liquid water

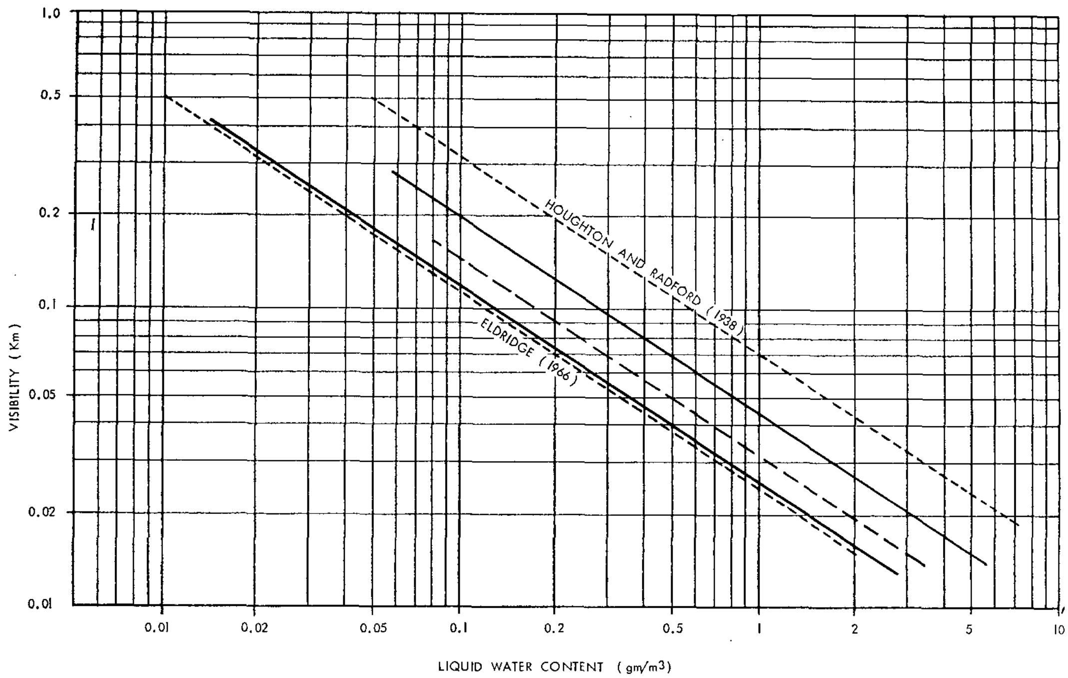

FIG. 3. Visibility as a function of liquid water content in a stable fog. The short dashed curves are the original published data, while the two solid curves represent modifications of the original data. The long dashed curve is the final translocation of the Houghton and Radford (1938) curve in accordance with the procedure presented in the text.

relationship. The discrepancy has been reduced to almost half of that pointed out by Platt. It might be of interest to speculate on the source of the remaining discrepancy.

Houghton and Radford indicate that some of the droplets $<10\,\mu$ radius are discriminated against by their fog microscope. If the lower droplet size is $4\,\mu$ radius, instead of the $3\,\mu$ values used above, the middle dashed curve of Fig. 3 would result. The discrepancy between this curve and the translocated curve of Eldridge is reasonably small.

The apparent inconsistency between the visibility-liquid water content relationship as presented by Houghton and Radford and by Eldridge is primarily a consequence of the different drop-size distribution intervals. The analysis is, however, limited to stable fogs and contains some inherent uncertainty because it is based on a model which has a visibility of 0.11 km and a liquid water content of 0.14 gm m^{-3}.

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