NOTES AND CORRESPONDENCE

Periodic Updating of Meteorological Variables

DAVID L. WILLIAMSON AND ROBERT E. DICKINSON

National Center for Atmospheric Research,[1] *Boulder, Colo. 80302*

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1. Introduction

Recent experiments with general circulation models (Charney *et al.*, 1969; Williamson and Kasahara, 1971) have shown that the large-scale wind field can be obtained by periodically introducing the correct temperature into a numerical model during the course of the time integration. The integrations of Williamson and Kasahara have also shown that the temperature field can be obtained from the wind field using the same procedure. The introduction of either a new temperature or a new wind field into the model without change of the other fields is referred to as updating. The degree to which the winds are determined by updating the temperatures or vice versa is measured by the difference of that field between two integrations. The first integration or control case provides the "correct" fields which are used to update the second case. This second integration starts with different initial conditions from the control case. At periodic intervals, the correct fields, either temperature or wind, are substituted into the model. The difference between the two cases will be referred to as the error.

The following conclusions concerning the process of updating were derived empirically from the study of Williamson and Kasahara:

1) For long-enough intervals between times of updating and large-enough error field, the decrease of the error field depends on the number of times the updating is carried out, and is essentially independent of the length of the time between updates, e.g., updating with a 6-hr interval takes half as long to reduce the error as 12-hr updating.

2) For shorter intervals between times of updating, the number of updates required for a given reduction of the error increases as the interval between updates decreases, e.g., 2- and 4-hr updating did not reduce the wind error six and three times as fast, respectively, as the 12-hr updating.

3) After a large number of updates, the error approaches an asymptotic value which, to some extent, increases with increasing update interval.

4) The rate of error reduction and the magnitude of the asymptotic error depend on the numerical model being used.

Our objective here is to derive the first of these conclusions and interpret the others using a simple linear perturbation theory of the updating process. The analysis is based on the following observations of the updating process in the general circulation models. The control or "correct" solution is nearly in geostrophic balance. After updating one field from the control case, the solution consists partially of gravity wave modes and partially of geostrophic (or Rossby) modes. The gravity wave amplitudes decrease with time in the numerical integrations. After a long-enough time interval, only the Rossby mode components of the correction remain. The errors in both wind and temperature fields of the Rossby mode components have been reduced, but not eliminated. Additional updating reduces the error further.

2. Analysis

The basis for our analysis is geostrophic adjustment theory as developed by Rossby (1938), Cahn (1945), Obukhov (1949), Bolin (1953), and others. We consider a two-dimensional, linear, incompressible fluid with a

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free surface on a constant f plane. The height and velocity fields are assumed to depend on one spatial variable only. The equations governing perturbations about a state of rest are

$$\frac{\partial \mathbf{z}}{\partial t}+L\mathbf{z}=0, \tag{1}$$

where

$$\left.\begin{aligned} \mathbf{z}&=\begin{bmatrix} u \\ v \\ h \end{bmatrix} \\ L&=\begin{bmatrix} 0 & -f & g\dfrac{\partial}{\partial x} \\ f & 0 & 0 \\ D\dfrac{\partial}{\partial x} & 0 & 0 \end{bmatrix} \end{aligned}\right\}, \tag{2}$$

u is the velocity in the x direction, v the velocity perpendicular to and to the left of u, h the deviation of the free surface from the mean depth D of the fluid, f the Coriolis parameter assumed constant, and g gravity. We can use these equations to discuss the adjustment of a stratified atmosphere provided we regard D as the "equivalent depth" separation parameter (Lindzen, 1967).

The dependent variables can generally be represented by the sum of wavenumber components of the form

$$\mathbf{z}(t)=\sum_j \mathbf{z}_k(t)\exp(ik_jx), \tag{3}$$

where the allowed wavenumbers k_j depend on the boundary conditions assumed. At $t=0$, the initial data are expanded as such a sum of wavenumber components

$$\mathbf{z}(t=0)=\sum_j \mathbf{Z}_k\exp(ik_jx). \tag{4}$$

Solution to the initial value problem (1) is carried out by finding the evolution in time of the amplitudes $\mathbf{z}_k(t)$, given their initial value $\mathbf{Z}_k$. Substitution of (3) into (1) and solution of the consequent third-order ordinary differential equation in time for the Fourier coefficients gives

$$\mathbf{z}_k(t)=\{\mathbf{T}_k^{(R)}+\mathbf{T}_k^{(+G)}\exp[i\nu_k^{(+G)}t] +\mathbf{T}_k^{(-G)}\exp[i\nu_k^{(-G)}t]\}\mathbf{Z}_k. \tag{5}$$

The solution for each wavenumber consists of three modes: $\mathbf{T}_k^{(R)}$, the Rossby mode, and $\mathbf{T}_k^{(-G)}$ and $\mathbf{T}_k^{(+G)}$, the gravity modes. The frequencies of the Rossby modes are zero and those of the gravity modes are

$$\nu_k^{(\pm G)}=\pm(gDk^2+f^2)^{\frac{1}{2}}. \tag{6}$$

The matrices defining the projection of the initial data onto these modes are

$$\mathbf{T}_k^{(R)}=\frac{1}{gDk^2+f^2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & gDk^2 & igkf \\ 0 & -iDkf & f^2 \end{bmatrix}, \tag{7}$$

$$\mathbf{T}_k^{(\pm G)}=\frac{-1}{2(gDk^2+f^2)} \times\begin{bmatrix} -gDk^2-f^2 & if\nu_k^{(\pm G)} & gk\nu_k^{(\pm G)} \\ if\nu_k^{(\pm G)} & -f^2 & igkf \\ Dk\nu_k^{(\pm G)} & -iDkf & -gDk^2 \end{bmatrix}. \tag{8}$$

The Rossby mode in component form is

$$\left.\begin{aligned} u_k&=0 \\ v_k&=\frac{1}{gDk^2+f^2}(gDk^2V_k+igkfH_k) \\ h_k&=\frac{1}{gDk^2+f^2}(-iDkfV_k+f^2H_k) \end{aligned}\right\}. \tag{9}$$

If the initial conditions are already in geostrophic balance, i.e.,

$$\left.\begin{aligned} V_k&=\frac{ikg}{f}H_k \\ U_k&=0 \end{aligned}\right\}, \tag{10}$$

the Rossby mode is seen to be just $u_k=U_k$, $v_k=V_k$ and $h_k=H_k$.

We first analyze the reduction of the error in a single wavelength component by updating. The procedure generalizes to updating all wavelengths present in a numerical model. Assume now an initial state U_k,V_k, H_k which satisfies (10). Suppose at this time the true height field is determined by observations to be $(1+\epsilon)$ H_k, where ϵ, which may be complex, gives the ratio of the error to the initial amplitude of the height field. If this height field is substituted into the model, but the wind field is left unchanged, it is seen from (5) that some gravity waves will now be present in the solution. In a numerical model, these waves are presumably damped after a long-enough time interval and the computed field corresponds to the remaining Rossby modes. In the simple model discussed here, there is no mechanism to damp the waves, so we simply remove them from the data. We then ask what is the amplitude of the remaining Rossby mode. It obviously will not be the true field $(1+\epsilon)$ H_k except in very special cases. Substitution of the new height $(1+\epsilon)$ H_k and the old wind which was in geostrophic balance with

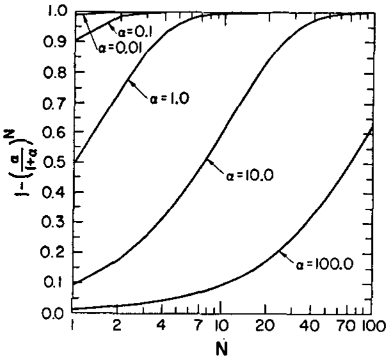

FIG. 1. Correction of Rossby mode as a function of the number of times updating is performed. For temperature updating, α is the square of the ratio of the radius of deformation to the mode length scale. For wind updating, α is the inverse of the square of the above ratio.

H_k into (9) gives the amplitude of the Rossby mode as

$$h_k^{(1)}=H_k\left[1+\epsilon\left(\frac{1}{1+\delta}\right)\right], \tag{11}$$

where

$$\left.\begin{aligned}\delta&=(kR)^2\\ R&=\frac{(gD)^{\frac{1}{2}}}{f}\end{aligned}\right\},$$

R being the radius of deformation as defined by Rossby (1938) and δ the square of the ratio of the radius of deformation to the mode length scale. The superscript 1 in (11) indicates the height field has been updated one time. The corrected height $h_k^{(1)}$ differs from the true height by a term, $\epsilon H_k\,\delta/(1+\delta)$.

This process can be repeated, say, N times, i.e., the true height field $(1+\epsilon)\,H_k$ is substituted into the model and the gravity waves are removed. The resulting amplitude of the Rossby mode is

$$h_k^{(N)}=H_k\left\{1+\epsilon\left[1-\left(\frac{\delta}{1+\delta}\right)^N\right]\right\}. \tag{12}$$

We first note that for finite $\delta\geqslant 0$, $\delta/(1+\delta)<1$, and the amplitude of the Rossby mode approaches the correct value as the number of updates N approaches infinity. Fig. 1 plots values of $1-[\alpha/(1+\alpha)]^N$ for various values of α and for integral values of N from 1 to 100. For updating height or, equivalently, temperature, we take $\alpha=\delta$. It is seen from the figure that the smaller α is, the faster this expression approaches unity as N increases. Thus, the longer the wavelength, the higher the latitude, and the smaller the equivalent depth, the closer the amplitude of the Rossby mode is to the correct amplitude of that mode for fixed N.

This updating procedure is similarly applied to winds. Suppose the true wind is $(1+\epsilon)\,V_k$, and suppose this true value is periodically substituted into the model and the gravity waves removed. After this process is repeated N times, the resulting amplitude of the Rossby mode is

$$v_k^{(N)}=V_k\left\{1+\epsilon\left[1-\left(\frac{1}{1+\delta}\right)^N\right]\right\}. \tag{13}$$

The dependence of this expression on N is inferred from Fig. 1 with $\alpha=1/\delta$. Thus, in this case the mode responds to the repeated updating better for smaller wavelengths, at lower latitudes, and for larger equivalent depths. All modes converge to their correct values as N approaches infinity provided δ does not equal zero.

As shown above, the closer the field achieved after geostrophic balance is to the correct field, the more rapid is the convergence of the updating procedure. The difference between the correct and the balanced field depends on the ratio of radius of deformation to the mode length scale as discussed in geostrophic adjustment theory (e.g., Matsumoto, 1961; Washington, 1964).

Eqs. (12) and (13) and Fig. 1 indicate that the reduction of error of the amplitude of each mode depends only on the number of times updating is performed. The updated field is expressed as a sum of the orthogonal modes according to (4). Hence, the rms error, $|h^{(\infty)}-h^{(N)}|$, where the brackets denote the square root of the integral of the square over the horizontal coordinates, is given by the square root of the sum over k of the square error of each kth mode, and thus also depends only on the number of updates.

3. Discussion

According to simple perturbation theory, the reduction of the error in a numerical model through updating should be independent of the time interval between updatings provided the interval is large enough that gravity waves decrease in amplitude in the model during the time between updates. For shorter intervals, when gravity waves introduced by previous updating have not been removed, there will be less error reduction resulting from each updating. We do not understand what processes in the NCAR general circulation model (GCM) are responsible for removing gravity waves on a 6-hr time scale in the updating experiments as summarized in the introduction. To verify the presence of some gravity wave removal mechanism in the model, we carried out the following further calculation. Data from one day of a GCM simulation experiment were decomposed into its six vertical structure normal modes and each vertical mode was further projected onto gravity wave horizontal structure modes. For this calculation, the actual GCM spherical geometry and finite-difference equations were used rather than the simple model discussed in this paper.

The total energy of each of the gravity wave modes was increased by a factor of 10, the modes recombined to form grid point data, and the simulation continued

TABLE 1. Relative energy of gravity waves.

	Equivalent depth (cm)					
Hours	1.02×10^6	5.29×10^4	1.50×10^4	4.09×10^3	1.62×10^3	3.17×10^2
0	10.0	10.0	10.0	10.0	10.0	10.0
2	3.7	8.2	8.1	7.9	7.9	7.5
4	3.2	5.9	6.0	6.0	6.5	6.1
6	3.4	4.6	4.3	4.7	5.4	5.2
12	2.7	4.5	3.7	4.2	5.1	4.5

for 12 hr. We then examined the energy in the gravity modes at 2, 4, 6 and 12 hr. The results are shown in Table 1 for each vertical mode. The internal modes are damped rapidly for the first 2–6 hr and slowly for the 6–12 hr period. The energy of the external mode is damped mostly in the first 2 hr, followed by a slight decrease for the next 10 hr.

The rate of gravity wave removal appears to depend on the modal amplitudes and somewhat on the horizontal wavenumber distribution. Thus, the inferred 6-hr time scale for removal of the gravity waves is specific to the gravity waves initially generated in the updating process and, of course, has only been determined for the NCAR GCM.

Our analysis does not include nonlinear interactions which result in error growth during the intervals between updating. This nonlinear error growth must also be important in determining the degree to which one field can be obtained by updating another. During the time between updating, the error in the Rossby mode grows. Thus, we would not expect to have the modes converge to the correct value with repeated updating. Rather, we would expect the error to approach some asymptote dependent on the error growth rate of the model and time interval between updates. Results from numerical experiments illustrating these dependencies are discussed in Williamson and Kasahara (1971).

It should be pointed out that while our analysis has assumed the availability of correct values everywhere at one time for updating, the future observational systems which are anticipated will only provide observations at limited spatial locations at a given time. The present analysis can be generalized to such a situation.

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Fluctuations in the Position of the ITCZ in the Atlantic and Pacific Oceans

ARNOLD GRUBER

National Environmental Satellite Service, NOAA, Washington, D. C. 20031

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1. Introduction

There has recently been renewed interest in the intertropical convergence zone (ITCZ), both theoretically (Bates, 1970; Holton *et al.*, 1971; Charney, 1968) and observationally (Hubert *et al.*, 1969; Bjerknes *et al.*, 1969; Godshall, 1968).

It is the purpose of this note to present some additional results, based on satellite observations, concerning the spatial and temporal variations of the ITCZ as well as other prominent cloudy zones over the tropical oceans, primarily the Atlantic and Pacific. Emphasis is being placed on these regions since they are least affected by continental influences.