Study of Evaporation and Instability of Charged Water Droplets

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ABSTRACT

The wind correction to the evaporation rate of electrically suspended water droplets has been measured for Reynolds numbers between 1 and 6. The results agree well with those of Kinzer and Gunn. During evaporation, Rayleigh instabilities have also been observed. At instability, the charge variation ΔQ is of the same order as that observed by other workers, but under the present experimental conditions there is no detectable change of mass.

1. Introduction

The study of evaporation of water drops moving relative to air is important in meteorology.

In the case of evaporation of droplets at rest, the time rate of change of mass is given by the Maxwell equation

$$\frac{dm}{dt}=I_0=4\pi rD(c_0-c_\infty), \tag{1}$$

where r is the radius of the droplet (cm); D the diffusion coefficient of water vapor in air, given by $D=0.22+0.0015\theta$ [$\text{cm}^2\ \text{sec}^{-1}$], θ being the temperature (°C); and c_0-c_∞ is the water vapor concentration gradient (gm cm^{-3}).

Indeed, if the droplets are sufficiently large ($r>50\ \mu$) the kinetic effect on evaporation can be neglected (Fukuta and Walter, 1970).

From (1), it can be seen that the droplet surface varies linearly with time, i.e.,

$$\frac{d(d^2)}{dt}=8D(c_0-c_\infty). \tag{2}$$

When droplets are ventilated, (1) is modified depending upon the flow rate around them. The flow is characterized by the Reynolds number $\text{Re}=vd/\nu$, v being the droplet velocity relative to the air, and ν the kinematic viscosity of the air.

For $\text{Re}<1$, Stokes flow exists and, in accordance with Fuchs' theory (1959), the loss of mass vs time is still given by (1).

For $\text{Re}>100$, a diffusion boundary layer exists around the drop. The radius R of this layer is related to the radius r of the drop by

$$R=A\,(r/v)^{\frac{1}{2}}. \tag{3}$$

Then (1) must be modified by a factor f (Kiveliovitch and Roulleau, 1951) such that

$$f=\frac{\lambda}{\lambda-1}, \tag{4}$$

with $\lambda=R/r$.

One can show that if $f=1+K\ \text{Re}^{\frac{1}{2}}$, the time rate of change of mass is then given by the Frössling (1938) equation

$$I-I_0f=4\pi rD(c_0-c_\infty)(1+K\ \text{Re}^{\frac{1}{2}}), \tag{5}$$

which can be written as

$$\frac{d(d^2)}{dt}=8D(c_0-c_\infty)(1+K\ \text{Re}^{\frac{1}{2}}), \tag{6}$$

where $K=\beta\text{Sc}^{\frac{1}{3}}$, $\text{Sc}=\nu/D$ being the Schmidt number, and the coefficient β has a value of about 0.3 (Fuchs, 1959).

For $1<\text{Re}<100$, there is no flow theory. In this range of Reynolds numbers, for water drop evaporation, only Kinzer and Gunn's measurements are available (1951). Their results show that the factor f rapidly increases when Re varies between 1 and 10, with the rate of increase becoming smaller as Re exceeds 10. The value of f given by the Frössling equation occurs for $\text{Re}\approx 100$. Kinzer and Gunn have also shown that the coefficient $\beta=(f-1)\text{Sc}^{-\frac{1}{3}}\text{Re}^{-\frac{1}{2}}$ increases to a maximum value of 0.46 for $\text{Re}=4$.

We set out to confirm these results under different experimental conditions. For this purpose, we chose to suspend electrically charged water drops in a laminar stream of humid air.

When charged droplets evaporate, they become unstable if their charge volume ratio becomes too great. This effect has been studied by Lord Rayleigh who has shown that instability occurs when

$$\frac{Q^2}{2\pi\gamma d^3}=1,$$

γ being the surface tension of water.

Such instability was easily observed in our experiments and the last part of this paper concerns its study.

2. Apparatus

To avoid uncertainties due to supporting drops on wires, the droplets are electrically suspended in a chamber similar to Owe Berg's apparatus (1970). This chamber, schematically represented on Fig. 1, is a simple vertical wind tunnel containing the necessary electrodes. A 50-Hz voltage applied on the central ring permits the stabilization of the charged droplet in the center of the device. This voltage can vary from 0 to 2000 V. To hold the charged droplet at the level where the ac field is zero, a continuously recorded dc voltage is applied between two electrodes located along the axis of the tunnel. This voltage can vary from +1000 to −1000 V. The upper electrode is spherical and the lower one, ground connected, is cylindrical to allow upward motion of humid air. The walls of the tunnel are grounded.

The equilibrium of forces exerted on the drop can be written as

$$QE=QkV=\frac{4}{3}\pi r^3(\rho-\rho')g-\frac{\pi}{2}r^2C\rho' v^2, \tag{7}$$

where Q is the droplet charge (esu), $E=kV$ (the dc field, also in esu) when V, the d.c. voltage is in volts, k being a coefficient due to the geometry of the field, r is the radius of the droplet (cm), ρ and ρ' are the respective densities of water and moist air (gm cm^{-3}), C the drag coefficient, and v the wind speed (cm sec^{-1}).

A variable-velocity humid air stream passes upward through the tube. The air is saturated by passing through a water bath, the temperature of which is regulated to within ±0.05C. The temperature of the cylinder and of the lower electrode is kept constant to ±0.05C by means of water circulation.

By means of a fine pipette connected to a 5000 V dc source, a charged droplet is injected into the center of the ring through a port made in the tube for this purpose. Afterward, the port is sealed by a sheet of plexiglass. The diameter of the droplets was initially of the order of 200 μ and finally of the order of 50 μ.

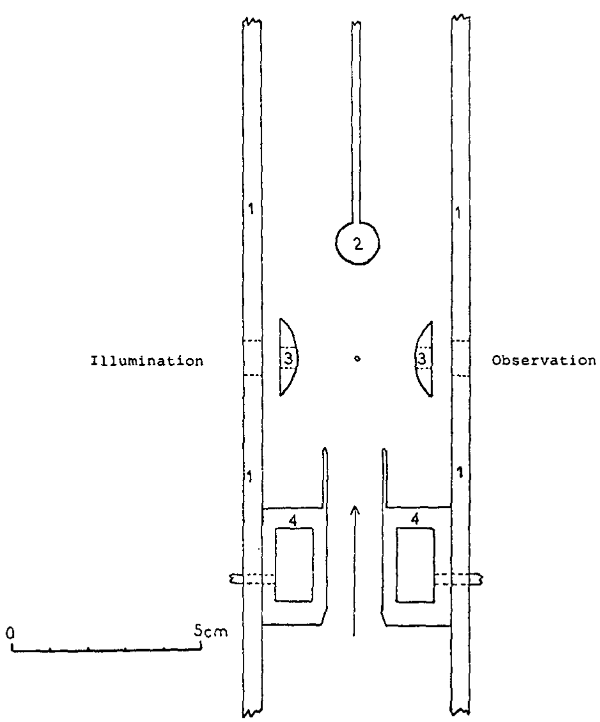

FIG. 1. Schematic diagram of apparatus showing the grounded thermo-regulated outside cylinder, 1, the upper electrode (variable dc voltage), 2., the central ring (ac voltage), 3., and the lower thermo-regulated electrode (grounded), 4.

3. Wind factor determination

a. *Experimental technique*

A single experiment consisted of determining the droplet diameter as a function of time by direct photography, and simultaneously recording the dc voltage necessary to levitate the drop. During this experiment, the wind velocity and relative humidity were held constant.

For a fixed relative humidity, a series of measurements were made with different wind speeds ranging from 5 to 50 cm sec^{-1}. Higher speeds could not be used because the droplets could not be held stationary by the electric field. At lower speeds, it was not possible to

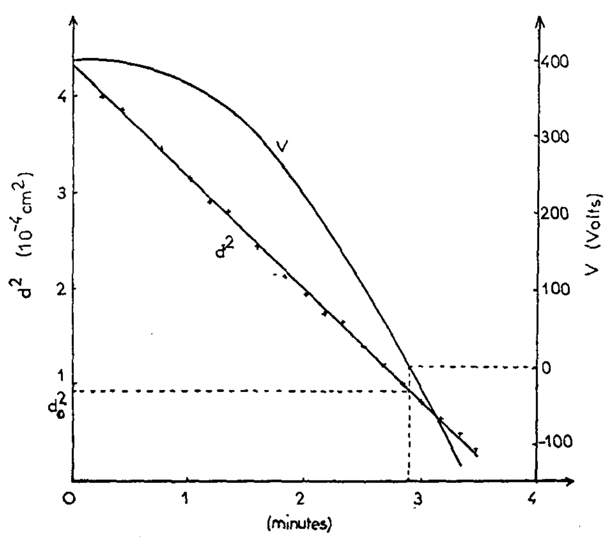

FIG. 2. Example of the variation of d^2 and V as a function of time.

control the humidity accurately. Indeed, external perturbations, like opening the port to introduce the drop, then become too important.

The actual wind speed was determined indirectly, making use of the fact that at $V=0$, $r=r_0$ (see Fig. 2) and the equation

$$\frac{4}{3}\pi r_0^3(\rho-\rho')g=\frac{\pi}{2}r_0^2C\rho' v^2, \tag{8}$$

from which v was determined. For one setting of the air flow, the variation of wind speed so determined was not more than 1 cm sec^{-1}.

In order to get adequately slow evaporation rates, it is necessary to have a rather moist atmosphere. In these experiments, the relative humidity is maintained between 80% and 98%. Since direct measurement of humidity with classical methods is not sufficiently precise, the vapor concentration gradient is determined from the evaporation rate of a drop at such a slow wind velocity that the wind factor can be neglected (Re<1). Under these conditions $d^2=f(t)$ has a slope p_0 and, from (2), we have

$$c_0-c_\infty=\frac{1}{8D}\frac{d(d^2)}{dt}=\frac{p_0}{8D}. \tag{9}$$

For a fixed humidity, p_0 was determined by the mean value of four experiments.

For higher wind speeds, changes in the slope of curves of d^2 vs time could not be determined within the accuracy of the measurements due to the Reynolds number variation. Straight lines of slope p are therefore obtained. For one experiment, the Reynolds number is based on the average value of the diameter of the drop.

It has been possible to compute the wind factor f for Reynolds numbers varying from 0 to 6. From (2) and (6), we have

$$f=\frac{p}{p_0}=1+K\,\mathrm{Re}^{\frac{1}{2}}. \tag{10}$$

b. Results

The results of a typical single experiment are illustrated in Fig. 2 which shows how d^2 and the dc voltage vary with time.

Fig. 3 gives an example of the variation of d^2 vs time for a given humidity and for different velocities of the air flow.

Fig. 4 gives the results obtained for the wind factor f vs the Reynolds number Re. The curve is drawn using average means of the experimental points. For Re>1, f rapidly increases up to 1.8 for Re=4.4, with the increase then slowing for greater values of Re.

From this curve, the coefficient β of the Frössling equation can be calculated (Fig. 5). The results show

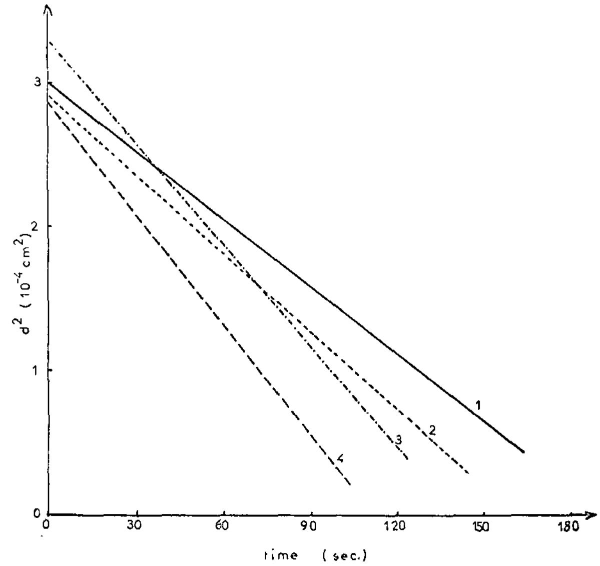

FIG. 3. The variation of d^2 vs time at constant humidity for different wind speeds:

1. $v=10$ cm sec^{-1}, Re=0.85, $p_0=1.57\ 10^{-6}$ cm^2 sec^{-1}.
2. $v=20$ cm sec^{-1}, Re=1.50, $p=1.82\ 10^{-6}$ cm^2 sec^{-1}.
3. $v=32$ cm sec^{-1}, Re=2.75, $p=2.25\ 10^{-6}$ cm^2 sec^{-1}.
4. $v=38$ cm sec^{-1}, Re=3.28, $p=2.57\ 10^{-6}$ cm^2 sec^{-1}.

that β is not constant: for Re>1, it increases up to a value of 0.45 at Re=4.6, and then possibly decreases slightly. When Re>5, only a few experiments are available because at such high wind speeds the drop is unstable and, as precise measurements of diameters are impossible, the slope p is not accurately known.

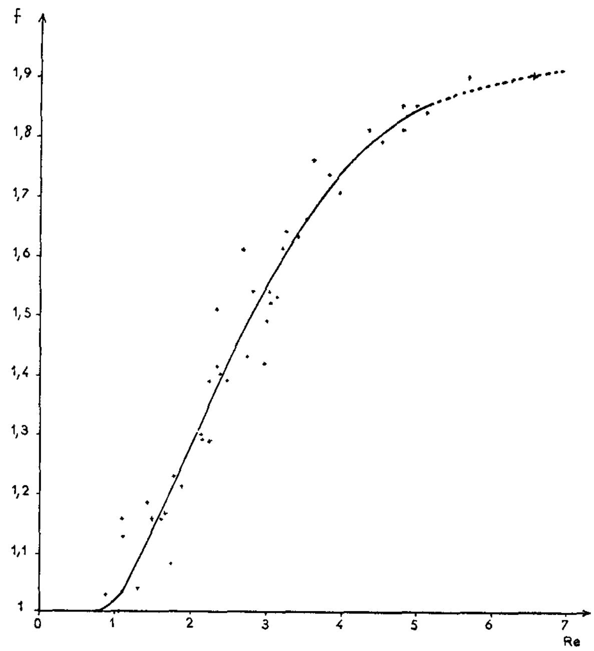

FIG. 4. Plot of f vs Re.

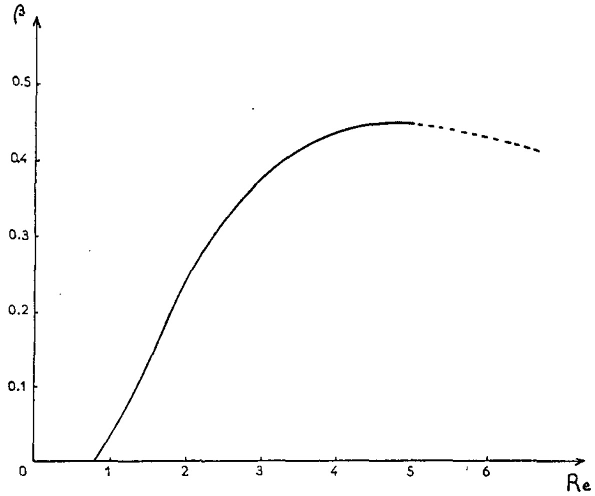

FIG. 5. Plot of β vs Re.

c. Conclusion

These results are in good agreement with the measurements of Kinzer and Gunn. The general tendency for the wind factor f to be a little lower probably occurs as a result of the determination of p_0, which, in our experiments, is evaluated for $0.4<\mathrm{Re}<0.9$, where f is not strictly negligible because the Stokes law is not exact. When $\mathrm{Re}<100$, the experiments show that the wind factor f is not equal to the value predicted by the Frössling equation. Of course, this equation is applicable in the domain of the boundary layer theory, i.e., for $\mathrm{Re}>100$. For small Reynolds numbers, the thickness of the boundary layer should be assumed smaller than in this theory. This can be seen in Fig. 6 where $\lambda=R/r$ has been calculated, using (4), from the Frössling equation, from our experimental data, and from the results of Kinzer and Gunn.

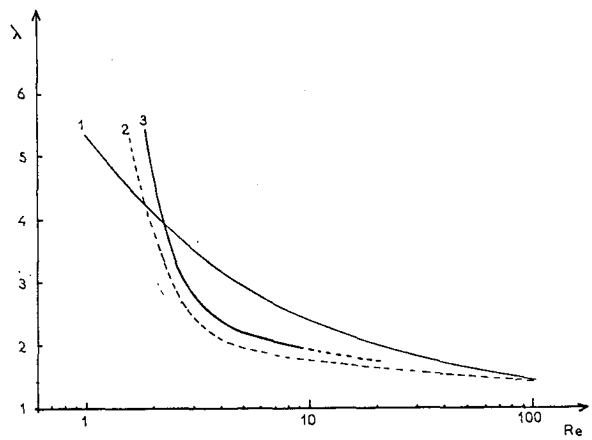

FIG. 6. Plot of $\lambda=R/r$ vs Re from Frössling's equation, 1, from Kinzer and Gunn's results, 2, and from our results, 3.

4. Charge variation observed during evaporation

a. Experimental technique

It is possible to compute the charge of the droplets from (7). The dc voltage is recorded, the radius r is known from photographs, and the wind speeds are calculated as above. It is necessary only to determine the geometrical factor k to know $E=kV$. For this purpose, we used the same method as that of Owe Berg. A charged drop is suspended in the apparatus without any air flow. Photographs are then taken continuously during evaporation. Instabilities predicted by Lord Rayleigh are observed. Visual observations give their exact time of occurrence. When instability arises, the dc voltage must be increased to hold the drop at a constant level. This increase is easily seen on the recording.

Just before instability, the charge of the drop is known from the Rayleigh criterion

$$\frac{Q_i^2}{2\pi\gamma d_i^3}=1. \tag{11}$$

Knowing Q_i, V_i and r_i at instability, k is given by

$$k=\frac{4}{3}\pi r_i^3(\rho-\rho')\frac{1}{Q_iV_i}. \tag{12}$$

Ten determinations of k were made with an average value of

$$k=2.50\ 10^{-4},$$

when V_i is in volts, r in centimeters and Q in esu. [V was estimated from the smoothed dc record and r from the linear approximation of $d^2=f(t)$.]

The error in the Q calculation depends on the accuracy of various factors:

1) The wind velocity must be known very accurately, especially when the aerodynamic force is close to the droplet weight, i.e., when the electric force is weak.

2) The dc voltage, in our device, is not known precisely for small values ($V<100$ V). This is a significant factor affecting the determination of Q.

3) The lack of precision in the diameter measurement is not an important consideration.

Taking all these errors into account, the precision in the measurement of Q is estimated to be $\pm10^{-3}$ esu, provided than $V>100$V.

b. Results

Fig. 7 gives an example of the charge variation occurring during the evaporation of a droplet.

Instabilities are observed provided Q decreases slowly or stays constant during the experiment. This happens only with small values of wind speeds ($v<20$ cm sec^{-1}). For greater speeds, Q decreases rapidly, probably be-

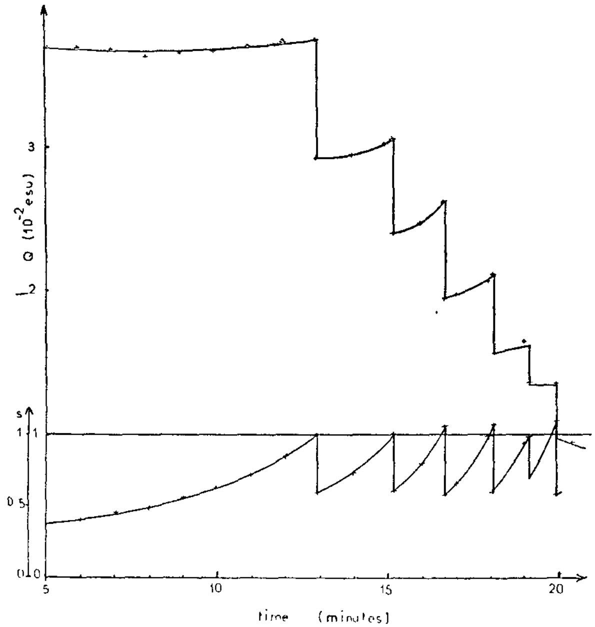

FIG. 7. Example of the time variation of Q and s.

cause of the greater number of collisions between the droplet and dust particles in air.

At instability, no variation of droplet diameter can be seen. Within the accuracy of our experiments, this means that d does not vary by more than $5\,\mu$.

On the other hand, the charge variation ΔQ, at instability, is great; $\Delta Q/Q_i$ varies from 0.16 to 0.40, the average value for 50 cases being 0.26.

In Fig. 7 we have also plotted $s = Q^2/(2\pi\gamma d^3)$. For the 50 cases of instabilities studied, there is a scatter of results from 0.83 to 1.10. The error in Q probably causes this spread.

When several successive instabilities occur for one drop, the charge generally grows between each instability, as in Owe Berg's experiments.

c. Conclusion

Our results, showing no detectable size variation at instability, do not agree with the observations of Abbas and Latham (1967) who found a 20–30% loss of mass in their experiments on charged droplets of the same size. On the other hand, our results could be consistent with those of Owe Berg who found a diameter variation of only about $5\,\mu$, and also with those of Doyle *et al.* (1964) who observed explosive-type ejections of one to ten $15\,\mu$ droplets, corresponding to a maximum variation of $2\,\mu$ for drops $100\,\mu$ in diameter.

The value of the charge loss ΔQ is in good agreement with that of the above authors. The s values obtained verify the Rayleigh criterion. There is no evidence for the existence of metastabilities similar to those observed by Owe Berg.

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