An Equivalent Disc for Calculating the Terminal Velocities of Plate-Like Ice Crystals

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ABSTRACT

Measurements of terminal velocities of metal models of plate-like ice crystals in a tank containing various concentrations of glycerol in water suggest that the terminal velocities of a given model can be predicted from that of a disc made of the same material having the same mass and thickness. The terminal velocities of the models are less than that of the corresponding disc by a factor determined by the shape of the model. This factor is a maximum at 25% for the star-shaped model. These results are independent of the material of the disc or liquid and cover a Reynolds number range of at least 0.5 to 200; they are therefore applicable to plate-like ice crystals falling in air.

1. Introduction

Terminal velocities of ice crystals can be calculated from the relationship between the Best and Reynolds numbers (hereafter referred to as Be-Re) of bodies having geometric similarity to that of the ice crystals. Due to the various forms an ice crystal can take, this would require a large number of such curves each corresponding to a particular geometric shape. Therefore, it has been customary to approximate columnar ice crystals to cylinders and plate-like ice crystals to discs. The validity of these assumptions is questionable and needs further investigation.

For columnar ice crystals the laboratory experiments of Jayaweera and Ryan (1972) confirmed that their terminal velocities can be calculated from the Be-Re of solid cylinders having the same mass and external dimensions as the crystals. However, except for solid hexagonal plates, such equivalence has not been confirmed for plate-like ice crystals.

Therefore, calculations of terminal velocities of plate-like ice crystals from Be-Re would require the availability of a set of curves to cover the known forms of planar ice crystals. The Be-Re curves of List and Schemanauer (1971) can be utilized for this purpose with interpolations assumed for in-between shapes. These authors give the Be-Re of five different shapes of typical plate-like ice crystals. However, it is preferable to reduce as much as possible the number of Be-Re curves that need to be utilized for the determinations of terminal velocities of ice crystals.

Because of an indication (see Jayaweera and Cottis, 1969) that discs having the same mass, thickness and density of ice have very much the same terminal velocities as plane dendritic ice crystals, a tank experiment was performed to test the accuracy of such an approximation for various shapes for plate-like ice crystals.

2. Experiment and results

A plexiglass tank (1 ft by 1 ft by 3 ft) was filled with a mixture of glycerol and water. Four plate-like ice crystal models as shown in Fig. 1 were cut out of 0.13-mm brass and 0.52-mm duraluminum sheets. The terminal velocities of these models falling in the liquid were measured and compared with those of discs made from the same material and having the same mass and thickness. A number of such comparisons were made by changing the concentration of the liquid from 100% to 60% glycerol by weight. With these combinations of discs and liquids it was possible to obtain a range of Reynolds numbers of the discs or ice crystal models defined in terms of the diameter from about 0.5 to 200.

The results of the comparison of terminal velocities are shown in Fig. 2. It is evident from the results that

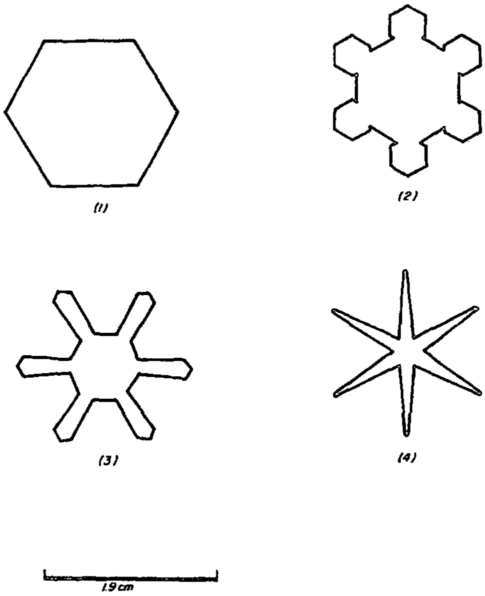

FIG. 1. The ice crystal models.

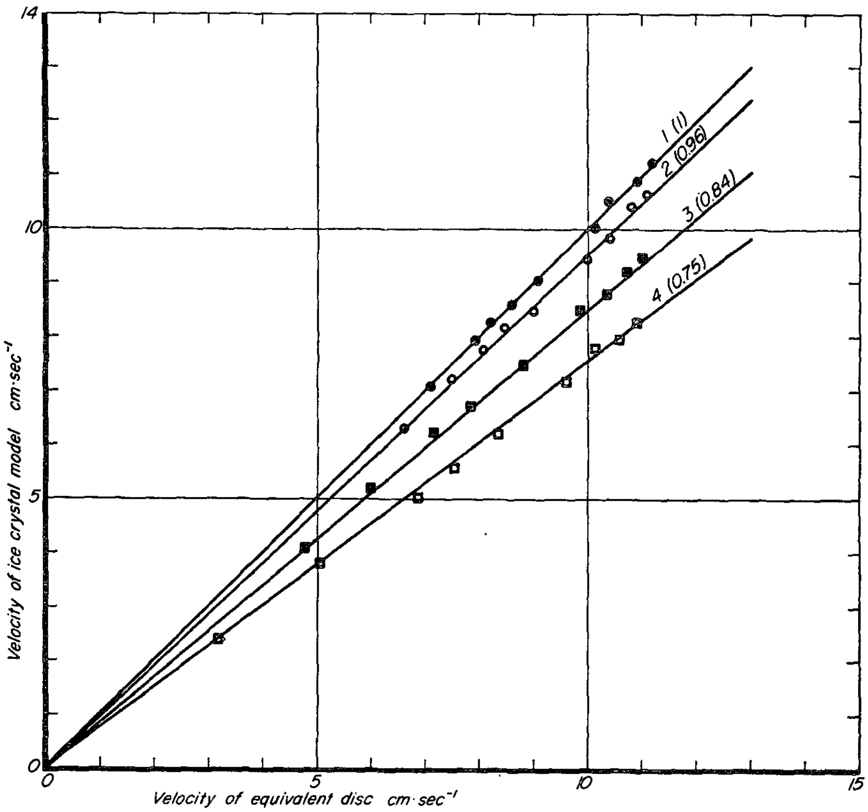

FIG. 2. Comparison between the terminal velocities of ice crystal models and their equivalent discs: model 1 (solid circles), model 2 (open circles), model 3 (solid squares), model 4 (open squares). The ratios of the terminal velocities are indicated in parentheses.

the ratios of the terminal velocities of ice crystal models to those of the equivalent discs are independent of the range of the Reynolds numbers or the properties of the material of the bodies or the liquid. Further, since the Reynolds numbers covered in the tank experiment correspond to those of ice crystals falling in air, these comparisons will be satisfied for the case of ice crystals falling in air, or for that matter, for any plate-like body falling freely in any fluid.

The comparison shown in Fig. 2 indicates that for hexagonal plates the terminal velocity is almost exactly the same as its equivalent disc, while for the star-shaped crystals (Fig. 1, model 4) the terminal velocity is 25% less than that of its equivalent disc. The ratio of the terminal velocities for the other models is indicated in Fig. 2. For a particular model the terminal velocity can be calculated from that of the equivalent disc by applying the necessary correction factor as indicated, but if accuracies within 25% can be tolerated, then as far as the terminal velocity calculation is concerned, all plate-like ice crystals can be replaced by discs of solid ice having the same mass and thickness. Such an equivalence provides a convenient way for calculating the terminal velocities of plate-like ice crystals by using the more accurately known Be-Re of solid discs, and avoids the use of a number of such curves for the different geometric shapes corresponding to those of ice crystals.

It may be necessary at this stage to clarify a confusion that may arise between the present velocity ratios and those which can be derived from the drag coefficient ratios between crystals and discs as given by List and Schemanauer [1971; Eq. (3) of their paper]. The latter ratio refers to ice crystals and discs having the same thickness and diameter, or in other words, the expression of List and Schemanauer gives the ratio of the terminal velocity of an ice crystal to that of a disc circumscribing the crystal. The present results, on the other hand, compare the fall velocities of crystals and discs having the same mass and thickness. The conclusion of List and Schemanauer that the data for discs are applicable to ice crystals only to an order of magnitude approximation is because of their special way of comparison. Such wide differences in the fall velocities between a crystal, say a dendrite, and its circumscribing disc is to be expected because of the great difference in their respective masses.

In conclusion, the results of the present experiments suggest that as far as terminal velocities are concerned the data for discs can be used to an accuracy of 25% provided the comparison is made between discs and crystals having the same mass and thickness. The ratios of the terminal velocities as given in Fig. 2 can be utilized to obtain with greater accuracy the fall velocities of plate-like crystals of a known shape.

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