Reply

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We are grateful for Illingworth's interest in our paper (Hoppel and Phillips, 1971) and for this opportunity to clarify some aspects of our model in more detail. We would like to preface this reply with a comment regarding the concept of electrical conductivity within a thunderstorm. The conductivity can be defined in the conventional manner as

$$\lambda = n_1 e k_1 + n_2 e k_2, \tag{1}$$

but for a cloud stressed by a high electric field, n_1 and n_2 have a strong dependence upon electric field (the notation is the same as in original paper and the comments by Illingworth). Therefore, the conductivity is "non-ohmic" in the sense that it is field-dependent. The term "conductivity" when applied to thunderstorms has meaning only when the electric field is also specified. To illustrate this point it is only necessary to consider a cloud with $Z=5\times10^6$ droplets m^{-3} and radius $b=45\,\mu$ which is possibly quite representative of the lower half of a thunderstorm cloud. Using the ion production rate and recombination rates given in the original paper results in an ion density of about $3\times10^9\ m^{-3}$ outside the cloud; inside the cloud with no electric field, the ion density decreases to about $1.4\times10^9\ m^{-3}$ as a result of ionic diffusion to the cloud particles. However, if the cloud is stressed with an electric field of $10^5\ Vm^{-1}$, then the cloud particles act as very effective centers for recombination and the conductive loss of ions to the cloud particles results in a depletion of ions to a value near $1\times10^7\ m^{-3}$. This latter value is determined by incorporating a field-dependent expression for the attachment coefficient at high fields:

$$\beta = \pi k 3 E b^2. \tag{2}$$

This can be obtained from Eq. (4) in our original paper where the influence from the charge on the droplet is small compared to the induced dipole field. If a cloud is suddenly stressed by an electric field of $10^5\ V\ m^{-1}$, the time for the cloud conductivity to relax to this lower value of conductivity is

$$\tau = \frac{1}{\beta Z} = 0.5\ \text{sec}.$$

We therefore conclude that the conductivity in a cloud stressed by a high electric field with no source of ionization other than cosmic rays is extremely small.

Illingworth raises the question as to the effect of the "non-conducting core." As previously, we would rather not refer to this region as a "non-conducting core" for two reasons: 1) we view the surface at 2 km as an arbitrary surface at which we apply certain boundary conditions and then study the shielding layer, which develops near this surface; and 2) in our solutions the negative ions always have an appreciable value at this boundary. The negative ion density will be continuous across this surface and hence the second term of Eq. (1) will not be zero inside the surface and there will be no "step function" in the conductivity as has been suggested.

Illingworth's question as to the appropriateness of this boundary condition is pertinent because the boundary condition does affect the solution and the width of the shielding layer as shown in Fig. 6 of our original paper. A point we attempted to make (and evidently were not fully successful) was that the solutions given in Figs. 2 and 3 represented steady-state solutions which show that the shielding layer that initially forms at the cloud boundary will move inward and come to rest at the inner boundary for our model at $t=\infty$, a condition which never occurs in thunderstorms. However, the important point to realize in applying this solution to the shielding layer is not that these boundary conditions hold on a sphere of 2 km radius, but that these boundary conditions are nearly satisfied on some arbitrary surface near the inner boundary of the shielding layer. The location of this surface is determined by the depth of the shielding layer at the given moment. The low positive ion density is insured by the capture of ions by polarized droplets in the highly stressed inner cloud region. The comparatively high density of negative ions at the inner surface of the shielding layer results because negative ions traverse the shielding layer. This occurs because the negative charge on the droplets (which are producing the screening layer) decrease the droplet capture cross section for negative ions, and thus the negative ions moving inward can traverse the heart of the screening layer. If the positive-ion density within the cloud is increased from zero to over 10^8 ions m^{-3} as shown in Fig. 6 of our original paper, then an increase in the width of the shielding layer must result. However, as was shown earlier, such a high ion density is not possible within a cloud stressed by a high electric field unless there is a source of ions other than cosmic rays. This, of course, is a distinct possibility and is explored in our paper.

For the reasons just given, the positive-ion density was set equal to zero and the negative-ion density was adjusted to satisfy the asymptotic boundary conditions. (It can be noted here that n_1 could have been taken to be 1×10^7 m^{-3} inside the cloud with no significant change in the time-dependent approximation.) The time assigned to a given solution is determined by the time required for the surface field to decay to the value found in that solution. The time for the field at the surface to decay is given by Eq. (13) in the original paper.

After the submission of our paper, a publication by Brown *et al.* (1971) on the same subject appeared. The approach is entirely different but the results regarding the screening layer for a quiescent cloud are in quite good agreement. Unfortunately, we were not aware of the earlier paper by Lecolazet (1948) and therefore did not reference that paper. However, Lecolazet considers only the diffusional loss of ions to the droplets and not the more important ion capture which results because of the effect of the electric field upon the ion-aerosol attachment coefficient. Therefore, his solution is not generally applicable to thunderstorm clouds. His solution precludes hyper-electrification of droplets which causes the very rapid decrease in field at the cloud boundary. The method of Lecolazet requires numerical integration as does our method. Illingworth suggests that Lecolazet's boundary conditions may be more appropriate. Lecolazet's boundary conditions for the case discussed in the first paragraph above would be that $n_1=n_2=\sqrt{q/\alpha}$ at infinity in the clear air and $n_1=n_2=1\times10^7$ deep within the electrically stressed cloud. The primary difference in the steady-state solution is our inclusion of field-dependent charging as it affects the inner boundary condition and sheathing charge distribution. For the case originally treated by Lecolazet of diffusional charging, a steady-state shielding layer exists; however, with the inclusion of field-dependent charging (as in our model), the shielding layer first develops at the interface and then slowly moves on into the cloud. The physical reason for the inward movement of the shielding layer is the relaxation of the electric field at the outer region of the shielding layer as droplets nearer the center of the cloud become charged by the inward flux of negative ions. Once the electric field is reduced at the outer edge the charge formerly bound to the droplet by the induced electric field will be discharged by ions formed by cosmic rays.

From a strictly mathematical viewpoint the assumption of steady state would require that Lecolazet's boundary condition hold infinitely deep within the cloud, and as such would have little influence on the formation of the shielding layer on the time scale of interest in a thunderstorm. Since the solutions generated are for the steady state, charge continuity requires that the current density must be constant both inside and outside the cloud. This means that the region deep within the low-conducting core (where boundary conditions must be applied in Lecolazet's case) determines the current density everywhere. However, this cannot be the case when the shielding layer is forming near the cloud boundary. The current density outside the cloud and within the boundary is much larger during this period of time. In our analysis the steady-state equations are also used, but the current density is determined by conditions within the shielding layer and not by conditions deep within the cloud where the time to establish steady state is extremely long.

The physics of the model suggested by Illingworth's final comment that no field change will result underneath a cloud from ionic flow across the cloud surfaces is not clear to us. The electric field at the ground reflects the integrated charge configuration aloft. For a spherically symmetric case with uniform ionization and ionic mobility outside the cloud, there will be a decay of field not only at the cloud surface but on every spherical Gaussian surface surrounding the cloud, including the surface drawn through the observer. The decay will be determined by the conductivity on the Gaussian surface in question. In the atmosphere the conductivity increases with height. The new charge configuration and the field at the surface will be altered on a time scale related to the integrated conductivity around the cloud.

Such a spherically symmetric model was not discussed in our original paper. We did describe the thunderstorm as having the primary charge dipole and shielding charge distributions. The electric field variation with time results from the thunderstorm generator current between the dipole charge centers and the mean conductivity of the free air surrounding the system. We did not mean to imply that the field-recovery curve at ground level is determined solely by the time associated with the screening layer aloft. Certainly if the field is sufficiently high, point discharge will be important at ground level beneath the storm; farther from the cloud the high conductivity aloft will influence the recovery times; and charge regeneration rates within clouds may sometimes be important. The thrust of our argument was that shielding-charge readjustments could account for recovery time constants as fast as 10–15 sec at ground level.

In his final comment Illingworth apparently questions our acceptance of a value of 15 sec for typical recovery times found under thunderstorms. Yet Illingworth (1971) published data wherein for field changes >1000 V m^{-1} (implying nearby or overhead storms) the average value of the number of seconds after the flash for a field recovery to $1/e^{\frac{1}{2}}$ of its value is given as 13.5

sec. The normally defined period required to recover to $1/e$ is, of course, longer.

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