A Comparison of Circulations in Transverse and Longitudinal Planes in an Unstable Planetary Boundary Layer

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ABSTRACT

Circulations in the vertical-lateral and vertical-longitudinal planes in an unstable planetary boundary layer are compared through the use of tetroon trajectories. On the average, the circulation in the transverse plane is 40% greater than in the longitudinal, but in the afternoon the transverse circulation is twice as great, providing evidence for the existence of longitudinal roll vortices at this time. The absolute magnitude of the transverse circulation increases uniformly with increase in wind speed and increase in the depth of the well-mixed layer, but the longitudinal circulation does not. The tetroon-derived stress increases with increase in wind speed and increase in absolute transverse circulation, suggesting that longitudinal roll vortices represent an efficient mechanism for the earthward transport of momentum. Comparisons are made between these tetroon results and results obtained by Deardorff from a three-dimensional numerical model of the unstable planetary boundary layer.

1. Introduction

Evidence for the existence of counter-rotating longitudinal roll vortices (helices) in an unstable planetary boundary layer has been presented by Angell *et al.* (1968), based on tetroon flights at the National Reactor Testing Station, Idaho Falls, in July 1966. In a series of impressive papers, Deardorff (1969, 1970a, b, 1972) presents the results of numerically modelling the planetary boundary layer in stable, neutral and unstable conditions. From the parcel trajectories deduced from integration of the equations of motion, he suggests that the helical loops delineated by tetroon flights result from a combination of strong circulations in vertical planes and downstream drift of the circulating parcels, and are not associated with the presence of longitudinal roll vortices per se.

Angell (1971) reexamined the Idaho Falls tetroon data and showed that the frequency with which transverse (vertical-lateral) circulations of opposite sense occurred along flights released simultaneously 0.5 km apart did not seen compatible with the concept of air parcels circulating more or less randomly in the transverse plane. During the review of this latter manuscript, Deardorff indicated (personal communication) that in order to clarify the degree and manner of organization of the flow in the planetary boundary layer it would be useful to compare the magnitude of the circulation in the vertical-longitudinal plane with that in the transverse plane. Such a comparison, based on the Idaho Falls tetroon flights, is the main theme of this paper. In addition, where possible, specific comparisons are made between the results obtained from the tetroon flights and the results obtained from the numerical experiments of Deardorff. Inasmuch as near noon on each day of the tetroon experiment the lapse rate was superadiabatic up to a height of about 1.5 km, whereas the mean tetroon height was about 1 km, most of the circulation data presented herein definitely refer to an unstable planetary boundary layer.

A problem worth noting at this juncture involves the imperfect manner in which tetroons follow air parcel trajectories. As is well known, because of the restoring force acting to return the constant-density tetroon to its equilibrium (isopycnic) float surface, the tetroon cannot depict an air parcel trajectory with fidelity for more than a few tens of minutes at most. Deardorff has clearly shown this by numerically calculating both "parcel" and tetroon trajectories in his model.[1] Of course, at every point along the tetroon trajectory the restoring force is known, and this allows us theoretically to estimate the parcel vertical velocity given the tetroon vertical velocity. This does not, however, let us reconstruct the air parcel trajectory without making some severe assumptions with regard to the variability of velocity in space and time. At this stage in the development of mesoscale meteorology, the tetroon probably represents as close an approximation to an air trajectory as it is possible to get, but it is far from an exact representation, and when discrepancies occur between tetroon-derived results and the results obtained from numerical models, it may not necessarily prove that the numerical results are inapplicable.

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[1] Personal communication.

2. Procedures

Tetroon positions at 1-min intervals were obtained on the flights at the National Reactor Testing Station in July 1966. Lateral displacements along the trajectories have been determined as deviations from 25-min running averages of the lateral distance from straight lines connecting tetroon launch sites and terminal positions, and similarly, longitudinal displacements have been determined as deviations from 25-min running averages of the longitudinal distance along the trajectories. This technique tends to filter out the longer period (>1 hr) lateral and longitudinal oscillations of limited interest in the present context. Tetroon trajectories in the vertical-lateral (transverse) plane and in the vertical-longitudinal (hereafter called longitudinal) plane were than plotted using tetroon height as the vertical coordinate. The coordinates have been plotted at 5-min intervals so that high-frequency oscillations are automatically eliminated. All tetroon flights between 1000 and 1700 local time of more than 1-hr duration were used in the analysis. Before 1000, such an analysis is made difficult by abrupt trajectory turnings associated with the development of upslope winds as well as by the very small tetroon height variations due to the stability prevailing at flight level.

As in the previous paper (Angell, 1971), we approximate the circulation in transverse and longitudinal planes by planimetering the area delimited by the trajectories, recognizing that, if the projection of the tetroon trajectories onto the two planes is circular, the circulation C associated with a planar velocity v is given by

$$C=\int_C \mathbf{v}\cdot\delta\sigma=2\pi r v=4\pi^2 r^2/t=4\pi A/t, \qquad (1)$$

where r is the radius and A the area of the circular trajectory, and t is the time taken to complete the cycle. The circulation is somewhat underestimated by this technique if the trajectory projection is not circular, but this should not be crucial to the following discussion. A positive circulation in the transverse and longitudinal planes is taken as that which would result if the winds varied with height in the manner of the Ekman spiral, i.e., a wind direction veering with height and a wind speed increasing with height. Thus, looking downwind, a clockwise circulation in the transverse plane is considered positive.

As an example, Fig. 1 shows trajectory projections onto the transverse plane (left) and longitudinal plane (right) of flights 7A (top) and 7B (bottom). These flights were released simultaneously at 1515 local time from sites 0.5 km apart in a direction normal to the mean wind, as were all flight pairs. Flights 7A and 7B exhibit typical afternoon circulations, neither unusually uniform nor unusually chaotic.

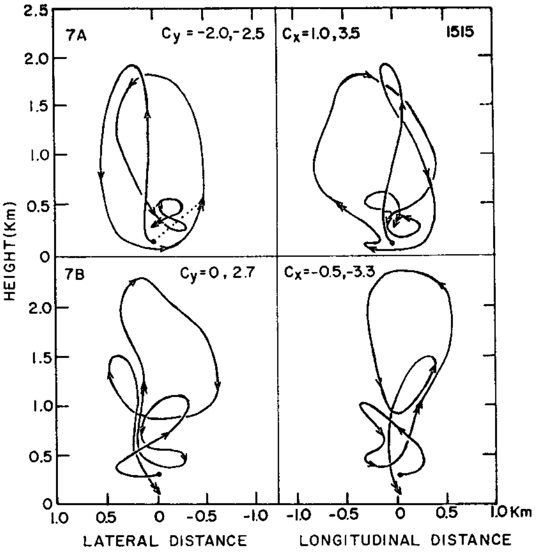

FIG. 1. Tetroon trajectories in the transverse plane looking downwind (left) and in the vertical-longitudinal plane (right) for flights 7A and 7B released simultaneously at 1515 local time. Arrows along the trajectories are at 15-min intervals, with double arrows at 1-hr intervals. Circulations in transverse and longitudinal planes, in units of 10^3 m^2 sec^{-1}, are indicated for successive hours. Positive circulations are defined as those in accord with the wind variation with height given by the Ekman spiral.

The arrows along the trajectory projections in Fig. 1 are at 15-min intervals, with double arrows at 1-hr intervals. It is seen that frequently the tetroons took more than one-half hour to complete a loop in the transverse and longitudinal planes; consequently, to ensure the obtaining of representative circulations, it seemed desirable to evaluate the circulation for 1-hr intervals along the trajectories. Plotted in Fig. 1 are transverse and longitudinal circulation estimates (units of 10^3 m^2 sec^{-1}) so obtained at hourly intervals along the two trajectories. The dotted line at upper left shows how the planimetered area was closed by drawing a straight line from initial to terminal tetroon position. Given the average vortex diameter of about 1 km, a circulation of 3×10^3 m^2 sec^{-1} corresponds to a mean velocity in the transverse or longitudinal plane of about 1 m sec^{-1}. Note that since we estimate the circulation from area measurements, small loops within large loops are taken into account (whether of the same or opposite sense of rotation) in determining the 1-hr average circulation. Along both flights 7A and 7B the sense of transverse and longitudinal circulation remained essentially the same for a 2-hr period, but this is only true for about two-thirds of the flights. Intercomparison of the two flights shows that both transverse and longitudinal circulations became more strongly of opposite sense as the flight time increased, and along both flights the sense of circulation was compatible with the concept of a "plane of circulation" extending to the left of the flow in the downwind direction and to the right of the flow in the upwind direction, where the plane of circulation is defined as perpendicular to the axis of the rolls.

TABLE 1. Percentage of time transverse and longitudinal circulations of opposite sense occurred the given number of flight hours after simultaneous tetroon releases 0.5 km apart in the crosswind direction, and, in parentheses, after sequential releases at about 30-min intervals from the same launch site.

Flight hours	Transverse circulation	Longitudinal circulation	Number of cases
0–1	35 (21)	58 (46)	24 (24)
1–2	67 (86)	43 (57)	15 (7)

3. Comparison of transverse and longitudinal circulations

Table 1 summarizes the information on the sense of transverse and longitudinal circulations obtained from flight groupings; that is, from either flight pairs released simultaneously or from sequential flights from the same site. In the case of simultaneous tetroon releases from sites 0.5 km apart in a crosswind direction, Table 1 shows that for the first hour of flight the transverse circulations were of opposite sense along the two flights 35% of the time, whereas during the second hour the transverse circulations were of opposite sense 67% of the time. It has been assumed that this increase in percentage was due to the laterally-spreading trajectories becoming embedded in longitudinal roll vortices of opposite sense. During the first hour of flight, a different sense of longitudinal circulation occurred 58% of the time, implying that the alongwind component was less coherent in the crosswind direction than was the crosswind component, a result which would be in agreement with turbulence theory. In the case of sequential tetroon releases from the same launch site at about one-half hour intervals, for the first hour of flight the sense of transverse circulation was different only 21% of the time, indicating considerable persistence in the transverse flow pattern over a given region. Such a persistence in sense of circulation was not found in the longitudinal plane.

Fig. 2 shows the cumulative frequency of occurrence of transverse and longitudinal circulations of given absolute magnitude, based on 115 hourly-mean values. Circulations in the transverse plane tend to be somewhat larger than in the longitudinal plane, and this difference becomes more pronounced with increase in circulation, as shown by the dashed line. Thus, circulations exceeding 4×10^3 m^3 sec^{-1} are found twice as often in the transverse plane as in the longitudinal plane.

The mean absolute and algebraic circulation values in the two planes are given at upper right in Fig. 2. On the average, the absolute value of the transverse circulation is 40% greater than the absolute value of the longitudinal circulation. This difference is not large enough to opt for the existence of longitudinal roll vortices on a regular basis, but it will be shown in a moment that under certain conditions the difference between transverse and longitudinal circulations is more significant. As anticipated, the mean algebraic value of the circulation in the longitudinal plane is an appreciable fraction of the absolute value of the circulation in this plane due to the usual increase of wind speed with height which tends to give a positive circulation (as here defined) in this plane. On the other hand, the mean algebraic value of the circulation in the transverse plane is zero, and accordingly, from these data there is no evidence in the mean of the wind direction veering with height specified by the Ekman spiral. This is in agreement with the numerical findings of Deardroff (1970b), who showed that in a moderately unstable planetary boundary layer there is no frictional veering of the wind with height and, consequently, transverse circulations of both signs occur with equal probability.

Fig. 3 presents the combined frequency distribution of transverse and longitudinal circulations of given algebraic magnitude. The numbers in the body of the figure indicate the percentage of the 115 circulations falling within given unit intervals of circulation; i.e., possessing transverse and longitudinal circulations of between 2 and 3×10^3 m^2 sec^{-1}, for example. Because of the smoothing applied, percentage frequencies below 1% were obtained. Of particular interest is the lack of circular symmetry in the distribution. Thus, when the longitudinal circulation is positive and large, the absolute transverse circulation tends to be relatively large. Inasmuch as the longitudinal circulation is apt to be large and positive when the wind speed shear with height is large and positive, there is evidence that transverse circulations (of either sign) are most pronounced when the wind speed shear with height is of above-average magnitude, and consequently that relatively large wind speed shears [at least 1 m sec^{-1} (100 m)$^{-1}$] may be a requisite for the development of

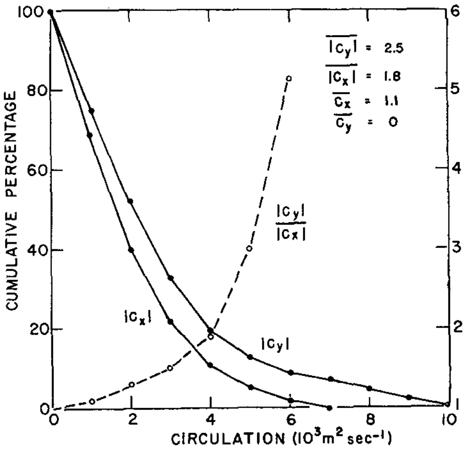

FIG. 2. The percentage of absolute values of transverse circulation, $|C_y|$, and longitudinal circulation, $|C_x|$, exceeding the given abscissa value, based on 115 hourly mean values of circulation. The dashed line (right-hand ordinate) shows the frequency ratio of these cumulative percentages. The absolute and algebraic means of the circulations (10^3 m^2 sec^{-1}) are given at upper right

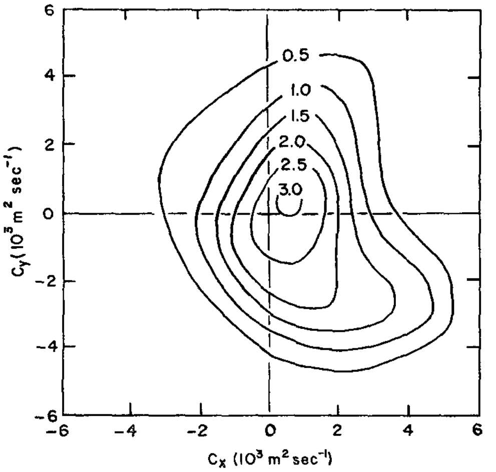

Fig. 3. Percentage of transverse and longitudinal circulations of given algebraic magnitude per unit circulation interval.

transverse circulations. The relation between large transverse circulation and large wind speed shear (large longitudinal circulation) is especially obvious when the transverse circulation is negative. Accordingly, a large wind shear appears to be associated not only with large transverse circulations, but with circulations which frequently are of opposite sense to that given by the Ekman spiral.

A weak but opposite relation emerges when the longitudinal circulation is negative. Under these conditions the transverse circulation is relatively small when the longitudinal circulation is relatively large, and vice versa, providing slight evidence for a plane of circulation aligned at varying angles to the flow.

Fig. 4 shows the smoothed variation with time of day of absolute and algebraic transverse and longitudinal circulation. The number of circulation estimates available at half-hour intervals during the day is given at the top of Fig. 4. Bearing in mind the possible bias resulting from the uneven distribution of estimates during the day, it is seen that the absolute magnitude of the transverse circulation is a maximum shortly after 1400 local time, or, according to Fig. 5, near the time of maximum convection (maximum mixing depth). However, the longitudinal circulation is a maximum at least 1 hr earlier. As a consequence, between 1300 and 1400, the transverse circulation is indicated as increasing at the same time the longitudinal circulation is indicated as decreasing, suggesting a systematic change in orientation of the plane of circulation at this time. The absolute magnitudes of transverse and longitudinal circulation are similar in the morning but in mid-afternoon the transverse circulation is twice as large as the longitudinal circulation. Thus, only in the afternoon is there evidence for the dominance of transverse circulation over longitudinal circulation and consequently for the existence of longitudinal roll vortices. Note that the paper of Angell *et al.* (1968), which provided evidence for the existence of such vortices, was based on tetroon data obtained during the afternoon.

Fig. 4 also shows that the mean algebraic transverse circulation is slightly negative (counterclockwise looking downwind) in early afternoon and tends to become slightly positive in late afternoon and, apparently, in early morning. Accordingly, the sense of transverse circulation seems to be weakly related to stability, with the transverse circulation in accord with the Ekman spiral under relatively stable conditions but not under relatively unstable conditions. These results are also in agreement with the numerical findings of Deardorff. In 30% of the cases, the longitudinal circulation is of opposite sense to that which would be expected from an increase in wind speed with height. A striking example of such a "reverse" circulation is provided by Flight 7B in Fig. 1. The temporal increase in the distance between the dashed lines in Fig. 4 suggests that the reverse circulation is most frequent in the late afternoon, or near the time of maximum convection.

4. Relations between transverse and longitudinal circulations and other meteorological parameters

A better idea of the significance and meaning of the circulations in transverse and longitudinal planes may be obtained by comparing the magnitudes of these circulations with other meteorological parameters. From the tetroon data themselves, estimates are available of wind speed and "mixing depth," the latter assumed equal to the maximum heights attained by the tetroons during the day. Fig. 5 shows the smoothed variation with time of day of mean mixing depth (Z_i), mean tetroon height ($\bar{Z}$), and mean tetroon-derived wind speed (V). The tetroons were ballasted to float at a height of 0.3 km, so that the considerable influence of

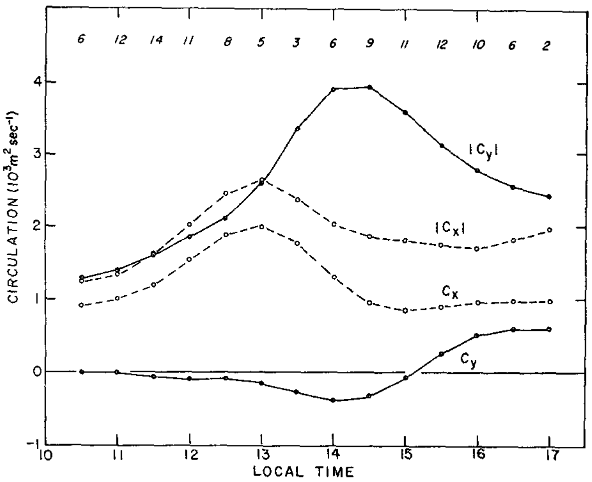

Fig. 4. Temporal variation of the absolute and algebraic magnitude of transverse and longitudinal circulation. The number of circulation values available at half-hourly intervals is given at top.

convection on mean tetroon height is apparent. The maximum wind speed occurs later than the maximum mixing depth and the most rapid increase in mixing depth occurs in the late morning whereas the wind speed increases fairly uniformly throughout the day. During the late morning, the mean tetroon height exceeds half the mixing depth but this is not so during the remainder of the day. This variation with respect to the midpoint of the mixed layer may have some influence on the tetroon-derived stress measurements to be considered later. It was mentioned earlier that rawinsonde ascents near noon showed that the lapse rate was superadiabatic to a height of about 1.5 km on the average. Fig. 5 shows that the convection extended at least 0.5 km above the height where the lapse rate became adiabatic.

Fig. 6 presents the absolute magnitude of transverse circulation, longitudinal circulation, and their ratio, as functions of tetroon-derived wind speed and mixing depth. On the basis of the observations that absolute transverse circulation, wind speed and mixing depth all tend to be a maximum in mid-afternoon (Figs. 4 and 5), one would anticipate a strong relation among these parameters, and this is seen to be the case, with the transverse circulation increasing uniformly with increase in wind speed and mixing depth, although the increase with mixing depth is more pronounced. A fourfold increase in mixing depth and a sevenfold increase in wind speed are associated with a nearly sixfold increase in absolute magnitude of the transverse circulation. Such a uniform variation is not at all apparent in the case of the longitudinal circulation, and in fact in this latter case the maximum circulation occurs at intermediate values of wind speed and mixing depth. At these intermediate values, the absolute magnitude of the longitudinal circulation is comparable with the absolute magnitude of the transverse circulation on the average, as shown in the bottom diagram. However, when the mixing depth is ~ 2 km and the wind speed ~ 12 m sec^{-1}, the transverse circulation is double the longitudinal circulation, in accord with Fig. 4.

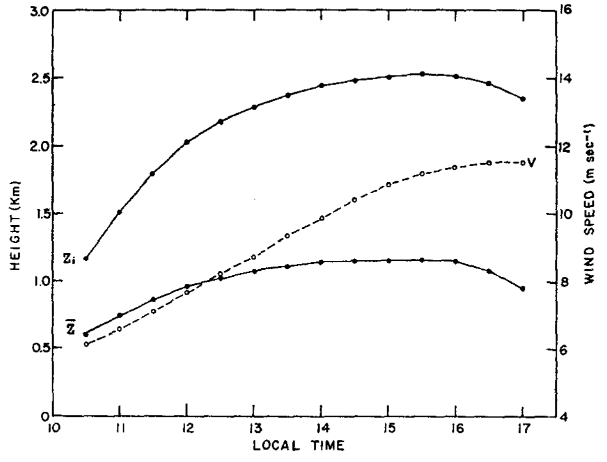

Fig. 5. Temporal variation of mean tetroon height ($\bar{Z}$) and mean tetroon derived values of mixing depth (Z_i) and wind speed (V). The solid lines refer to the left-hand ordinate, the dashed line to the right-hand ordinate.

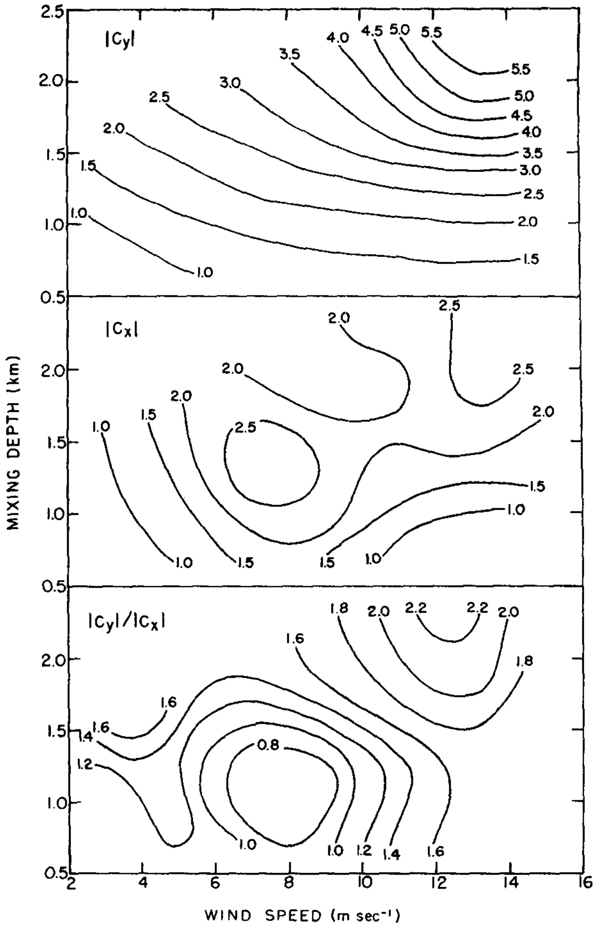

Fig. 6. Variation of the absolute value of transverse circulation, longitudinal circulation, and their ratio, with tetroon-derived mixing depth and wind speed. Circulation units are 10^3 m^2 sec^{-1}.

It would have been useful to have had heat and momentum flux measurements at the time of the tetroon flights in 1966. This was not the case, but such measurements were obtained near the surface in August and September of 1968.[2] Because of the nearly steady-state meteorological conditions at the National Reactor Testing Station in summer, it is likely that these latter measurements are fairly representative of conditions during the time of the tetroon flights.

Based on the 1968 data, the bottom diagram of Fig. 7 shows the smoothed variation with time of day of the mechanical rate of production of turbulent energy $(\tau/\rho)(du/dz)$, and the rate of production by buoyancy $gH/(\rho C_p T)$, where τ is stress (vertical momentum flux), H is vertical heat flux, and the other terms have their

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[2] The writer is indebted to C. R. Dickson, Chief, Air Resources Field Research Office, National Reactor Testing Station, Idaho Falls, for making these data available.

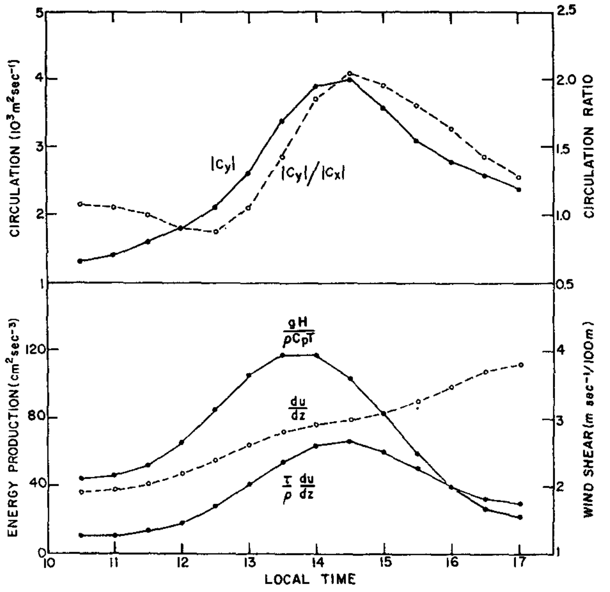

Fig. 7. Comparison between the temporal variation of absolute transverse circulation and the ratio of transverse and longitudinal circulation obtained from tetroon flights in the summer of 1966 (top), and the temporal variation in wind shear between 1 and 60 m and rate of production of turbulent energy by mechanical and buoyant means at a height of 30 m in the summer of 1968 (bottom). The solid lines refer to the left-hand ordinate, the dashed lines to the right-hand ordinate.

customary meaning. This average is based on 5 days of around-the-clock measurements at a height of about 30 m. At this height the buoyant production of turbulent energy is indicated to be somewhat larger than the mechanical production and its maximum value occurs about one hour earlier.

The top diagram of Fig. 7 shows again the smoothed variation with time of day of the absolute value of the transverse circulation, as well as the ratio of transverse to longitudinal circulation. The transverse circulation is a maximum slightly more than one-half hour after the maximum rate of production of turbulent energy near the surface by buoyancy, a not unreasonable time lag if the driving mechanism for the transverse circulations is assumed to be buoyancy. On the other hand, the transverse circulation is a maximum just before the maximum rate of production of near-surface turbulent energy by shearing stresses, and a reason for this near-simultaneity will be presented in Section 5 in connection with tetroon-derived stress estimates. Because of these small time lags, it is difficult to determine whether the magnitude of the transverse circulation is most closely attuned to heat flux, or to momentum flux and wind shear in the surface boundary layer.

It is also useful to compare the absolute magnitude of the transverse circulation with the vertical wind shear obtained from conventional vertical soundings, although there is the interesting point as to the representativeness of individual vertical soundings if large transverse circulations of opposite sense exist. Unfortunately, vertical soundings near midday were obtained on only 7 days of the experiment so that the data sample is small. With such a small sample, the most logical procedure is to consider separately days with relatively strong transverse circulation and relatively weak transverse circulation (based on an average

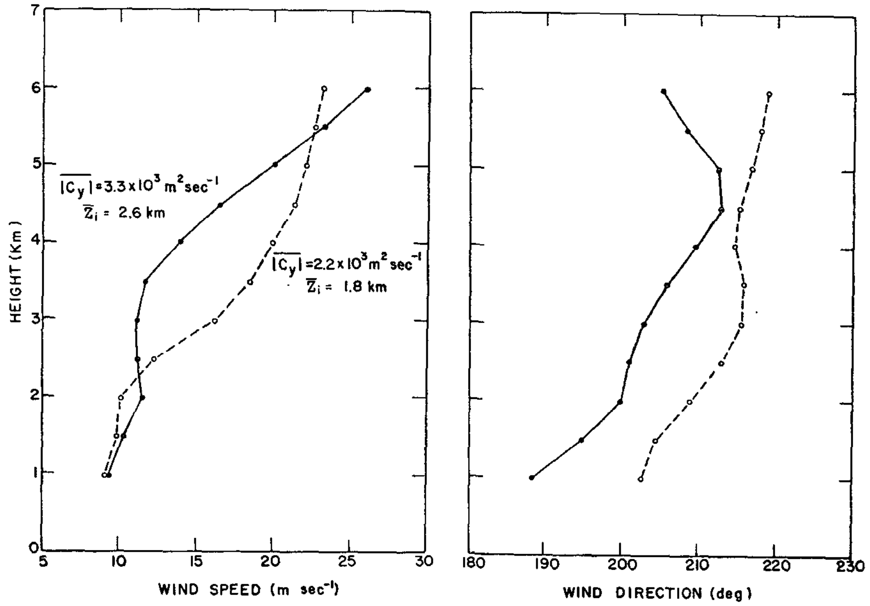

Fig. 8. Variation of rawinsonde-derived wind speed and direction with height near noon on 3 days when the absolute magnitude of the tetroon-derived transverse circulation was relatively large (solid lines) and on 4 days when it was relatively small (dashed lines). The mean tetroon-derived values of mixing depth on these days are also indicated.

for all the flights from 1000 to 1700 local time on each day) and to determine the average variation of wind speed and direction with height for each group. Because of difficulties in radar acquisition of the ascending radiosonde balloon, wind data were not obtained below 1 km.

The left-hand diagram of Fig. 8 shows the mean variation of wind speed with height when the absolute value of the transverse circulation is relatively large (solid line) and relatively small (dashed line). Since the layer of nearly-constant wind speed is probably indicative of the depth of the well-mixed layer, there is evidence that on days with a relatively strong transverse circulation the air was convectively mixed up to a height of about 3.5 km, whereas on days with a relatively weak transverse circulation, the convective mixing extended to a height of only about 2 km. For comparison, the mean tetroon-derived mixing depth for the two data groups was 2.6 and 1.8 km, respectively. Thus, as expected, because of the buoyancy force acting to return the tetroon to its equilibrium float level, the tetroon underestimates the mixing depth, the more so the more it is forced above its equilibrium level. There were negligible differences in lapse rate for the above two data groups, hardly surprising in view of the fact that at this site in summer day-to-day lapse rate changes are trivial and not easily detectable with the radiosonde equipment (then) in use. Any study of the effect of stability on transverse circulations probably would be better accomplished by means of tall towers, or perhaps aircraft with very accurate instrumentation.

Because of the small data sample, wind speed difference <1 m sec^{-1} may not be meaningful. Thus, the slight wind speed maximum at a height of 2 km in the strong transverse circulation case is probably not significant, although it is noted that Kuettner (1971) has suggested, based on theoretical considerations, that a convex wind speed profile of this nature should preferentially be associated with the formation of longitudinal roll vortices, i.e., with strong circulations in the transverse plane.

The right-hand diagram of Fig. 8 shows the variation of wind direction with height in the case of relatively large and small transverse circulations. When the transverse circulations are large (solid line) the wind has a more southerly component to a height of 6 km. Furthermore, under these conditions, the rate of wind veering with height decreases suddenly at a height of 2 km, or just where the slight wind speed maximum occurs. This may reflect the presence of large transverse circulations of negative (counterclockwise looking downwind) sense. While Fig. 4 shows that during most of the day there is a slight tendency for a mean negative circulation in the transverse plane, the vertical soundings show a tendency for a veering of the wind with height, which should be associated with a positive circulation in this plane. This suggests that the counter-rotating longitudinal roll vortices may be nearly fixed in space, and that the rawinsonde launch site happened to be located in a region where roll vortices of positive sense predominate.

Along this line, it might be noted that Big Southern Butte, an obstruction to the air flow some 4 km in diameter and more than 500 m high, is located 10 km from the tetroon launch sites on a bearing from the launch site of 192°. Thus, the strong transverse circulations of Fig. 8 occur when the mean air flow at a height of 1 km is almost exactly from Big Southern Butte to the launch sites. This raises the possibility that the large transverse circulations result at least partly from air being forced over and around Big Southern Butte. The anchoring of the transverse circulations to a terrain feature would explain why successive tetroon releases usually exhibited the same sense of transverse circulation (Table 1). Further experimentation with tetroons in the vicinity of the Butte could clarify the role of the obstruction in producing transverse circulations.

5. Relations between tetroon-derived stress and transverse and longitudinal circulations

The covariance of vertical and longitudinal eddy velocity components along tetroon trajectories permits, formally, an estimate of the tangential Reynolds stress in the manner of fixed point data. The accuracy of the stress so obtained is controversial (and under investigation) because the tetroon does not follow the vertical air motion exactly and because, unlike tower measurements, the individual eddy products are not obtained at the same height. It will be assumed here that, while the magnitude of the stress derived from tetroon flights may be unrepresentative, the change in magnitude with change in circulation is probably meaningful.

The solid lines in Fig. 9 show the smoothed variation with time of day of absolute and algebraic magnitude of tetroon-derived stress in the latitudinal direction. The variation is similar to that found for the transverse circulation (Fig. 4), with the tetroon stress exceeding 6 dyn cm^{-2} between 1400 and 1500 local time on the average. From the difference between absolute and algebraic stress values it is seen that during the morning hours the stress is frequently negative (upward momentum flux) whereas during the afternoon the stress is almost always positive. In a review of this paper, Deardorff suggested that the negative stress value during morning hours might be related to the observation that the tetroons were floating in the upper portion of the mixed layer in the morning (Fig. 5), and hence perhaps in the negative portion of the stress profile. However, it was found that the difference between mixing depth and mean tetroon height in the morning was nearly 200 m greater when the stress was negative than when it was positive, so that proximity of mean tetroon height and mixing depth does not seem to be a factor in the negative stress values.

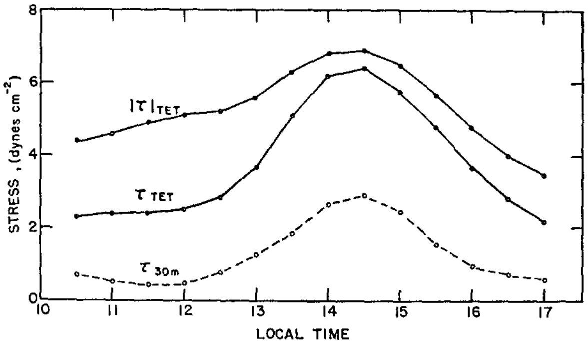

Fig. 9. Temporal variation of mean tetroon-derived absolute and algebraic value of the tangential stress in 1966 (solid lines), and the mean variation obtained in 1968 from fixed-point stress measurements at a height of 30 m (dashed line).

The dashed line in Fig. 9 shows the smoothed variation with time of day of the stress evaluated at a height of 30 m at the National Reactor Testing Station during 5 days in summer 1968. The temporal variation in stress is almost identical to that obtained from the tetroon flights, but the tetroon-derived stress is greater by a factor of 2–4. Since numerous other investigations make it seem unlikely that the stress increases with height, either the atmospheric conditions during limited periods in the summers of 1966 and 1968 were not compatible or either or both of the stress evaluations were unrepresentative. It is not possible at this stage to be sure where the difficulty lies, although there is evidence that the tetroon-derived stress is too large, perhaps by a factor of 2.

It is of considerable interest to see if there is a relation between absolute magnitude of the transverse circulation and tetroon-derived stress. The top diagram of Fig. 10 shows that this stress does indeed increase uniformly with increase in tetroon-derived wind speed (which of course would be expected) and with increase in transverse circulation. For wind speeds of ~12 m sec^{-1}, the stress increases from 3 to 12 dyn cm^{-2} as the transverse circulation increases from 1 to 6×10^3 m^2 sec^{-1}. Thus, there is little doubt that large transverse circulations are associated with large downward fluxes of momentum, and hence, if one associates large transverse circulations with longitudinal roll vortices, these vortices are indicated to be very efficient transporters of momentum earthward. Accordingly, the strong correlation between temporal variation in absolute transverse circulation and mechanical rate of production of turbulent energy at a height of 30 m (Fig. 7) may be explained by the increase in downward momentum flux brought about by the increase in transverse circulation, as well as the indicated increase in low-level wind shear, the latter presumably partly the result of momentum-flux convergence above the surface boundary layer.

An investigation was also made to see if the magnitude of the tetroon stress is a function of the sense of circulation in the transverse plane. It was found that the tetroon stress averages about 0.5 dyn cm^{-2} more when the transverse circulation is positive (clockwise looking downwind) than when it is negative, but this small difference can hardly be considered significant.

The bottom diagram of Fig. 10 shows that there is a systematic increase in stress with increase in absolute magnitude of the longitudinal circulation only at the larger wind speeds, while at the lower wind speeds there is actually a decrease in stress with increase in longitudinal circulation. Thus, perhaps contrary to *a priori* expectation, the tetroon-derived stress is much more closely related to the magnitude of the transverse circulation than it is to the magnitude of the longitudinal circulation.

6. Comparison with Deardorff's three-dimensional planetary boundary layer model

On the basis of his three-dimensional numerical model of the planetary boundary layer, Deardorff has proposed that the relevant indicator of the degree of thermal instability in the boundary layer is $-Z_i/L$ not $-u_*/fL$, where Z_i is the height of the convecting (mixed) layer, u_* the friction velocity $(\tau/\rho)^{\frac{1}{2}}$, f the Coriolis parameter, and $-L$ the Monin-Obukhov length equal (using an approximation for the wind shear in the surface boundary layer) to the ratio of turbulent energy production by mechanical and buoyant means at a given height, or $u_*^3\rho C_pT/(kgH)$, where k is von Kármán's constant. Deardorff found from his

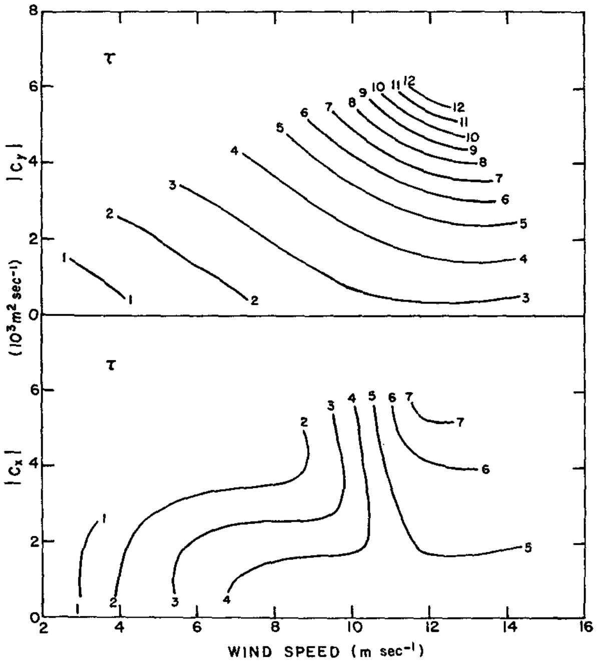

Fig. 10. Variation of tetroon-derived stress with absolute transverse circulation and wind speed, and with absolute longitudinal circulation and wind speed. Stress units are dyn cm^{-2}.

numerical model that in slightly unstable conditions $(-Z_i/L<5)$ there is considerable downwind "elongation" of the vertical and along-stream velocity eddies, yielding circulations similar in form to longitudinal roll vortices. However, when $-Z_i/L>50$ he finds that the eddy structure is one of free convection with no preferred horizontal eddy orientation above a small height comparable to $-L$.

In order properly to intercompare the results obtained from the Idaho Falls tetroon flights and from Deardorff's model, it is necessary to know the value of $-Z_i/L$ appropriate to the tetroon flights. An approximation to the variation of Z_i with time of day has been given in Fig. 5. However, from Fig. 9, it is possible to derive two quite different values of $-L$ depending upon whether one uses the tetroon-derived value of stress or the value of stress at a height of 30 m. The solid lines in Fig. 11 show that the smoothed values of $-L$ vary from 100 m in the morning to 300 m in the afternoon if the tetroon stress is used, and from 10 to 60 m if the stress at a height of 30 m is used. The corresponding values $-Z_i/L$ hover between 10 and 20 if the tetroon stress is used and between 150 and 50 if the 30 m stress is used.

Regardless of which of the above stress values is more correct, it quite obviously is unlikely that $-Z_i/L$ is less than 5, and consequently that longitudinal roll vortices should occur with any regularity according to Deardorff's numerical calculations. Thus, there is a discrepancy between the tetroon-derived results and the numerical results if it is assumed that the twofold excess of transverse circulation over longitudinal circulation obtained from the afternoon tetroon flights is sufficient to indicate the frequent existence of longitudinal roll vortices. If we use the ratio of absolute transverse and longitudinal circulation as a measure of the frequency of occurrence of longitudinal roll vortices, then the top diagram of Fig. 7 shows that the frequency would be greatest near 1500 local time and least near 1200 local time. The evaluation of $-Z_i/L$ using the stress at 30 m results in a maximum value of this parameter at 1200 and a below-average value at 1500 (Fig. 11), so that from this point of view there *is* an inverse relation between the frequency of longitudinal roll vortices and the value of $-Z_i/L$, in agreement with Deardorff's results.

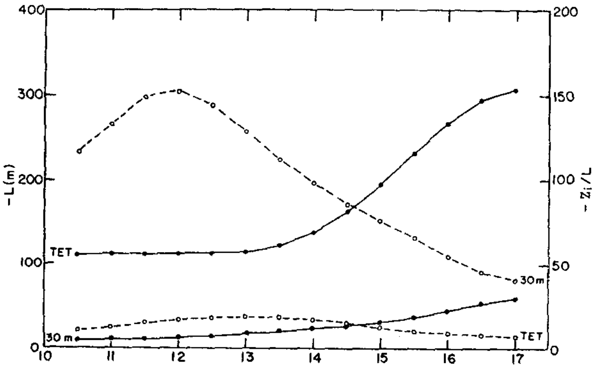

FIG. 11. Temporal variation of Monin-Obukhov length L (solid lines), and the ratio of mixing depth to this length (dashed lines), using 1966 tetroon-derived stress estimates and stress estimates at a height of 30 m in 1968. The solid lines refer to the left-hand ordinate, the dashed lines to the right-hand ordinate.

It is apparent that, with the data available, it simply is not possible categorically to confirm or deny the planetary boundary layer flow patterns deduced from Deardorff's numerical model. This would require tetroon flights in horizontally homogeneous terrain with concomitant heat and momentum-flux measurements. Even then, the acknowledged difference between air parcel and tetroon trajectories could lead to difficulties in interpretation and deduction.

7. Conclusion

Further analysis of constant volume balloon (tetroon) flights made at the National Reactor Testing Station indicates that, in an unstable planetary boundary layer, the circulation in the transverse plane averages 40% greater than the circulation in the longitudinal plane. In the afternoon, or under conditions of relatively strong wind and deep convection, the ratio of transverse circulation and longitudinal circulation exceeds a factor of 2. Such a circulation ratio seems sufficiently large to support the concept that longitudinal roll vortices frequently exist at this site at this time, but possible terrain influences on the flow make generalization to other locales hazardous. The transverse circulation appears strongest at about the same time as the maxima in turbulent energy production near the surface by buoyant and mechanical means, but these findings, based on non-simultaneous observations, should be confirmed.

Tetroon-derived stress values increase strongly with increase in transverse circulation, suggesting that longitudinal roll vortices are an efficient mechanism for the earthward transport of momentum. The formation of such vortices may be nature's way of ensuring a large flux of momentum from atmosphere to earth.

It would be useful to repeat the tetroon experiment in non-mountainous terrain with simultaneous surface measurements of heat and momentum flux in order better to compare the tetroon results with the results obtained by Deardorff[3] from his three-dimensional numerical model of the planetary boundary layer.

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[3] *Editor's note.* See Comments by Deardorff on pp. 1394–1395 of this issue.

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