Calculations of Hailstone Growth and Trajectories in a Simple Cloud Model

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ABSTRACT

A hailstone model is developed which simulates both wet and dry hailstone growth and partial or complete melting as the hailstone falls to the ground. The simulation is obtained by a consideration of the mass and heat budgets of the hailstone as a function of its size and of the environmental conditions. Competition among hailstone embryos is not treated in this model.

The hailstone model has been run in over 50 versions of a one-dimensional, time-dependent cloud model to derive estimates of maximum hailstone size from a wide variety of convective storms. The model suggests that hailstone diameter at the ground is determined closely by the strongest updraft encountered by the stone and the temperature at that level. In the model, most hailstones spend some time above the maximum updraft and significant growth occurs during descent from that level to the freezing level. A comparison of model predictions to hailstone observations in the Rapid City area during 1968 and 1969 shows fair agreement.

The model has been used to test the concept of hail suppression through artificial glaciation of cloud water and rainwater. It suggests that results would vary with temperature at the level of maximum updraft, but that artificial cloud glaciation would lead in a majority of cases to a reduction in maximum hailstone size at the ground.

1. Introduction

A fall of hailstones is a most impressive phenomenon and scientists have long speculated on the processes which lead to its formation. While the hail formation process is difficult to observe directly, the application of basic physical laws to our admittedly inadequate knowledge of conditions inside hailstorms has led to considerable insight into the formation of hail. The contributions of Schumann (1938), Ludlam (1958), List (1960), Douglas (1963), and of Sulakvelidze *et al.* (1967) to the modeling of hailstone growth processes are worthy of special note.

The hailstone model developed and applied in this paper is an elaboration of one developed by Musil (1970) to study in detail the growth of individual hailstones in developing convective cells of Great Plains thunderstorms. Musil's (1970) results indicated that hailstones in commonly observed size ranges can be produced from embryos of 50–100 μm diameter in a single pass (ascent and descent) through a vigorous convective cell. The total hailstone size distribution is not considered in the Musil model, as the objective was to explore the largest stones which can be produced in a given situation. The model has shortcomings in its inability to model either the depletion of cloud water by the growing hailstones or the feedback of precipitation processes upon cloud dynamics, but it has the advantage of modest computer requirements.

The original version of the Musil model distinguished between wet and dry hailstone growth, but assumed that all excess water was shed from the wet hailstones. This assumption is unrealistic, as List (1959) has found in wind tunnel experiments that growing hailstones can incorporate liquid water into an outward developing ice lattice ("spongy" growth); Macklin (1961) has reached similar conclusions.

The present version of the model simulates spongy hailstone growth, the subsequent freezing of the incorporated water should the hailstone move to sufficiently cold regions, and partial or complete melting as the hailstone returns to warmer regions during its fall. The application of the model has permitted tentative identification of the cloud parameters determining maximum hailstone size at the ground and makes it possible to evaluate (within the model's limitations) one suggested means of hail suppression, namely, artificial glaciation of supercooled cloud water and rainwater to reduce hailstone growth rates (e.g., Howell, 1966).

2. Description of the model

The hailstone model consists of two parts. The first part, the model proper, consists of equations which describe the mass and heat budgets of a hailstone in terms of hailstone size and cloud environment. The second part consists of tables or equations prescribing the hailstone environment.

Discussion in this section is limited to the model proper. As the equations constituting the model are given in Appendix A, discussion in this section will be qualitative.

a. Mass budget

The model treats a hailstone as a sphere with density between 0.9 and 1.0 gm cm^{-3}, depending upon the fraction of liquid water (F_w) contained within it, and at a uniform temperature throughout.

The hailstone gains mass through the collection of cloud water, cloud ice particles, and rainwater. (Hereafter, collection of cloud water or rainwater will be called accretion and collection of cloud ice will be referred to simply as collection.) There are changes in mass associated with deposition of water vapor upon the hailstone or evaporation/sublimation from it, but they are so small compared to accretion and collection for particles[1] $>100\ \mu$m that they can be neglected.

Determination of accretion and collection rates requires a consideration of fallspeeds and collection efficiencies (discussed in Appendix A). All cloud water accreted by the growing embryo or hailstone is assumed to be retained within a spongy ice structure. Therefore, the dry growth equations of Musil (1970) are applicable for determining the rate of mass increase. However, an upper limit is imposed on the amount of water which can be collected by a hailstone through the breakup mechanism described below.

b. Breakup mechanism

In the original version of the model, referred to hereafter as Musil 70, raindrops broke up when they reached 5 mm. The question of breakup of hailstones did not arise since growing stones were assumed to shed all excess liquid water and the melting process was not handled.

In the present version of the model (Musil 72), a hailstone is assumed to break up when it exceeds a critical diameter as shown in Fig. 1. For an unfrozen or completely melted hydrometeor, the breakup diameter is 5 mm in agreement with Musil 70. As the fraction of water decreases from 1 to 0.5, the breakup diameter increases from 5 mm to 6 cm. The relationship shown in Fig. 1 is a guess; experimental data to confirm or refute the suggested variation in breakup diameter are lacking. The breakup mechanism assumes that all of the ice and half of the water remain with the "hailstone." The remainder of the water is thrown off in one or more droplets and is not considered further.

The concept of an ice lattice is inappropriate to a stone melting during fall below the freezing level, but the breakup mechanism (Fig. 1) has been used for both ascent and descent in our standard Musil 72 runs. Some special runs have been made in which cloud water and rainwater accreted *during fall* form a film with a specified upper limit on its thickness, say 2 mm. The result is much the same as with the breakup mechanism, namely, repeated shedding of water drops, and the size of the hailstone at the ground differs little between the two treatments.

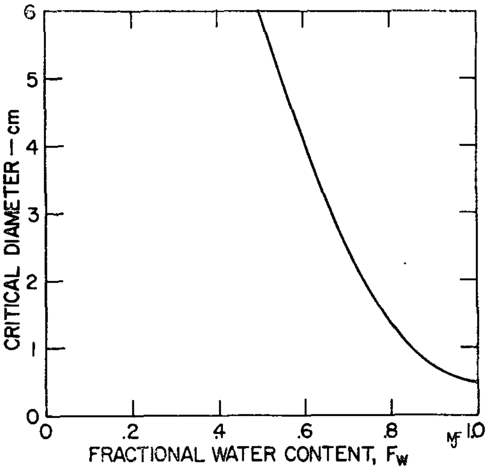

Fig. 1. Assumed critical diameter for breakup of spongy hailstones as a function of fractional water content.

c. Heat budget

Heat transfers to and from a hailstone consist of four terms: conduction, evaporation/sublimation, accretion, and collection. The convention has been adopted that heat transfers to the stone are positive.

The conduction term is readily understood. Heat is conducted away from the hailstone when it is warmer than the environment, which is usually the case for a hailstone growing in a supercooled cloud. Heat is conducted to the hailstone when it is colder than the environment; this condition holds for a hailstone melting during fall through the lower part of a cloud.

The evaporation/sublimation term is quite important to the heat budget, especially for small hailstone embryos, even though evaporation and sublimation are insignificant in the mass budget. The contribution of this term to the heat budget can be either positive or negative.

The accretion term involves both the latent heat and sensible heat in the accreted water. This term is positive, that is, it leads to warming of the stone, under all conditions ordinarily encountered. The collection term involves only the sensible heat of the collected ice crystals and always serves to chill wet hailstones and those dry hailstones which are warmer than the environment (the usual situation).

At each time step in the computer program, the heat added or taken away from the hydrometeor through the four processes just described is calculated and an appropriate adjustment made in the characteristics of the hailstone. The time steps are normally 1.0 sec but in regions of rapid change, as when a small particle first

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[1] In this paper, size will refer to diameter.

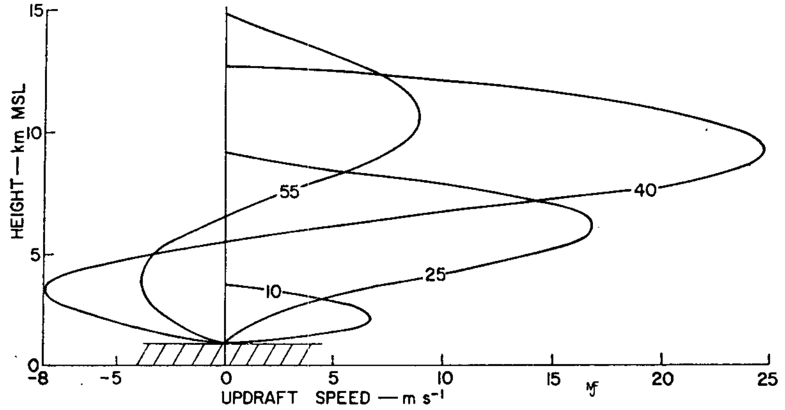

FIG. 2. Sample set of vertical velocity profiles generated by Fourier series program. Profiles are shown at 10, 25, 40 and 55 min for a run with cloud life of 60 min, maximum cloud height of 15 km, time of onset of second harmonic of 20 min, peak amplitude of first harmonic of 14 m sec^{-1}, and peak amplitude of second harmonic of 16 m sec^{-1}.

freezes on ascent, 100-msec time steps are required to prevent computational instability. For dry hailstones and for liquid drops the adjustment to hydrometeor characteristics takes the form of an increase or decrease in temperature. For wet hailstones it takes the form of an increase or decrease in the fraction of liquid water within the stone. Time lags in the transfer of heat within the stone are neglected, so that the possibility of a film of water at a temperature above 0C surrounding ice at or below 0C is not considered. This appears to be an acceptable procedure for all but the very large stones (e.g., Hitschfeld and Stauder, 1967).

3. Hailstone environments

The Musil model is applicable with any cloud model which provides information on temperature, air density, cloud water content, rainwater content, and cloud ice content. For example, Orville[2] has used it to investigate hail formation in his two-dimensional field-of-motion cloud model (Liu and Orville, 1969). We have used the Musil 72 model to study the growth of hailstones in many possible cloud environments, with particular reference to updraft profile, cloud water concentration, and temperature.

a. Updrafts

In order for the model to give useful results, the assumed updraft profiles must correspond to those of actual clouds. Convective cells contain updrafts throughout the early stages of their existence. After considering various observations, Sulakvelidze *et al.* (1967, p. 25) adopted a model updraft profile along the cloud axis peaking roughly midway between the cloud base and cloud top. In the later stages of a cell's existence, it is common for the updrafts to persist in the upper part of the cloud while downdrafts appear in the lower part and below cloud base (e.g., Byers and Braham, 1949; Battan and Theiss, 1966). Numerical modeling suggests that the downdrafts are due in part to the drag forces exerted by the accumulation of precipitation in and below the cloud (e.g., Orville and Sloan, 1970; Wisner *et al.*, 1972).

This commonly observed pattern suggests that the updraft profile along the vertical axis of a cell can be approximated by the first two terms of a Fourier series, provided that the second harmonic is suppressed during the early part of the cell's lifetime. By specifying the cloud (cell) life, the maximum cloud top, the amplitudes of the two harmonics, and a lag time for the onset of the second harmonic, updraft profiles with various desired maximum values and shapes can be readily generated (Appendix B).

A sample of one set of these updrafts is given in Fig. 2. The speed and height of the maximum updraft both increase with time to $\sim$50 min, after which the updraft diminishes. Due to the action of the second harmonic, downdrafts begin to appear around 30 min and reach their maximum intensity around 40 min. The profile ends with vertical velocities at 60 min, the prescribed cloud lifetime in this case, of zero.

b. Cloud models used

As our first objective has been to model typical hailstorms of the Great Plains region, all runs to date have assumed a ground elevation[3] of 1 km. As a start, the model has been run for the 36 basic types of hailstorms shown in Fig. 3, which extend over the range of cloud base conditions commonly encountered in hailstorms of the northern Great Plains (Dennis, 1971a). A cloud lifetime of 60 min has been assumed in each case. The total set constitutes a 2 by 2 matrix of two pseudo-adiabats (θ_w, 15 and 23C) and two cloud base heights

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[2] Private communication.

[3] All heights MSL.

(2 and 3 km). The conditions correspond to surface temperatures of 16–32C, cloud base temperatures of 0–15C, and saturation mixing ratios in the cloud air at cloud base of 6–13 gm kg^{-1}.

Each element of the 2 by 2 matrix is further divided into a 3 by 3 matrix of short, medium and tall clouds versus weak, moderate and strong updrafts. Heights of 9, 12 and 15 km are assigned for short, medium and tall clouds, respectively, and updraft maxima of 18, 25 and 32 m sec^{-1} for weak, moderate and strong updrafts, respectively (Dennis *et al.*, 1970; Dennis, 1971b).

The parameters described in Appendix B have been adjusted through trial and error to yield sets of updraft profiles conforming to the desired specifications. It should be understood that the probability of actually encountering some of the modeled combinations of cloud base conditions and updraft profiles could be small.

Additional runs have been made to investigate hailstone sizes to be expected in unusually severe storms with updrafts approaching 50 m sec^{-1} (Dennis, 1971a).

c. *In-cloud parameters*

Given the cloud-base temperature and mixing ratio, the corresponding pseudo-adiabatic lapse rate is found using methods developed by Hirsch (1971), which give a representation of cloud temperature and mixing ratio as a function of height. Conditions are taken to be saturated with respect to water or ice as appropriate. For partly frozen clouds, the ambient mixing ratio is based on a linear interpolation between the water and ice saturation values.

The Musil 72 model involves both cloud liquid and cloud ice. After consideration of observational data in South Dakota clouds (Williamson and Miller, 1967; Hirsch and Schock, 1968) and the laboratory studies by Vali (e.g., Vali and Stansbury, 1965), we have assumed freezing of cloud water in natural clouds to begin at −20C and be completed at −40C. This happens to agree closely with the natural glaciation range of −15 to −40C selected independently by McCarthy (1972) for models of convective clouds of Missouri.

The percent of cloud water (PC) frozen in the Musil 72 model is given by

$$PC=0.008(1.274)^{T'}, \qquad (1)$$

where T' is the difference between cloud temperature and the temperature at which freezing of cloud water begins. The exponential relationship allows the freezing process to begin slowly at warmer temperatures, with the rate increasing rapidly as the temperature approaches −40C, in agreement with actual observations (e.g., Vali and Stansbury).

The temperature increase within the cloud due to the freezing of the cloud water is taken into account (Saunders, 1957). It amounts to a few degrees of warming depending upon the amount of cloud water.

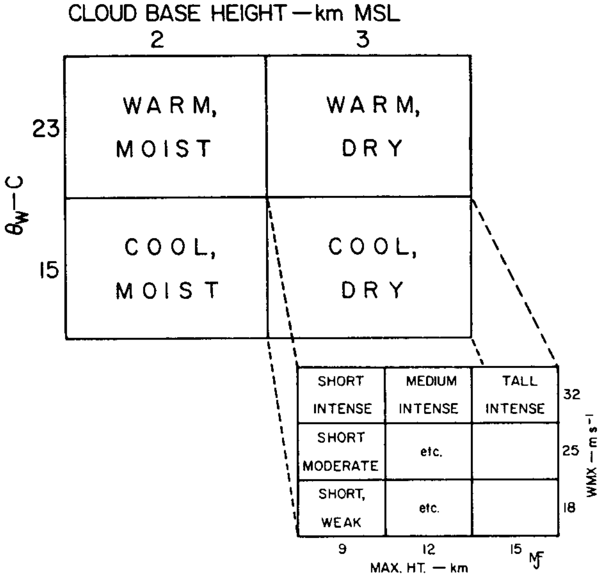

Fig. 3. The 36 basic hailstorm types used in the initial studies with the Musil 72 model.

The total water content at any level is obtained by comparing the saturation mixing ratio with that at cloud base. The product of air density and the difference in mixing ratios between cloud base and that level yields the adiabatic water content. This is arbitrarily adjusted downward early in the life of the cloud to allow for entrainment, but adiabatic values are assumed for the mature cloud. Rainwater content was assumed to be zero for the first runs of the model.

4. Model predictions

a. *Sample results*

Hailstone embryos of 100 μm diameter were introduced at the base of each specified version of the cloud model every 5 min. Fig. 4 shows as an example the results of the embryo introduced at time 20 min at the base of the cloud model with the following characteristics: base at 3 km, $\theta_w=23$C, maximum cloud height of 15 km, and maximum updraft of 32 m sec^{-1}. Initially, the embryo is wholly liquid ($F_w=1$) and its temperature (T_s) is equal to the ambient cloud temperature (T_c), which is about 10C at cloud base. The embryo grows slowly and remains close to thermal equilibrium with the environment.

As the embryo passes through the −5C level, it is assumed to freeze and F_w drops abruptly to 0. It should be noted that we have assumed the hail embryo to freeze at a higher temperature than do the cloud droplets. The timing of the freezing of the hail embryo is not critical to its subsequent growth unless it is approaching breakup size. Freezing is accompanied by

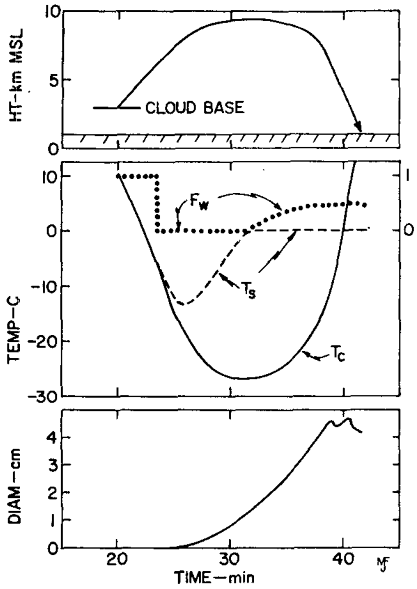

FIG. 4. Fate of a 100-μm hailstone embryo introduced at time 20 min at the base of the tall, intense cloud model with warm, dry base of Fig. 3. Hailstone location, diameter, temperature, and fractional water content are shown as a function of time. Environmental temperature (T_c) is shown for comparison.

a sudden upsurge of temperature toward 0C, but this lasts only 100 msec or so and does not appear on the figure because of the compressed time scale. From this point onward the temperature of the stone, T_s, departs from that of the cloud environment, T_c, and the departure increases as the stone accretes cloud water at an increasing rate because of its larger size and the greater liquid concentrations higher in the cloud.

The stone reaches its lowest temperature of −13C when it is about 1 mm in diameter at a height of 8.3 km. Beyond that point the stone begins to warm even though it is being carried to colder regions. At 31.5 min, T_s reaches 0C and wet growth begins, even though T_c is near −26C; F_w then increases slowly while T_s remains at 0C. The stone falls rapidly, reaching the ground at about 42 min with a diameter near 4 cm and $F_w=0.47$. This value of F_w exceeds values based on calorimetric analysis of freshly fallen hailstones (Gitlin *et al.*, 1968; Browning *et al.*, 1968) which suggests that some further adjustment of the model's breakup/shedding process would be desirable for falling stones.

b. *Effect of spongy growth*

Spongy growth could lead to larger hailstones at the ground if stones moved from a wet to a dry growth region where the trapped water could be frozen. In fact, most of the trajectories studied showed stones shifting from dry to wet growth because of their increasing size, as in the example of Fig. 4, and never going back to dry growth. Therefore, the spongy growth (Musil 72) model usually does not lead to bigger stones at the ground than does the Musil 70 model, if one considers the masses of ice only. A different result might occur if the stones were recycled in the updraft and carried very high on the second ascent. In some cases the addition of water (spongy growth) to the stone causes it to fall from the lower part of the cloud without ever rising to the updraft maximum, or freezing completely, thus leading to smaller stones at the ground than are developed by the Musil 70 model for the same embryo-cloud combination.

c. *Parameters controlling hailstone size*

The results of the various Musil 72 computer runs have been used to compare hailstone sizes at the ground with such cloud parameters as liquid water content, maximum updraft speed, and height of the freezing level. As observations of hailstones at the ground show liquid water contents to be small (e.g., Browning *et al.*, 1968; Gitlin *et al.*, 1968), diameters for this study have been computed referring *only to the ice content* of each hailstone. This adjustment suppresses random variations due to the time of a stone's impact with respect to water shedding (breakup) events. In these studies consideration has been given to all hailstones that had already reached the ground at the end of the cloud's lifetime ($T=60$ min) and all hailstones still above the ground at that time were assumed to reach it quickly due to disappearance of the updraft.

Little success has been achieved in relating computed hailstone diameter to cloud height or to cloud base conditions alone. The best results have been obtained by plotting computed hailstone diameter at the ground

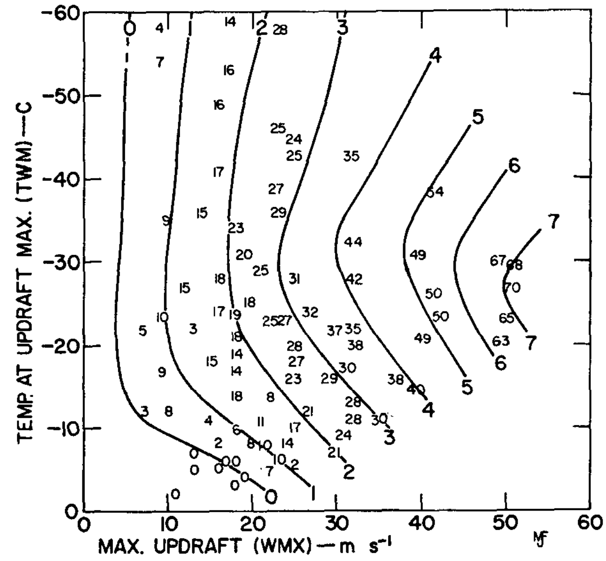

FIG. 5. Computed hailstone diameters at the ground as a function of the maximum updraft experienced and the temperature at level of maximum updraft. Diameters of solid particles at ground are plotted in millimeters for all model runs using breakup assumptions of Fig. 1. Isopleths of computed hailstone diameter are fitted by eye and labeled in centimeters.

as a function of the maximum updraft experienced by the stone (WMX) vs TWM, the cloud temperature at WMX (Fig. 5). In the one-dimensional model, the maximum updraft encountered is usually the maximum updraft in the cloud at that point in time. The hailstone diameter increases as WMX increases, but is strongly influenced by TWM, with these two parameters determining hailstone diameter very closely.

The indication from the model that hailstone diameter is controlled largely by WMX and TWM agrees with Sulakvelidze *et al.* (1967), but a comparison of Fig. 5 with their Fig. 59 shows some differences. In particular, our results indicate that a value of TWM just below 0C is not conducive to large hail at the ground. If the updraft maximum is near the freezing level, the stone descends from it as a part-water hydrometeor and is often completely melted before reaching the ground.

Our model suggests that the largest hail is favored by values of TWM near −30C. This reflects the importance of the supercooled water accreted and *frozen* during the descent from the updraft maximum. The limitation on hailstone diameter at the colder values of TWM is related to the trapping of stones in the cold region above the updraft maximum and their slow growth in the low-liquid-water environment. The hailstone diameter at the ground in such a case is often limited by the finite lifetime of the cloud.

The liquid water contents in the model clouds of Fig. 3 range between 2 and 5 gm m^{-3}. Additional runs have been made with the Musil 72 model to extend Fig. 5 to higher updraft maxima, as noted in Section 3, and to study such factors as the melting of hailstones by introducing rain in quantities from 1–3 gm m^{-3} between −20C and the ground. In special runs with excess liquid (rain) water available, embryos often accrete mass so quickly that they fall from the cloud prior to the time of its maximum development and without ever rising above the updraft maximum. In these cases, the stones usually melt prior to reaching the ground. Large stones are not greatly affected by rainwater encountered during fall, but there does appear to be an optimum amount of liquid water that will produce the largest hailstone for any given set of updraft conditions.

The conclusion that there is no great dependence of hailstone diameter upon liquid water content provided the adiabatic value is met, has been reached independently by English (1972). If substantiated, it would tend to downgrade the importance of rainwater accumulation zones, which figure prominently in some hail growth models (e.g., Douglas, 1963; Sulakvelidze *et al.*, 1967). The prediction from the Musil 72 model that higher liquid water contents in a cloud may lead in certain cases to smaller hailstones is interesting in view of the suggestion by Appleman (1959) that raindrop growth by coalescence may act to suppress hail in wet clouds. Furthermore, the shedding of excess water in wet clouds could be an important source of precipitation embryos.

The above comments are valid within the context of the model; however, they are only speculative until one establishes how well the model corresponds to nature.

5. Comparison with hailstone observations

As a first check on the validity of the hailstone model for Great Plains hailstorms, it has been used to compute a maximum hailstone diameter for each of the 33 hail cases observed during the Hailstorm Models Project, which was conducted in the Rapid City area in 1968 and 1969 (Dennis *et al.*, 1970; Dennis, 1971b). Although hailstone observations were collected by several means on the Hailstorm Models Project, most reports of maximum hailstone diameter were obtained by the mobile observer who was directed by radio into the paths of the most severe cells as identified on radar.

Although aircraft were used to probe the environments of the hail cases, no measurements of updraft speeds within the storms were obtained except for a few observations with transponder-equipped balloons. In the absence of direct observations, we have turned to cloud models as a means of estimating the updrafts in the storms. The particular model used has been developed by Hirsch (1970, 1971). The Hirsch model has been run with the afternoon soundings at Rapid City for each hail case day with an updraft radius

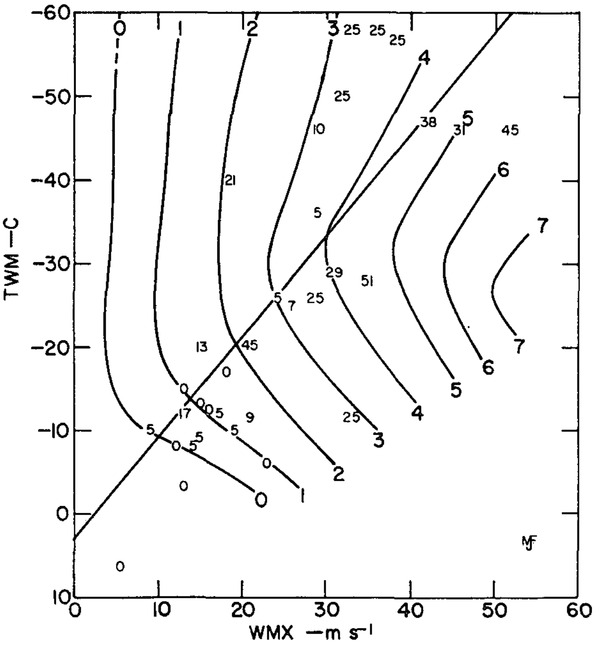

Fig. 6. Maximum observed hailstone diameters for Hailstorm Models Project cases as function of updraft maximum and temperature at level of maximum updraft indicated by Hirsch model runs. Isopleths of computed hailstone diameter based on Fig. 5 are included for comparison. The regression line is fitted to locations of plotted points; the Hirsch model predicts updraft maxima to occur at low temperatures in intense storms.

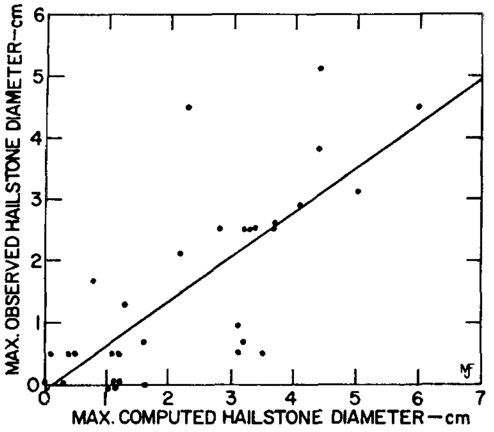

FIG. 7. Scatter diagram comparing maximum observed hailstone diameters in Hailstorm Models Project cases to those computed by use of Hirsch and Musil 72 models in tandem.

selected to provide agreement between predicted cloud top and the maximum radar echo height observed in each hail case. It yields predicted profiles of updraft speed and in-cloud temperatures, from which the maximum updraft speed and the in-cloud temperature at the level of its occurrence have been taken.

The maximum observed hailstone diameter for each hail case has been entered on a diagram of TWM vs WMX at the appropriate point indicated by the Hirsch model output for that case (Fig. 6). Fig. 6 also reproduces the predicted hailstone diameter isopleths of Fig. 5, permitting a direct comparison of observed and predicted hailstone diameters. The distribution of points is of some interest: the Hirsch model predicts that the updraft in intense storms of the northern Great Plains tends to peak at low temperature, with the correlation coefficient between TWM and WMX being −0.80. The straight line on Fig. 6 is a regression line calculated using WMX as the predictor and TWM as the predictand.

Comparison of the predicted and observed maximum hailstone diameters in the Hailstorm Models Project cases gives some reason for optimism concerning the value of both the Hirsch model and the Musil 72 model, but the predictions of hailstone size are no better than those of Foster and Bates (1956). Fig. 7 is a scatter diagram of the observed vs predicted maximum hailstone diameter for the 33 hail cases plotted on Fig. 6; the correlation coefficient between the observed and predicted diameters is 0.76. In comparison, Foster and Bates (1956), using an improved version of the empirical method of forecasting hailstone diameter originated by Fawbush and Miller (1953), computed a correlation coefficient of 0.75 for predicted vs actual hailstone diameter in a sample of 41 hail observations from U. S. Weather Bureau climatological records.

Fig. 7 suggests that the Hirsch and Musil 72 models used in tandem tend to overpredict maximum hailstone diameter. In no case was hail observed where it is not predicted by the models, but there are cases where small hail is predicted that was not observed. More important, the regression line of observed diameter upon predicted diameter falls below the 1:1 line. This could be due to inadequacies in the observing system or to an actual tendency for overprediction. Such a tendency could be due to overpredictions of WMX by the Hirsch model, but we suspect that effects not included in the Musil model are responsible. Among these may be mentioned melting of small hailstones in associated heavy rain (neglected in our standard runs) and the possibility of tilted updrafts permitting stones to fall before achieving their maximum indicated size. In application of the models, one could make an empirical correction based upon the regression line of Fig. 7 to allow for these unknown factors.

6. Numerical test of the cloud glaciation concept of hail suppression

The indication above that hailstone size at the ground is influenced considerably by growth on the final descent from the updraft maximum to the 0C level suggests a possibility for hail suppression by the artificial glaciation of the cloud or rainwater on which hailstones grow, as suggested by Howell (1966) and others. The Musil 72 model can test this *concept.* Whether or not the results would apply in nature depends upon choosing realistic glaciation ranges for both natural and modified clouds and upon the adequacy or inadequacy of the hailstone model.

We have set the glaciation range for modified clouds

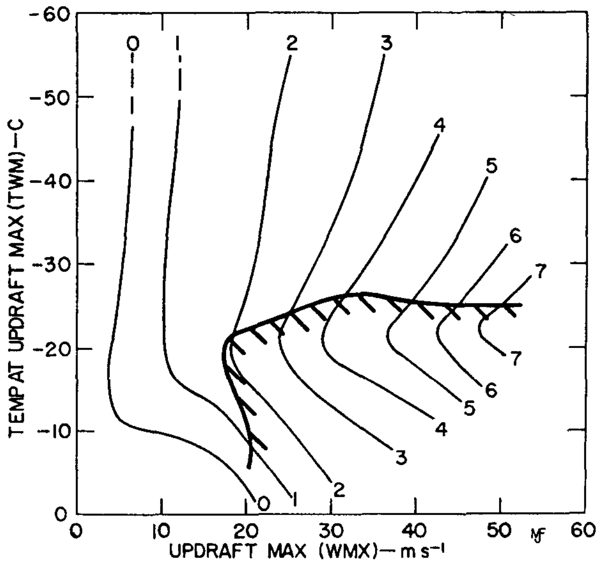

FIG. 8. Computed hailstone diameters (cm) at ground as a function of maximum updraft and temperature at level of maximum updraft for a "seeded" cloud with complete glaciation at −25C. In the region below and to right of heavy line, the model predicts that maximum hailstone diameters would be increased by artificial glaciation; in the region above and to left, the model predicts decreases in hailstone diameter.

at −5 to −25C, in line with the observation that many silver iodide smokes become active ice nucleating agents near −5C. The exponential form of the glaciation formula (1) has been retained. The model in no way tests one's ability to produce such glaciation. However, the early glaciation introduced in the model does not seem unreasonable, and should be easier to implement than such proposals as, for example, total glaciation between −4 and −8C (McCarthy, 1972). Cloud seeding for hail suppression in the northern Great Plains has sometimes provided visual evidence of heavy glaciation in the −20 to −30C range, and Williamson and Miller (1967) have reported snow and a scarcity of supercooled water at −10C in convective clouds in South Dakota following AgI seeding.

The runs described earlier for the various cloud models have all been repeated for the "seeded" models. The results can be seen by comparing Figs. 5 and 8. Hailstone diameters at the ground are smaller for the seeded than for the unseeded cloud model over a wide range of conditions, but artificial glaciation results in larger hailstones at the ground for cases with strong updrafts peaking not far above the freezing level. Here the hailstones descending in the seeded cloud model grow upon an ice-water mixture at a more rapid rate than they would if growing upon supercooled water alone, a possibility pointed out by H. K. Weickmann and B. B. Phillips in a 1968 communication to the Hail Suppression Research Steering Committee.

The model calculations to this point indicate that the validity of hail suppression by artificial cloud glaciation hinges upon whether or not the updraft maximum in intense storms occurs at sufficiently low temperatures. Some insight into the possible value of artificial cloud glaciation for hail suppression in western South Dakota can be obtained by examining Fig. 6. As noted in Section 5, there is a negative correlation between WMX and TWM (both determined from the Hirsch cloud model) for the 33 hail cases of the Hailstorm Models Project, with the correlation coefficient being −0.80. The location of the regression line of TWM vs WMX, above and to the left of the region of possible hail growth enhancement, indicates that in a majority of these cases maximum hailstone diameter at the ground would tend to be suppressed by artificial glaciation of cloud and rainwater.

The prospects of moving the threshold for cloud water glaciation above −5C do not appear good. One could test with the model the importance of compressing the glaciation range to, say, 10C rather than 20C, but this test has not yet been made. Again, one would need some assurance from actual experiments that such a change could be produced before accepting the result as real.

The hail suppression concept which is perhaps most popular at the present time is that of reducing hailstone diameter by increasing competition for the available supercooled water (Howell, 1966). Embryo competition, like artificial glaciation, would enter the Musil model through changes in the specified environmental conditions. These environmental changes cannot be calculated from the model itself. Additional studies of hail suppression concepts with the aid of various cloud models appear desirable to help determine the relative importance of the many effects that can be postulated.

7. Conclusions

The model indicates that hailstone diameter at the ground is determined principally by the strength of the maximum updraft encountered by the hailstone and the temperature at which it occurs. Therefore, the maximum updraft in a storm and its location set an upper limit on the size of hailstones which the storm can produce. High concentrations of supercooled water do little to change the maximum hailstone diameter.

According to the model, many hailstones spend some time above the level of maximum updraft in a cloud and grow significantly on their descent from that level to the 0C level. Changing the glaciation range of cloud and rainwater affects hailstone growth rates during this final descent and so influences computed hailstone diameters at the ground. If the updraft maximum is in the −10 to −20C region, computed hailstone diameters at the ground are increased by artificial freezing of supercooled cloud water. If the updraft maximum is at lower temperatures, which is more common for Great Plains hailstorms, the model indicates that maximum hailstone diameter at the ground can be reduced by as much as 10–20%.

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APPENDIX A

Development of Equations Governing Mass and Heat Budgets

1. Mass exchanges

The change in mass (M) per unit time of a growing hailstone can be expressed as the sum of the mass changes due to the accretion of supercooled cloud droplets and the collection of ice crystals. This can be written as

$$\frac{dM}{dt}=\frac{dM_w}{dt}+\frac{dM_i}{dt}, \tag{A1}$$

where the subscripts *w* and *i* refer to the water and ice masses, respectively.

If it is assumed that all intercepted water and ice remain with the growing hailstone, the change in mass per unit time due to collection of liquid droplets can be written as

$$\frac{dM_w}{dt}=\frac{\pi D^2}{4}V_tX_wE_w, \tag{A2}$$

and for collection of ice crystals by

$$\frac{dM_i}{dt}=\frac{\pi D^2}{4}V_tX_iE_i, \tag{A3}$$

where $(\pi D^2/4)V_t$ is the volume of the cylinder swept out by a hailstone falling at terminal velocity V_t, X is the concentration of the water substance encountered, and E the collection efficiency.

Terminal velocity V_t is a nonlinear function of diameter. It varies as D^2 for cloud particles, as D for intermediate sized particles, and as $D^{\frac{1}{2}}$ for hailstones. For $D<80$ μm the Stokes expression has been assumed to apply, i.e.,

$$V_t=\frac{(\rho_e-\rho_a)gD^2}{18\eta}, \tag{A4}$$

where ρ_e is the particle density, ρ_a the air density, g the gravitational acceleration, and η the dynamic viscosity of the air.

For particles outside the Stokes region, a knowledge of the drag coefficient C_D is required. Terminal velocities in this region are not amenable to analytic solution because the drag coefficient is a function of terminal velocity through its dependence on the Reynolds number, which for spheres is given by

$$\mathrm{Re}=\frac{\rho_aV_tD}{\eta}. \tag{A5}$$

The problem is overcome by using a method suggested by McDonald (1960), where the product $C_D\,\mathrm{Re}^2$ is given as a function of Re. Curves have been fitted to the McDonald relationship to find a value for Reynolds number that can be used in (A5).

It is recognized that this method may lead to errors for large raindrops because of their nonspherical shape (Foote and duToit, 1969). Similar errors may occur for nonspherical hailstones, but this is not considered critical to the model.

For diameters $\gtrsim 2$ cm, C_D has an essentially constant value of 0.6, ranging roughly between 0.45 and 0.75 (Macklin and Ludlam, 1961). With this simplifying assumption, the expression for terminal velocity becomes

$$V_t=\left(\frac{4g\rho_eD}{3\rho_aC_D}\right)^{\frac{1}{2}}. \tag{A6}$$

The collection efficiency for water droplets, E_w, is assumed to be unity for all embryos considered in the present study, as these embryos start at 100 μm diameter and grow, and Atlas (1966) considered E_w to be unity for all values of D above 90 μm.

The collection efficiency for ice particles, E_i, is set at unity for wet hailstones. For dry growth, E_i takes on a value of 0.25 (Saunders, 1968). This may be high and, if so, our results overstate the growth rates of dry hailstones in environments containing large amounts of ice.

2. Heat exchanges

For convenience, hailstone heat contents are expressed with respect to a dry hailstone at 0C.

The heat content of a wet hailstone is

$$MF_wL_f, \tag{A7}$$

where M is its total mass, F_w the fraction (by mass) of water in the stone, and L_f the latent heat of fusion. By differentiating,

$$\frac{d}{dt}(MF_wL_f)=\frac{dQ_T}{dt}, \tag{A8}$$

where dQ_T/dt is the rate at which heat is added to the stone from all sources. From (A8),

$$\frac{dF_w}{dt}=-\frac{F_w}{M}\frac{dM}{dt}+\frac{1}{ML_f}\frac{dQ_T}{dt}, \tag{A9}$$

where the first term on the right is related to the increase in heat content required to maintain the existing water-ice ratio as the stone grows.

The heat content of a dry hailstone is

$$MC_iT_s, \tag{A10}$$

where C_i is the specific heat of ice and T_s is its temperature (°C), always negative. In this case

$$\frac{d}{dt}(MC_iT_s)=\frac{dQ_T}{dt}, \tag{A11}$$

so that

$$\frac{dT_s}{dt}=-\frac{T_s}{M}\frac{dM}{dt}+\frac{1}{MC_i}\frac{dQ_T}{dt}. \tag{A12}$$

For both wet and dry hailstones, the total heat exchange is made up of four terms, i.e.,

$$\frac{dQ_T}{dt}=\frac{dQ_k}{dt}+\frac{dQ_s}{dt}+\frac{dQ_a}{dt}+\frac{dQ_c}{dt}, \tag{A13}$$

where Q_k is a conduction term; Q_s a sublimation, condensation or evaporation term as appropriate; and

Q_a and Q_c account for the heat contained in the accreted water and the collected cloud ice, respectively.

Heat exchanges due to conduction are given by

$$\frac{dQ_k}{dt}=2\pi DaK(T-T_s), \qquad \text{(A14)}$$

where D is the diameter of the stone, a the ventilation coefficient, K the thermal conductivity, and T used here for the ambient temperature. Expressions for the ventilation coefficient and thermal conductivity have been discussed previously by Musil (1970) and will not be reproduced here.

Heat exchanges due to sublimation or evaporation/condensation are given by

$$\frac{dQ_s}{dt}=-2\pi DaL_sD_i\Delta\rho_w, \qquad \text{(A15)}$$

where L_s, the latent heat of sublimation, is used in the dry growth regime and L_v, the latent heat of vaporization, is used in the wet growth regime; D_i is the diffusivity and can be written as

$$D_i=D_0(T/T_0)^n\left(\frac{p_0}{p}\right), \qquad \text{(A16)}$$

where p is pressure in millibars and the subscript 0 denotes conditions at some reference point, in this case taken to be 1000 mb and 273.2K. The exponent n is an empirical constant given as 1.81.

The last variable in (A15), $\Delta\rho_w$, is the difference in vapor density between the hailstone surface and the cloud air. The vapor density in the cloud will assume a value ranging between saturation with respect to water and saturation with respect to ice, depending on cloud composition. We have assumed that ρ_w varies linearly between these values as a function of the amount of cloud water substance frozen. The vapor density at the hailstone surface is taken as saturation with respect to ice in dry growth and as saturation with respect to water in the wet growth regime.

The heat added by accretion of cloud droplets or raindrops is given by

$$\frac{dQ_a}{dt}=\frac{dM_w}{dt}(L_f+C_wT), \qquad \text{(A17)}$$

where L_f is the latent heat of fusion at 0C and C_w the specific heat of water. The sensible heat term (C_wT) is negative for supercooled water, but accretion always results in heat being added to the hailstone because the latent heat term is larger.

Heat added by the collection of ice crystals can be written as

$$\frac{dQ_c}{dt}=\frac{dM_i}{dt}C_iT. \qquad \text{(A18)}$$

This term is always negative.

Combining the various heat transfer terms and substituting in (A9), and noting that $T_s=0$ for wet stones, we obtain

$$\frac{dF_w}{dt}=-\frac{F_w}{M}\frac{dM}{dt}+\frac{1}{ML_f}\left[2\pi Da(KT-L_sD_i\Delta\rho_w)+\frac{dM_w}{dt}(L_f+C_wT)+\frac{dM_i}{dt}C_iT\right]$$

for changes in the liquid fraction (F_w) of wet hailstones.

Substitution of the heat transfer terms into (A12) leads to

$$\begin{aligned}\frac{dT_s}{dt}&=-\frac{T_s}{M}\frac{dM}{dt}+\frac{1}{MC_i}\left[2\pi Da(KT-KT_s-L_sD_i\Delta\rho_w)+\frac{dM_w}{dt}(L_f+C_wT)+\frac{dM_i}{dt}C_iT\right]\\&=\left(\frac{T-T_s}{M}\right)\frac{dM}{dt}+\frac{L_f+(C_w-C_i)T}{MC_i}\frac{dM_w}{dt}\\&\quad+\frac{2\pi Da}{MC_i}(KT-KT_s-L_sD_i\Delta\rho_w) \qquad \text{(A20)}\end{aligned}$$

for the rate of temperature change for dry hailstones. Consideration of the final form of (A20) for the case where $dM_w/dt=0$ shows that collection of ice crystals does not change T_s if T_s is equal to the environmental temperature T.

APPENDIX B

Generation of Updraft Profiles

We have found it possible to simulate a wide range of realistic time-dependent vertical velocity profiles through the specification of five parameters (Table B1). These parameters describe the first two terms of a Fourier series defined along the vertical axis of the storm between the ground and the rising cloud top.

The height of cloud top *above ground*, Z, is assumed to rise as the first quarter of a sine wave during the time interval between $t=0$ and $t=\tau$, reaching the

TABLE B1. Cloud parameters for specifying updraft characteristics

Parameter	Symbol
Cloud lifetime	τ
Maximum cloud height above ground†	HMX
Precipitation load start time, the time of onset for the second harmonic	τ_2
Peak amplitude of first harmonic	PAMP1
Peak amplitude of second harmonic	PAMP2

† As we are considering ground elevation of 1 km MSL, HMX for the 15-km clouds of Fig. 3 is 14 km, and so on.

maximum height at the end of the cloud's lifetime. That is,

$$H=\mathrm{HMX}\sin(\pi t/2\tau). \tag{B1}$$

The amplitude of the first harmonic at any given time is given by

$$\mathrm{AMP1}=\mathrm{PAMP1}\sin(\pi t/\tau). \tag{B2}$$

The amplitude of the second harmonic at any time is defined by

$$\left.\begin{array}{ll}\mathrm{AMP2}=0, & t<\tau_2\\[1ex] \mathrm{AMP2}=\mathrm{PAMP2}\sin\dfrac{\pi(t-\tau_2)}{\tau-\tau_2}, & t>\tau_2\end{array}\right\}. \tag{B3}$$

Both AMP1 and AMP2 complete the first half of a sine wave and return to zero at $t=\tau$.

Finally, the updraft W at any height Z is obtained from

$$W=\mathrm{AMP1}\sin(\pi Z/H)+\mathrm{AMP2}\sin(2\pi Z/H). \tag{B4}$$

The peak updraft generated by (B4) usually occurs sometime after $\tau/2$ and somewhat above the midpoint between cloud base and cloud top. As noted above a sample of the updraft profiles generated by this method is shown in Fig. 2.

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