Thunderstorm Electrification by the Inductive Charging Mechanism: I. Particle Charges and Electric Fields

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(Manuscript received 3 July 1972, in revised form 9 April 1973)

ABSTRACT

Expressions are derived for ice particle charges acquired through successive collisions in a changing electric field. Our computations give lower particle charges, slower maximum field growth rates, and higher maximum fields than reported in recent literature. The minimum ice particle sizes and concentrations required for efficient electrification appear to lie well within the range observed in thunderstorms. If the mean radius of the large ice particles (graupel, hail, etc.) exceeds 0.2 cm, fields higher than 5000 V cm^{-1} can be generated.

1. Introduction

Charge separation between colliding and rebounding cloud particles which are polarized by a vertical electric field (commonly referred to as inductive charging) is one of a number of charging mechanisms that have been proposed to explain thunderstorm electrification.

Although conceptually this mechanism is simple, many problems arise when attempts are made to evaluate it quantitatively. Some of the problems are due to lack of quantitative data on the physical process of charge transfer (see Paluch and Sartor, 1973, Section 2), others are mathematical. We shall here discuss the latter.

Since the amount of charge separated in a collision is a function of the particle sizes, the local electric field strength (which depends on the spatial distribution of all other charged particles), the particle previous charges (which depend on the amount of charge transferred in previous collisions), and the particle fall velocities (which depend on particle mass, charge and the local electric field strength), the problem of computing the electric field growth is complex.

Various simplifications have been used for approximating particle charges acquired through successive collisions in a changing electric field. Sartor (1967) computes the amount of charge transferred in a collision assuming the terms involving particle previous charges are negligible; whereas Latham and Mason (1962), Mason (1972), Kamra (1970) and Kamra and Vonnegut (1971) compute charges assuming they depend on the existing (but not the previous) electric field strength. We shall here compute the particle charges without making the above assumptions.

[1] The National Center for Atmospheric Research is sponsored by the National Science Foundation.

To simplify our computations we shall, however, use the infinite parallel plate approximation as found in previous works. Although this simplification overestimates the electric field growth rate, it can give some information about the minimum particle sizes and concentrations required for efficient electrification and the equilibrium fields that can be produced.

Our computations are intended to apply to charge separation in ice particle collisions that rebound, and in collisions between ice particles and supercooled droplets where part of the droplet mass splashes off the bottom surface of the ice particle [see Scott and Levin (1970) and Aufdermaur and Johnson (1972)]. We do not consider collisions between liquid drops, since as discussed elsewhere (Paluch and Sartor, 1973) these are not likely to be effective in separating charge in a systematic way.

2. Charges carried by particles undergoing successive collisions in a growing electric field

A general theory of charge distribution on the surfaces of two charged conducting spheres in an external electric field has been developed by Davis (1962, 1964).

The electric field on the near surface of the smaller of the two spheres is given by Davis (1964) as

$$E_a=\frac{1}{\epsilon R_i^2}(E_1Q_{j0}+E_2Q_{i0}+E_3E\cos\theta),$$

where:

- ϵ specific inductive capacity (in esu, $\epsilon=1$)
- R_i radius of small sphere
- E electric field strength (esu), positive downward
- θ collision contact angle, measured with respect to the positive direction of the electric field

Q_{j0} the charge before collision on the larger sphere
Q_{i0} the charge before collision on the smaller sphere.

The terms E_1, E_2, E_3 are nondimensional constants for each position and size combination of two spheres. They are given in Legendre polynomial form for the general situation and in tabular form for selected sphere size ratios and separations by Davis (1964). When the two spheres are moved together until they are an infinitesimal distance apart and the separation is bridged by an infinitesimally small conductor so that $E_a=0$, the sum of the charges becomes $Q_{j1}+Q_{i1}$, the same as the sum before, $Q_{j0}+Q_{i0}$. Thus, using the above equation with $E_a=0$ and

$$Q_{j1}+Q_{i1}=Q_{j0}+Q_{i0},$$

we find the charge transferred, in the positive direction of the field, to be

$$\Delta q=\frac{1}{E_2-E_1}(E_3ER_i^2\cos\theta-E_1Q_{j0}-E_2Q_{i0}).$$

The special case of the two spheres touching is not treated by Davis (1964) explicitly. This case has been solved by Gordon (Latham and Mason, 1962) who gives the following expression for the amount of charge transferred when a charged and an uncharged sphere come in contact:

$$\Delta q=\gamma_1R_i^2E\cos\theta+\frac{\gamma_2A^2}{1+\gamma_2A^2}Q_{j0}-\frac{1}{1+\gamma_2A_2}Q_{i0},$$

where R_j is the radius of the large sphere (cm), $A=R_i/R_j$, and γ_1 and γ_2 are positive nondimensional coefficients that are weak functions of A:

$$\gamma_1=\frac{E_3}{E_2-E_1}\quad\text{and}\quad\gamma_2=-\frac{E_1}{E_2A^2}.$$

The above equation is valid for the static case or when the sphere contact time is much larger than the electrical relaxation time. Extended to the nonstatic case where, due to a short contact time, only some fraction of charge is transferred, we can express the charges carried by the two spheres after a collision in the form

$$Q_j=Q_{j0}-K\Delta q=-\gamma_1KR_i^2E\cos\theta+\left(1-\frac{K\gamma_2A^2}{1+\gamma_2A^2}\right)Q_{j0}+\frac{K}{1+\gamma_2A^2}Q_{i0},\qquad(1)$$

$$Q_i=Q_{i0}+K\Delta q=\gamma_1KR_i^2E\cos\theta+\left(1-\frac{K}{1+\gamma_2A^2}\right)Q_{i0}+\frac{K\gamma_2A^2}{1+\gamma_2A^2}Q_{j0},\qquad(2)$$

where K is the charge transfer efficiency, $1-\exp(-t_2/t_1)$, where t_2 is the contact time and t_1 the electrical relaxation time.

In typical thunderstorm clouds the number of small ice particles or droplets greatly exceeds the number of large graupel or hailstones (Jones, 1960), and the latter undergo many collisions during the time of the electric field growth. Consider a simplified cloud that contains 0.5 cm^{-3} small spherical ice crystals 50 μm in radius, and 10^{-4} cm^{-3} graupel particles 2 mm in radius with a terminal velocity of 8 m sec^{-1}. A graupel particle will, on the average, collide with an ice crystal every 0.02 sec, whereas an ice crystal will, on the average, collide with a graupel particle every 100 sec (assuming the collision efficiency is near 1). If instead of ice crystals the cloud contains 500 cm^{-3} supercooled droplets 10 μm in radius and if the separation probability is 0.005, then the average time between collisions without complete coalescence for the graupel particles is about 0.004 sec. Thus it can be seen that if the electric field growth time is on the order of minutes, successive collisions must be considered in computing charges on graupel or hailstones but that it is questionable whether successive collisions are significant in the case of small ice crystals.

To make the problem of computing cloud particle charges due to successive collisions more manageable, we shall assume that the small ice crystals that are carried along with the updraft in the charging region have a sufficiently narrow size spectrum and such small terminal velocities that the amount of charge separated in collisions among themselves is insignificant compared to the charge separated in collisions between large ice particles and ice crystals or supercooled droplets. We shall also suppose that the large particles falling against the updraft are so few that the number of collisions among them are insignificant and do not contribute to the charge separation process. We shall then treat the interactions between large ice particles and ice crystals or droplets, assuming that these particles can be represented by some constant average size and number density.

To further simplify the problem, we shall assume that all particles are spherical, that charge transfer efficiency K remains constant, and approximate the cosine of the collision contact angle by some average value. If particle trajectories are straight lines parallel to the electric field, $\overline{\cos\theta}=2/3$. However, because of some outward bending of the trajectories due to flow field interactions, the value of $\overline{\cos\theta}$ can be expected to be somewhat less. From here on the symbol E refers to the vertical component of the electric field strength.

If a large particle undergoes successive collisions with smaller neutral particles in a changing field, then after successive use of (1) the resulting charge can be expressed as

$$Q_j=-\gamma_1KR_i^2E_n\overline{\cos\theta}\left[1+\left(1-\frac{K\gamma_2A^2}{1+\gamma_2A^2}\right)\frac{E_{n-1}}{E_n}+\left(1-\frac{K\gamma_2A^2}{1+\gamma_2A^2}\right)^2\frac{E_{n-2}}{E_n}+\cdots+\left(1-\frac{K\gamma_2A^2}{1+\gamma_2A^2}\right)^{n-1}\frac{E_1}{E_n}\right],$$

where E_m is the field strength at the time of the collision.

To evaluate the above expression we must know how the electric field has varied with time. Since observations show that the electric field, at least initially, appears to exhibit exponential growth, we shall approximate the field over some time interval by

$$E(T)=E(0)\exp(T/\eta), \tag{3}$$

where T is the time and η some unspecified time constant (i.e., the time it takes for the field to increase by a factor of 2.72); we shall determine its value later [see (8)]. Let us express the time as $T=n\Delta t$, where n is the number of collisions and Δt the average time interval between collisions without coalescence for the large sphere, i.e.,

$$\Delta t=[\pi N_i R_j^2 \epsilon P(V_j(T)-V_i)]^{-1},$$

where:

- ϵ collision efficiency normalized to the radius of the large sphere
- P separation probability
- N_i number of small neutral spheres (cm^{-3})
- V_i fall velocity of small neutral spheres
- $V_j(T)$ fall velocity of large spheres at time T.

At the time of the nth collision the electric field can be written as

$$E(n\Delta t)=E_n=E_0\exp\left(\frac{n\Delta t}{\eta}\right).$$

The charge on the large sphere after nth collision can now be expressed as

$$Q_j=-\gamma_1 K R_i^2 E_n \overline{\cos\theta}\frac{\left[1-\left(1-\frac{K\gamma_2 A^2}{1+\gamma_2 A^2}\right)^n \exp\left(\frac{T}{\eta}\right)\right]}{\left[1-\left(1-\frac{K\gamma_2 A^2}{1+\gamma_2 A^2}\right)\exp\left(\frac{\Delta t}{\eta}\right)\right]}.$$

Generally, for rapidly growing thunderstorm fields $\Delta t/\eta \ll 1$, so that $\exp(-\Delta t/\eta)\approx 1-\Delta t/\eta$. For $A^2\ll 1$, $\gamma_1/\gamma_2=3.0$, and

$$\left(1-\frac{K\gamma_a A^2}{1+\gamma_2 A^2}\right)^n \approx \exp(-n\gamma_2 K A^2)$$
$$=\exp(-\gamma_2 K A^2 T/\Delta t)\equiv\exp(-T/\tau).$$

[The time constant $\tau\equiv\Delta t/(\gamma_2 K A^2)$ can be physically interpreted as being the relaxation time for charging of the large particles; when $K=1$ it is the same as the τ used by Latham and Mason (1962), and by Mason (1972).] After making the above substitutions and rearranging terms, we get

$$Q_j(T)=3R_j^2 E(T)\,\overline{\cos\theta}\frac{\eta}{\eta+\tau}[1-\exp(-T/\eta-T/\tau)], \quad \text{for } A^2\ll 1. \tag{4}$$

TABLE 1. Values of τ and η computed from (8) for different particle sizes and concentrations. In all cases $\lambda=10^{-4}$ sec^{-1}, $K=\epsilon=P=1$, $\overline{\cos\theta}=0.5$, and M_j is the total large ice particle content.

R_j (cm)	R_i (cm)	N_j (cm^{-3})	N_i (cm^{-3})	M_j (gm m^{-3})	τ (sec)	η (sec)
0.1	5×10^{-3}	1.25×10^{-4}	0.5	0.314	28.6	116.0
0.5	5×10^{-3}	5×10^{-6}	0.5	1.57	11.4	41.8
0.5	5×10^{-3}	1×10^{-4}	0.5	31.4	11.4	5.08
0.3	5×10^{-3}	1×10^{-4}	0.5	6.78	14.2	11.9
0.2	5×10^{-3}	1×10^{-4}	0.5	2.0	17.9	26.7
0.2	5×10^{-3}	1×10^{-5}	0.5	0.2	18.1	213.0
0.2	2.5×10^{-3}	1×10^{-5}	0.1	0.2	358.0	435.0

It can be seen that in a constant field after a long time (when $\eta=\infty$ and $T/\tau\rightarrow\infty$) the charge on the large sphere will approach an equilibrium value Q_j^*:

$$Q_j^*=-3R_j^2 E\,\overline{\cos\theta}.$$

In the extreme case, when the electric field growth rate is very slow compared to the large particle charging rate so that $\eta\gg\tau$, (4) reduces to

$$Q(T)_j=3R_j^2 E(T)\,\overline{\cos\theta}[1-\exp(-T/\tau)].$$

The above equation is the same as the equation for large particle charge derived by Latham and Mason (1962). In their derivation it is implicitly assumed that the rate of change of the electric field is negligible.

In the opposite extreme case, when the field growth rate is very rapid compared to the particle charging rate so that $\tau\gg\eta$, (4) reduces to

$$Q(T)_j=3R_j^2 E(T)\,\overline{\cos\theta}(\eta/\tau)[1-\exp(-T/\eta)].$$

This equation gives the same expression for particle charge density in an exponentially growing field as that derived by Sartor (1967); to simplify his computations he neglected terms involving previous particle charges in the charge transfer equation.

Table 1 shows some values of τ and η for what may be "typical" cloud particle sizes and concentrations (computation of η is explained in Section 4). It can be seen that, depending on cloud conditions, τ and η can vary over a considerable range of values and that neither time constant is consistently larger or smaller than the other. Thus, both Sartor's (1967) and Latham and Mason's (1962) equations will generally tend to result in an overestimate of the large particle charges. This, in turn, will result in an overestimate of the electric field growth rates and, if changes in particle terminal velocities are not neglected, also will result in an underestimate of the maximum field strength that can be produced.

Eq. (4) can be used to compute the average charge transferred to an initially neutral small sphere in a collision at time T:

$$Q(T)_i=-\frac{dQ(T)_j}{dt}\Delta t$$

$$=\gamma_1 K R_i^2 E(T)\,\overline{\cos\theta}\frac{\tau}{\tau+\eta}\left[1+\frac{\eta}{\tau}\exp\left(-\frac{T}{\tau}-\frac{T}{\eta}\right)\right]. \quad (5)$$

It should be noted that $Q(T)_i$ is the charge the small particle acquires during a collision at time T in field $E(T)$. At any time after this collision the small sphere will carry that same charge unless it undergoes another collision.

Let us now consider a collision in which the small sphere is not initially neutral. If the initial charge on the small sphere was acquired in a collision at some earlier time T', then the charge at time T can be obtained by substituting $Q_{i0}=Q(T')_i$ from (5) and $Q_{j0}=Q(T)_j$ from (4) into (2); we then have

$$Q(T)_i=\gamma_1 K R_i^2 E(T)\,\overline{\cos\theta}\frac{\tau}{\eta+\tau}\left\{1+\frac{\eta}{\tau}\exp\left(-\frac{T}{\tau}-\frac{T}{\eta}\right)\right.$$
$$\left.+\frac{E(T')}{E(T)}\left(\frac{1}{K}-\frac{1}{1+\gamma_2 A^2}\right)\left[1+\frac{\eta}{\tau}\exp\left(-\frac{T'}{\tau}-\frac{T'}{\eta}\right)\right]\right\}.$$

It can be seen that the above equation differs from (5) in that it has an additional term; this term becomes negligible when $A^2\ll 1$ and the charge transfer efficiency $K\approx 1$, or when $E(T')\ll E(T)$.

In our computations all second and subsequent collisions of the small particles will be neglected. As long as $K\approx 1$, no significant errors are expected, since in all cases considered $A^2\ll 1$. Furthermore, since the time between collisions for the small particles is on the order of 100 sec, we expect that for fairly rapid field growth rates the factor $E(T')/E(T)$ will also be small. However, when K is small and the growth rate of the electric field is slow, the computations will tend to underestimate the charge on the small particles and to overestimate the charges on larger particles.

3. Maximum growth rate of the electric field

Because of the interdependence of particle charges and the local electric field strength, the problem of computing the electric field growth due to the inductive charging mechanism is formidable. We can, however, relatively easily obtain an upper limit for the field growth rates using the infinite parallel plate approximation as found in the works by Sartor (1961, 1967), Latham and Mason (1962), Kamra (1970) and Mason (1972).

According to the parallel plate approximation, the growth rate of the electric field can be expressed as

$$\frac{d}{dt}E(T)=-4\pi[\rho(T)_j V(T)_j+\rho(T)_i V(T)_i]+4\pi\lambda[E(T)-E_0],$$

where $\rho(T)_j$ is the charge density of the large particles, $\rho(T)_i$ the charge density of the small particles, E_0 some constant superimposed field (such as the fair weather field), and $4\pi\lambda[E(T)-E_0]$ the discharge current density[2] due to conduction and other leakage currents, all of which are assumed to be proportional to the field strength due to the charged cloud particles.

For the large particles $\rho(T)_j V(T)_j=N_j Q(T)_j V(T)_j$, where N_j is their number density. Since the average time between collisions for the small particles is of the same order of magnitude as the time it takes the field to grow, we expect that generally only a fraction of the small particles will carry charge and that their charge at any time T will not depend on the field strength $E(T)$, but rather will be a function of some previous field strength $E(t)$ where t is the time when charge has been transferred. To find the total charge flux due to the small particles, it is necessary to integrate the particle charges, their number density, and their fall velocity over the time during which collisions have taken place. Thus the equation for the rate of change of the electric field is

$$\frac{d}{dt}E(T)=-4\pi\left\{N_j Q(T)_j V(T)_j+\int_0^T-\left[\frac{d}{dt}N(t)_i\right]Q(t)_i V(t,T)_i dt+\lambda[E(T)-E_0]\right\}, \quad (6)$$

where the collision rate at time t is

$$-\frac{d}{dt}N(t)_i=N_j/\Delta t,$$

and where $V(t,T)_i$ is the terminal velocity of the small particles carrying charges $Q(t)_i$ in the presence of a field $E(T)$. Thus, the total force acting on the particle is $m_i g+Q(t)_i E(T)$.

The electric field grows at a maximum rate when T is sufficiently large so that

$$\exp\left(-\frac{T}{\eta}-\frac{T}{\tau}\right)$$

is negligible (typically this will occur when T is larger than 10 or 100 sec) and before the electrical forces get strong enough to affect particle terminal velocities.

[2] Anderson and Freier (1971) have pointed out that the discharge current density term should contain a factor of 4π. In most previous works this factor has been omitted. The error originated in an early paper by Sartor (1961) and has been carried through his later work as well as in work by Latham and Mason (1962) and Kamra (1970).

Under these conditions (6) reduces to

$$\frac{d}{dt}E(T)=4\pi\left\{3N_jR_j^2E(T)\,\overline{\cos\theta}(V_j-V_i)\frac{\eta}{\eta+\tau}-\lambda[E(T)-E_0]\right\}. \quad (7)$$

When $E(T)\gg E_0$, then after substituting

$$\frac{1}{E}\frac{dE}{dt}=\frac{1}{\eta},$$

Eq. (7) can then be solved for η to give

$$\eta=\frac{(1+4\pi\lambda\tau)+\{(1+4\pi\lambda\tau)^2+16\pi\tau[3N_jR_j^2\,\overline{\cos\theta}(V_j-V_i)-\lambda]\}^{\frac{1}{2}}}{8\pi[3N_jR_j^2\,\overline{\cos\theta}(V_j-V_i)-\lambda]}. \quad (8)$$

4. Computations of the minimum cloud particle sizes and concentrations that are required for efficient electric field growth and the equilibrium fields that can be produced

Although the infinite parallel plate approximation gives only an upper limit for the electric field growth rate, it can provide some information on the minimum cloud particle sizes and concentrations that are required for efficient electric field growth.

In all computations that follow, ice particle density $\rho=0.6$ gm cm^{-3}, particle terminal velocities are calculated using McDonald's (1960) drag table, and $\overline{\cos\theta}=0.5$ if not specified otherwise. Since the value for discharge current density due to cloud conductivity and other leakage currents is uncertain, we have assumed values for λ ranging from 10^{-4} to 2×10^{-3} sec^{-1}.

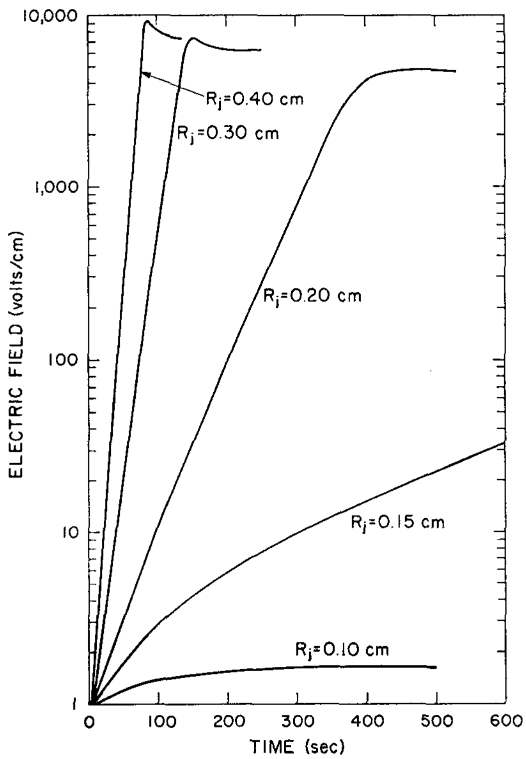

FIG. 1. Electric field strength vs time for different large ice particle radii for case 2 (Table 2).

Typical curves of electric field strength vs time can be seen in Fig. 1. The electric field growth was computed by numerical integration of Eq. (6). As long as the τ/η changes slowly, particle charges were computed for Eqs. (4) and (5) with values for η and τ computed in the preceding time step. This procedure could not be used when the field just starts to grow or when the field growth rate approaches zero, since there the values for τ/η change rapidly; at these times we had to resort to direct collision-after-collision computations using Eqs. (1) and (2). Our curves show that most of the time, except in the very beginning and when fields are close to their equilibrium values, the field grows at a constant exponential rate, whose time constant η is the same as given by (8). When the field becomes so strong that the electrical forces on the particles are no longer insignificant compared to the gravitational forces, the field growth rate decreases because the relative velocities between the particles decrease. As the growth of the field ceases, the charges on the large particles tend to approach their equilibrium values, while at the same time leakage currents decrease the existing field. This continues until an equilibrium field is reached in which the large particles carry their equilibrium charge and the gravitational charge separation just counterbalances the leakage currents. For $R_j=0.3$ cm and 0.4 cm we see that the maximum field exceeds the equilibrium field. This is because in these cases the initial growth rates are high and $\eta/(\eta+\tau)$ is small so that the charges on the large particles lag considerably behind their equilibrium charge. Here, gravitational charge separation becomes inefficient at a higher field strength than it would have if the charges on the large particles were near their equilibrium values.

Since we find that the time constant η as given by (8) is fairly representative of the field growth rate during most of the time the field is growing, we have used (8) to compute values for η for various cloud conditions. These are plotted in Figs. 2 and 3. The cases in Fig. 2 deal with collisions between ice crystals and large ice particles. The curves show that for the larger ice particle concentrations in the range of 10^{-5} to 10^{-4} cm^{-3}, rapid electrification can take place only if the particles have reached sizes of several millimeters in radius. It is interesting to note that relatively fast field growth rates

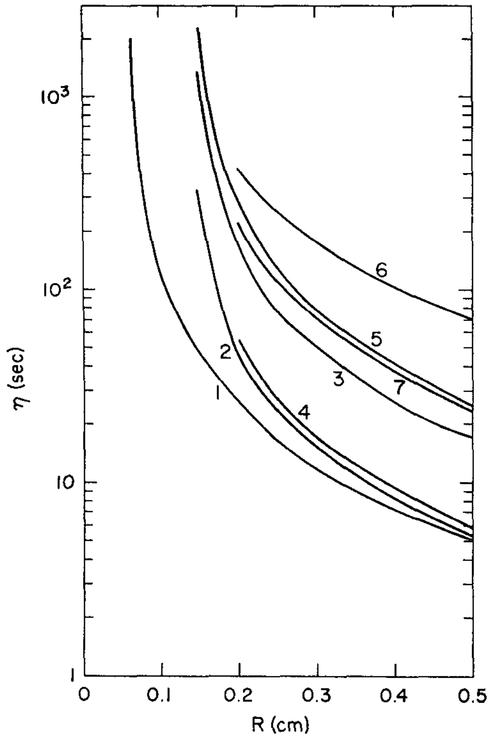

FIG. 2. Minimum time required for the field to increase by a factor of 2.72 as a function of large ice particle radii for collisions between ice crystals and large ice particles, as listed in Table 2.

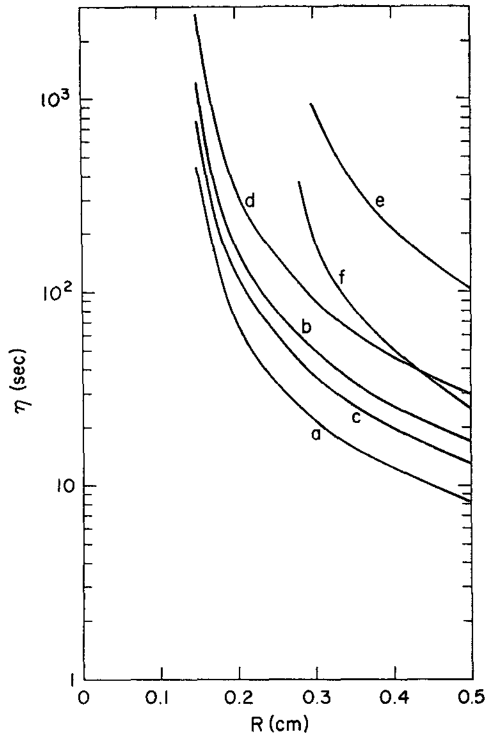

FIG. 3. As in Fig. 2 except for collisions between supercooled droplets and large ice particles, as listed in Table 3.

can be obtained with a charge transfer efficiency as low as 0.1 (case 3, Fig. 2). The cases in Fig. 3 are for collisions between supercooled droplets and large ice particles. It can be seen that here again large particle sizes and high concentrations are needed for rapid electric field growth. For curves e and f we have assumed that the separation probability is higher for near-grazing collisions than for head on collisions so that $\cos\theta = 0.1$. These cases give considerably lower values for the electric field growth rates.

Fig. 4 shows the time required for the field to increase by one order of magnitude plotted versus large ice particle content (gm m^{-3}). It can be seen that for rapid electric field growth the total weight of the large ice particles must be at least several grams per cubic meter. It is interesting to note that at higher large particle contents the exponential field growth rates become insensitive to the large particle sizes; thus, as long as the total large ice particle content is high, detailed knowledge of the particle size spectrum is not essential for estimating the maximum electric field growth rate.

Fig. 5 shows equilibrium electric fields that can be produced under different cloud conditions. Curves 1, 2 and 6 are the ice-ice collisions with $\overline{\cos\theta} = 0.5$, and curve f is for supercooled droplet-ice particle collisions with $\overline{\cos\theta} = 0.1$. Curves similar to 1, 2 and 6 were obtained for ice particle-supercooled droplet collisions when $\overline{\cos\theta} = 0.5$ (however, these curves are not included in Fig. 5). It can be seen that $\overline{\cos\theta}$ and the large ice particle sizes are of primary importance in determining the strength of the equilibrium fields, and that factors such as charge loss due to leakage currents, size and

TABLE 2. A list of constants specifying cloud conditions for collisions between ice crystals and large ice particles for curves in Figs. 1, 2, 4 and 5. In all cases $\overline{\cos\theta} = 0.5$.

Case	N_j (cm^{-3})	$N_i P_\epsilon$ (cm^{-3})	R_i (cm)	λ (sec^{-1})	K
1	10^{-4}	0.5	0.0050	10^{-4}	1.0
2	10^{-4}	0.5	0.0050	2×10^{-3}	1.0
3	10^{-4}	0.5	0.0050	2×10^{-3}	0.1
4	10^{-4}	0.1	0.0100	2×10^{-3}	1.0
5	10^{-4}	0.1	0.0025	2×10^{-3}	1.0
6	10^{-6}	0.1	0.0025	10^{-4}	1.0
7	10^{-5}	0.5	0.0050	10^{-4}	1.0

TABLE 3. A list of constants specifying cloud conditions for collisions between supercooled droplets and large ice particles for curves in Figs. 3 and 5. In all cases $K=1$.

Case	N_j (cm^{-3})	$N_i P_\epsilon$ (cm^{-3})	R_i (cm)	λ (sec^{-1})	$\overline{\cos\theta}$
a	10^{-4}	5.0	10×10^{-4}	2×10^{-3}	0.5
b	10^{-4}	5.0	5×10^{-4}	2×10^{-3}	0.5
c	10^{-4}	0.5	20×10^{-4}	2×10^{-3}	0.5
d	10^{-4}	0.5	10×10^{-4}	2×10^{-3}	0.5
e	10^{-4}	0.5	10×10^{-4}	2×10^{-3}	0.1
f	10^{-4}	5.0	10×10^{-4}	2×10^{-3}	0.1

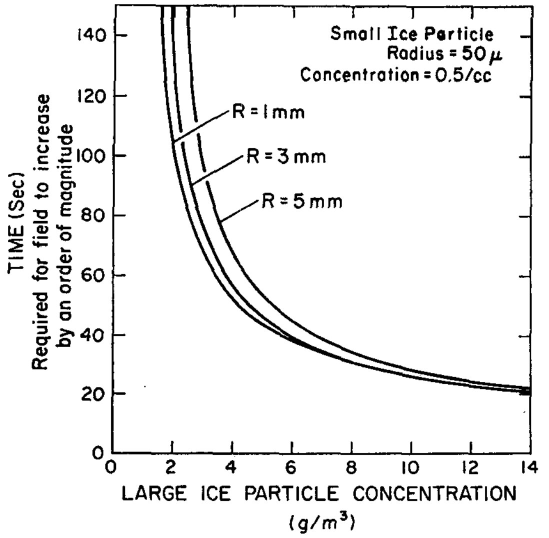

FIG. 4. Minimum time required for the field to increase by a factor of 10 as a function of total large ice particle content for different particle radii; other conditions are as in case 2.

concentration of the smaller particles, or concentration of the large particles affect it to a considerably lesser extent.

It should be noted that the obtained equilibrium fields depend on the parallel plate approximation only to the extent that it affects the positive charge distribution among the small cloud particles. If we had considered a finite charge distribution, the field would be growing at a slower rate, the positive charge would be distributed among more cloud particles, and thus each particle would carry less charge. Consequently, higher field strength would be required to affect the small positively charged particle terminal velocities and the resulting equilibrium fields would be slightly higher.

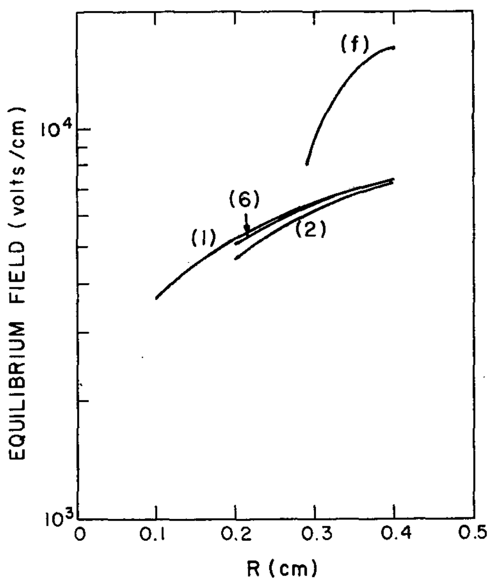

FIG. 5. Equilibrium fields vs large ice particle radii. Curves (1), (2), (6) and (f) correspond to cases listed in Tables 2 and 3.

Examination of the field growth equation shows that a stable equilibrium field is characteristic of this charging mechanism. If through relative motions within the cloud the charge centers are rearranged so as to increase or decrease the vertical field component, this charge separation mechanism will operate in a direction that will tend to reestablish the equilibrium field.

The equilibrium fields we obtain are higher than those obtained by Kamra (1970) and Kamra and Vonnegut (1971); on the basis of their computations they concluded that fields higher than 4000 or 5000 V cm^{-1} are unlikely to be produced by a charging mechanism involving gravitational charge separation. We believe that the field strength obtained by these authors is too low because they overestimate the small particle charges by implicitly assuming that 1) in all collisions the amount of charge transferred is the same as when the large particle carries no previous charge, and 2) at any given time the charge on the small particles is proportional to the field strength at that time and not at some earlier time when the collision took place. The discrepancy between our results and those of Kamra and Kamra and Vonnegut can also be partly attributed to the different numerical values chosen for $\overline{\cos\theta}$; in our computations we have set $\overline{\cos\theta}=0.5$, which is somewhat less than its maximum value (which is 2/3), whereas Kamra (1970) takes $\overline{\cos\theta}=\cos\bar{\theta}=\sqrt{2}/2$, and Kamra and Vonnegut (1971) set $\cos\theta=1$.

5. Conclusions

Although our computations give lower electric field growth rates and thus require higher particle concentrations for rapid electrification than reported in previous works, the required concentrations still appear to lie well within the range observed in thunderstorms.

Recently Kamra (1970), Kamra and Vonnegut (1971) and Vonnegut (1972) have questioned whether a charging mechanism based on gravitational charge separation can account for electric fields above 5 kV cm^{-1} which some scientists believe may exist in thunderstorms. Our present computations indicate that such fields may be generated by the inductive charging mechanism, if the mean radius of the large ice particles exceeds 0.2 cm or if the average cosine of the collision contact angle is small.

APPENDIX

List of Frequently Used Symbols

A — R_i/R_j

$E(T)$ — vertical component of the electric field strength (esu), positive downward

K — charge transfer efficiency $[=1-\exp(-t_2/t_1)$, where t_2 is the contact time and t_1 the electrical relaxation time$]$

n	the number of collisions
N_i	number of small neutral particles (cm^{-3})
N_j	number of large particles (cm^{-3})
P	separation probability
Q_{i0}	the charge before collision on the smaller particle
Q_{j0}	the charge before collision on the larger particle
$Q(t)_i$	charge on a small particle due to a collision of time t
$Q(T)_j$	charge on a large particle at time T
Q_i^0	the charge transferred to the smaller particle in a single collision between two initially neutral particles
$Q(T)_j^*$	equilibrium charge of a large particle in field $E(T)$
Q_n	charge on larger particle aften n collisions
R_i	radius of small particle
R_j	radius of large particle
T, t	time
Δt	average time interval between collisions without coalescence for the large particles
U	updraft velocity
V_i	fall velocity of small neutral particles
$V(T)_j$	fall velocity of large particles at time T
γ_1, γ_2	nondimensional coefficients as defined by Latham and Mason (1962)
ϵ	collision efficiency normalized to the radius of the large sphere
η	time required for the electric field to increase by a factor of 2.72
θ	collision contact angle, measured with respect to the positive direction of the electric field
ρ	particle density
τ	relaxation time for charging of the large particle [$\equiv \Delta t/(\gamma_2 K A^2)$]

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