A Numerical Model of Deep Moist Convection: Part II. A Prototype Experiment and Variations Upon It

ROBERT E. SCHLESINGER

Dept. of Meteorology, University of Wisconsin, Madison 53706
(Manuscript received 19 December 1972, in revised form 7 June 1973)

ABSTRACT

This study involves the dynamics of a deep convective cloud in conditionally unstable surroundings with moderate mid-tropospheric shear. A two-dimensional, anelastic numerical model is used to simulate convection of the squall-line type.

A prototype experiment is run with liquid water drag, liquid precipitation, and effects of pressure perturbations upon the buoyancy included. The roles of these forces are inferred by running variations upon the prototype with one force suppressed in each case. In another variation, a sharp upper jet replaces a flat upper wind maximum. It is found that:

1. The prototype storm exhibits a quasi-steady mature stage characterized by nearly time-independent streamline patterns for both air parcels and precipitation particles in and near the cloud core. Potentially warm low-level air feeds the updraft from downshear, while potentially cool middle-level air feeds the downdraft from upshear.

2. During maturity, thermal buoyancy is the dominant vertical force, but is strongly opposed by the vertical perturbed pressure gradient force. The buoyancy due to pressure perturbations is appreciable, with a maximum value about one-fourth that for the thermal buoyancy. Liquid water drag is intermediate in importance between the two buoyancy components. The vertical and horizontal net accelerations are comparable to each other and to the pressure buoyancy.

3. Dynamic entrainment of potentially cool air into the sides of the cloud eventually contributes to dissipation as the downdraft spreads laterally and isolates the updraft.

4. Liquid water drag limits updraft intensity but is not necessary for downdraft formation, which is due instead to evaporative cooling.

5. Fallout of precipitation is essential to storm dissipation; without fallout, liquid water accumulation at low levels is insufficient for significant downdraft development, and the cloud core evolves to a steady state.

6. Negative pressure buoyancy in the upper portion of the cloud slightly limits the intensity of the developing updraft, but positive pressure buoyancy at and near the foot of the updraft reinforces its intensity during maturity.

7. A sharp upper-level jet in place of a broad upper-level wind maximum delays and slightly prolongs the mature stage, but does not lead to a more intense updraft.

1. Introduction

In a previous paper (Schlesinger, 1973; henceforth referred to as Part I), a theoretical investigation was made into the joint influence of low-level moisture supply and mid-tropospheric ambient wind shear upon the intensity and persistence of precipitating cumulonimbus. These two parameters were systematically varied in a series of comparative experiments using a two-dimensional, anelastic numerical model.

It was found in Part I that updraft intensity, rainfall rate and cloud size each decreased with increasing shear, but that with sufficient ambient moisture a mature storm might persist relatively long with moderate or even strong shear. The case involving moderate shear and intermediate moisture supply, designated M2 in Part I, exhibited during maturity a quasi-steady flow pattern resembling more than the other cases the pattern inferred by radar for storms in Wokingham, England, and Geary, Okla. (Browning and Donaldson, 1963). This model storm will henceforth be referred to as the “prototype” case.

In the first of two principal thrusts of this article, several aspects of the prototype not studied in depth in Part I are analyzed in order to investigate the following questions:

1) What are the relative importances of various individual forces, especially those which one-dimensional convection models (e.g., Squires and Turner, 1962; Das, 1964; Srivastava, 1967; Weinstein, 1970) have been unable to include, namely, perturbed vertical pressure gradient forces and the buoyancy component due to pressure perturbations?

2) What is the pattern of dynamic entrainment (Riehl, 1954), i.e., systematic horizontal acceleration
of air into the cloud from its surroundings due to the horizontal component of the pressure gradient force?

3) What are the main differences between the configurations of airflow streamlines and precipitation streamlines resulting from the relative fall of precipitation?

4) During the mature stage of convection, what are the trajectories of individual air parcels whose initial positions inside or outside the cloud are specified?

The other main thrust of this article involves four variations on the prototype experiment, the first three involving the basic dynamics and the fourth involving the initial environment, in order to investigate the following points:

1) It is uncertain whether downdraft formation in severe convective storms is due mainly to evaporative cooling or to the downward drag of the liquid water itself. Schematic storm models motivated by observations (Browning and Ludlam, 1962; Browning, 1964; Newton, 1967; Goldman, 1968; Fankhauser, 1971) have emphasized the apparent importance of evaporating precipitation as a cooling agent for dry middle-level air. However, Byers and Braham (1949) indicated that liquid water drag should initiate the downdraft; the one-dimensional numerical models of Das (1964) and Srivastava (1967) have also indicated that a downdraft can be initiated at levels where the liquid water concentration, and hence its drag, is large. Nevertheless, a two-dimensional nonprecipitating cumulus model (Murray and Anderson, 1965) which ignored liquid water drag yielded significant maximum downdraft velocities, about one-third the maximum updraft velocities. Is the liquid water drag essential to the initiation of the downdraft, or does it only restrict the updraft intensity?

2) The effects of liquid water drag and of liquid water fallout might differ substantially, since liquid water exerts a drag whether or not it falls fast relative to the air. But the relative fall speed of water drops affects the spatial distribution of drag by changing the drop distribution. Since this in turn alters the distribution of evaporative cooling, fallout should affect the location and strength of the downdraft. Storm duration might also be affected, depending on whether the downdraft isolates the updraft from its low-level source. Is precipitation essential to either the downdraft or the dissipation of the storm?

3) Observations aloft within convective storms have not provided comprehensive data on pressure disturbances. Barnes (1970), however, has described a balloon ascent through an updraft in a severe Oklahoma thunderstorm indicating pressure differences \( \geq 3 \text{ mb} \) between updraft and environment near 500 mb. Citing such evidence, List and Lozowski (1970) have sug-

1375

suggested that the pressure contribution to the buoyancy in deep clouds may be a significant fraction of the thermal contribution. Does the pressure contribution significantly affect either the severity or persistence of the storm?

4) The wind shear included in the comparative experiments of Part I was of only one sign. Much importance has been recently attached to the apparent role of well-marked jets in the vertical wind profile in promoting severe or persistent storms. Isentropic mass convergence, resulting from translation of a descending polar jet into a region of weaker winds, is regarded as an important dynamic destabilization mechanism, as borne out by a detailed analysis of a severe weather outbreak in Ohio during April 1968 (Johnson and Sechrist, 1970). As noted in Part I, Takeda (1971) concluded from numerical experiments that a jet in the ambient wind supports a quasi-steady storm, provided that the jet altitude is neither too high nor too low. In this connection, does a sharp upper-level jet, in comparison to a flat upper-level wind maximum, help to either intensify or prolong the convection?

To investigate the first three of these variations [points 1), 2) and 3)'], the vertical equation of motion is respectively altered by omitting the liquid water drag, by assuming zero relative fallspeed for all water drops, and by omitting the pressure term in the gravitational buoyancy. Point 4) is investigated directly by altering the initial ambient wind.

2. The prototype storm

Before summarizing the further study of the prototype storm, it is appropriate to recall the chief properties of the numerical cloud model:

1. The model is two-dimensional and rectangular with open aperiodic lateral boundaries and rigid free-slip upper and lower boundaries.

2. The grid is 55×21 with \( \Delta x = 3.2 \text{ km} \) and \( \Delta z = 700 \text{ m} \) (172.8 km long and 14 km deep). The entire grid is translated downwind systematically in steps of length \( \Delta x \) to keep the leading cloud edge within the domain; the origin of the horizontal coordinate is at the initial upwind boundary.

3. Motions are assumed anelastic, admitting gravity waves but filtering out acoustic waves.

4. Frozen water is not included, but liquid water is partitioned into cloud droplets and precipitation following Takeda (1966).

5. The effect of pressure perturbations upon the gravitational buoyancy is included.

6. The initial ambient conditions are conditionally and convectively unstable with mid-tropospheric vertical wind shear, while the initial impulse is assumed to be a shallow buoyant saturated
region with a weak convective circulation already present.

In order to study the trajectories of selected air parcels and streamlines of precipitation particles during the quasi-steady and dissipating stages of the prototype storm, diagrams are presented showing these features at 20-min intervals from 40 through 80 min. The equivalent potential temperature field at 60 and 80 min is also examined in order to further reveal the contrast between the quasi-steady and dissipating stages. Force fields are examined at 60 min.

a. Air parcel trajectories and precipitation streamlines

The positions of 12 selected air parcels, together with the cloud and perturbation streamlines, are shown at the initial time in Fig. 1. Forward trajectories for each of parcels 1–12 were computed by a straightforward method presented in the Appendix. Figs. 2–4 show each parcel, together with a few relative precipitation streamlines represented by dotted curves, at successive 20-min intervals from 40 through 80 min. Also shown in these figures are the streamlines of airflow relative to the moving cloud core and the region of significant rainfall, plotted as in Part I. Although storm motion appeared to be slowing somewhat between 70 and 80 min, the average velocity between 40 and 70 min (about 12 m sec⁻¹) was taken to approximate the displacement rate at each time shown.

Despite the continuous horizontal elongation of the overall circulation, the closeness of the updraft to a steady state is strikingly evident from the paths of parcels 11 and 12. Initially in the moist lower layer to the right of the cloud, these slow-moving parcels are overtaken by the faster-moving storm. They rise...
through the updraft within about 20 min and then are carried rapidly toward the right in the anvil. Note that parcel 11 remains in the first channel to the right of the dividing streamline from 40 through 60 min, while parcel 12 remains in the second channel to the right. This shows how closely streamlines and trajectories correspond in the updraft core, further justifying the use of the term “quasi-steady state” for describing the mode of the storm; in true steady-state flow, streamlines and trajectories would coincide. Although the closed counterclockwise circulation in the upper rear part of the updraft widens with time, parcel 7 remains within this feature.

Since parcel 5 is initially in the transition zone between the moist and dry layers, and since some air above this parcel descends at the left edge of the cloud at 60 min, this indicates that inflow of potentially cool air is important to downdraft formation.

Much like parcels 11 and 12, parcel 10 is drawn into the updraft from ahead of the cloud and then curves at upper levels into the anvil. Parcels 8 and 9, initially in the core of the cloudy updraft, remain well within the main updraft at later times until they curve into the anvil, and apparently do not leave the cloud throughout the run.

The trajectory of parcel 2 indicates that some air from far above and upwind of the shallow initial cloud is drawn into the upshear branch of the developing circulation, just skirting the upper limits of the downdraft on entry but then rising into the rear of the updraft. Although parcel 1, initially even further upwind and also well up in the middle levels, has not yet entered the cloud at 80 min, it appears destined to overtake the cloud and also become part of the back of the updraft. Parcel 3, initially also above and upshear of the cloud but downshear of parcels 1 and 2, is drawn into the top of the deepening updraft some time before 40 min and subsequently becomes part of the anvil. Some of the air initially above and upshear of the cloud drifts nearly horizontally without really becoming part of the updraft; parcels 4 and 6, caught within the relatively inactive region of horizontally elongating circulation along the underside of the anvil, just skirt the cloud edge.

The precipitation streamlines, calculated from the relative wind field and the assumed terminal veloc-
ties, show a quasi-steady configuration as long as the updraft is also quasi-steady, and therefore approximate trajectories of precipitation particles. Near the 8-km level, at the rear edge of the updraft, precipitation spirals outward to surrounding parts of the cloud. Suspended light precipitation ascends by as much as 2–3 km, and a minor part of the precipitation falls into the downshear lobe of the anvil. The bulk of the precipitation falls into the comparatively narrow cloud trunk extending up to ~4 km, with rain first reaching the ground shortly after 40 min. Note that no precipitation streamlines extend outside of the cloud, but terminate slightly short of the cloud edges (unless they reach the ground). This is due to the way in which precipitation has been handled in this model, as explained in Part I. The outermost parts of the cloud contain no precipitation since the liquid water content decreases outward from the interior.

We next emphasize the contrasts between the quasi-steady and dissipating stages of the prototype storm. At 40 and 60 min, the updraft is nearly erect, marked by a dividing streamline which ascends from the surface into the anvil. The rapid weakening of the updraft toward 80 min is attended by a radical change in the streamline pattern. At 80 min, the updraft appears cut off from the low-level moist air downshear of the storm. The downshear leaping of the updraft indicates partly that ω has decreased and partly that the horizontal momentum within it has increased, apparently because it is now being infiltrated by fast-moving air entering the back of the cloud near 6 km. The dividing streamline has been replaced by an arched separating streamline about 4 km high; this indicates a widening of the downdraft into the cloud interior. The reverse eddy at the rear of the cloud is still present, and the secondary cloud within it has stretched horizontally, but the old separating streamline has disappeared. The decrease in rainfall intensity is much less striking than the decrease in updraft intensity. This is largely because precipitation formerly suspended aloft is now approaching the ground, even though the decaying updraft is producing much less new liquid water to add to the existing accumulation. Also, the surface wind maximum under the cloud trunk still maintains a value of 15–16 m sec⁻¹ although its location has shifted from the zone of heaviest surface rain to the much lighter precipitation nearer the right edge of the cloud.

b. Equivalent potential temperature fields

The sharp contrast between the quasi-steady and dissipating stages of the prototype storm is further brought out by the change in the equivalent potential temperature distribution between 60 min (Fig. 5) and 80 min (Fig. 6). At 60 min, the contrast between the equivalent potential temperature distributions within and away from the storm is immediately apparent. Away from the cloud, the isopleths of θₑ are nearly horizontal even though the flow pattern is appreciably disturbed; this reflects the unsteady character of the widening overall circulation. The previous subsidence of the more distant surroundings is evident up to at least 8 km in the lowering of the isopleths from their initial positions (shown at the extreme right of the figure), but their orientation is only slightly changed. In sharp contrast, the 335K isopleths within the cloud core are nearly vertical between 4 and 8 km, closely paralleling the erect updraft and revealing its saturated adiabatic structure throughout that layer. Note the relatively low surface θₑ, immediately in back of the storm, due to evaporation of precipitation into air which has descended from higher levels.

In a study of a small synoptic-scale disturbance in the Line Islands, Zipser (1969) concluded that a pronounced local decrease of equivalent potential temperature observed near 900 mb was due to evaporation of the precipitation into an unsaturated downdraft originating from between 400 and 600 mb. In that

![Figure 5](attachment:image.png)

**Fig. 5.** Equivalent potential temperature field for case M2 at 60 min. The heavy solid curve represents the cloud boundary. Thinner solid curves denote equivalent potential temperature (excess in °K above 300K), with half-intervals indicated by dashed curves. At the extreme right of the diagram, initial base values of the equivalent potential temperature are indicated by horizontal line segments, with numerals and dashed segments having the same meaning as in the remainder of the diagram.
case, the downdraft had much greater horizontal extent, not being associated with an individual cumulonimbus. Returning to the numerical model, the equivalent potential temperature at the cloud edges around 4–6 km is fully 10 K lower than in the updraft core, indicating systematic large-scale entrainment by the inflow just beneath the circulation centers seen in Fig. 3. At 80 min (Fig. 6), the potentially cool air has penetrated considerably further into the cloud interior, especially from the right. Since the precipitation in the right-hand part of the cloud interior is light, the loss of upward momentum of the air entering the cloud cannot be attributed mainly to liquid water drag but rather to its low equivalent potential temperature which facilitates loss of thermal buoyancy. The unsteadiness of the rapidly decaying updraft is graphically indicated by the loss of similarity between the streamlines and the $\theta_\alpha$ isopleths in the cloud core. Most noticeably, both in and outside of the updraft seen in Fig. 4, the 330K isopleth around 7–8 km is almost horizontal, even though the instantaneous flow is still much more nearly vertical in the cloud than outside of it.

c. Horizontal forces

1) Pressure gradient force

The distribution of the horizontal pressure gradient force per unit mass, $-(1/\rho_0)\partial \bar{p}/\partial x$, at 60 min, is shown in Fig. 7. In all diagrams showing individual or net accelerations, the acceleration is given in thousandths of gravity. This is almost equivalent numerically to cm sec$^{-2}$. For convenience and brevity, forces per unit mass will be frequently referred to as “forces” as has frequently been done in the literature.

It is clear from Fig. 7 that the strongest horizontal pressure gradient forces are located in the cloud interior on either side of the updraft core. Here air is being accelerated into the anvil in an asymmetric manner paralleling the development of the anvil. The maximum rightward force, nearly 4 cm sec$^{-2}$, is slightly more than twice the maximum leftward force. Large-scale entrainment near the 6-km level, already inferred from the equivalent potential temperature distribution in Fig. 5, is verified by the distribution of the horizontal pressure gradient force. From about 2 km to near the tip of the anvil, outside air is being accelerated into the cloud from both sides, with maximum forces of slightly over 1 cm sec$^{-2}$ on the left and 1.5 cm sec$^{-2}$ on the right. At upper levels, there is very weak detrainment near the ends of the anvil; more significantly, air at the lower rear edge of the cloud is accelerated leftward at up to 1 cm sec$^{-2}$ with a very shallow region of comparable rightward acceleration in the region of heavy rain. Therefore, once the high-pressure dome at the back of the storm has developed, the resulting horizontal pressure gradient forces tend to maintain the outflow induced by divergence in the downdraft.

2) Friction

The horizontal friction force $F_x$ in the cloud core, calculated in the model as described in Part I, was only a few percent as great as the horizontal pressure gradient force, so that net horizontal accelerations were almost wholly due to the horizontal pressure gradient force and hence well represented by the field in Fig. 7.

d. Vertical forces

The vertical equation of motion used in the model is

$$\frac{dw}{dt} = -\frac{1}{\rho_0} \frac{\partial \bar{\rho}}{\partial z} + g \left( \frac{\bar{T}_s}{T_{ref}} \frac{\bar{\rho}}{\rho_0} \right) + F_z,$$  \hspace{1cm} (I)

where $Q_0$ refers to initial base state values for each variable $Q$ involved, and $\bar{Q}$ denotes deviations from this base state. Note that the first three individual forces on the right-hand side—perturbed vertical pressure gradient force, thermal buoyancy, and pressure buoyancy—are evaluated in (I) with respect to the initial base state. However, as stressed in Part I, the surroundings of the storm were considerably modified
as it matured. Considering the extreme case in which
a storm might dissipate completely and result in a
new stratification with zero net vertical acceleration
everywhere, (1) would still yield non-zero individual
forces despite a net balance.

To remove the effects of environmental modifica-
tion, as well as to ensure zero individual vertical
forces in (1) should the above situation occur, \( \dot{T}_s \)
and \( \dot{p} \) were split into mean components \( \bar{T}_s \) and \( \bar{p} \),
and eddy components \( T'_s \) and \( p' \):

\[
\dot{T}_s = \bar{T}_s + T',
\]
\[
\dot{p} = \bar{p} + p',
\]

where the mean values were the horizontal averages
at 60 min, and only the eddy components of the
thermal buoyancy \((g \dot{T}_s/T_0)\), pressure buoyancy \((-g p'/\rho_0)\), and vertical perturbed pressure gradient force
\[-(1/\rho_0)\partial p'/\partial z\] have been considered.

1) THERMAL BUOYANCY

The thermal buoyancy is shown in Fig. 8. As would
be expected from the thermal patterns described in
Part I, the greatest upward buoyancy \((\sim 18 \text{ cm sec}^{-2})\)
is located in the fastest part of the updraft. There
are three significant regions of negative thermal buoyancy: 1) at and near the rear of the cloud trunk,
especially below 1.5 km, 2) along the base of the
anvil from 3 to 8 km, and 3) throughout the left
half of the cloud above the thin stable layer \((9.1-10.5 \text{ km})\).

The first two of the above negatively buoyant
regions are due to evaporative cooling; in the first
instance, heavy precipitation which has just fallen at
the lowest levels is evaporating as the storm recedes
toward the right, and in the second case cloud dro-
plets have been evaporating along the underside of
the anvil. The third, and most strongly developed,
region of negative buoyancy has resulted from moist
adiabatic ascent through the stable layer; since the
moist adiabatic lapse rate is nearly as steep as the
dry adiabatic at these high altitudes, parcels which
enter the stable layer warmer than their surroundings
emerge from this layer colder than their surroundings.

2) PRESSURE BUOYANCY

The term \(-g p'/\rho_0\), the eddy contribution of press-
ure perturbations to the buoyancy, is plotted in

\[
\text{Fig. 8. Thermal buoyancy force per unit mass for case M2 at 60 min. Otherwise the same as Fig. 7.}\]
Fig. 9. Pressure buoyancy per unit mass (contribution to the gravitational buoyancy due to pressure perturbations rather than temperature perturbations) for case M2 at 60 min. Otherwise the same as Fig. 7.

Fig. 9. In the cloud core, this force is at most only about one-fourth of the greatest thermal buoyancy. For convenience, we refer to this quantity as the "pressure buoyancy" or "pressure buoyancy force."

Two significant regions of upward pressure buoyancy should be noted. One is located at low levels directly underneath the leading edge of the anvil; the other is found at middle levels, most noticeably just to the right of the main updraft but also projecting into the accelerating part of the updraft itself.

The first of the above regions, the meso-low ahead of the storm, thus contributes to the ascent of the low-level inflow once it has passed under the leading edge. The upward pressure buoyancy in the second region is positive together with the thermal buoyancy, and thus contributes to the strength of the updraft.

Significant downward pressure buoyancy is found throughout the upper part of the main updraft and near the surface in and behind the cloud trunk. The first of these regions, associated with the pressure excess in the upper half of the cloud core, may partly explain the relatively low altitude (∼7 km) of the updraft maximum. As is evident from Fig. 9, the updraft core passes through this region not far above that altitude. Within the shallow meso-high at and near the back edge of the storm, the resulting negative pressure buoyancy acts together with the negative thermal buoyancy to perpetuate the downdraft.

3) Pressure gradient force

A substantial part of the thermal buoyancy force is canceled by the vertical pressure gradient force, \(-\frac{1}{\rho_0} \frac{\partial \theta}{\partial z}\). A comparison of Fig. 10 with Fig. 8 shows that these two forces tend to oppose one another throughout the part of the domain shown. The strong opposing tendency has a definite physical significance. If the horizontally averaged pressure were hydrostatic, the thermal buoyancy and pressure gradient forces equal and opposite, and the buoyancy purely thermal, the pressure perturbations would be hydrostatic. In this case, the horizontally averaged conditions are nearly hydrostatic; we have just seen that the buoyancy in the model is mainly thermal, although the contribution due to pressure perturbations is appreciable. Thus, despite the crucial importance of net vertical accelerations to the convection, significant parts of the pressure perturbations are still hydrostatic.

Fig. 10. Vertical component of the perturbation pressure gradient force per unit mass for case M2 at 60 min. Otherwise the same as Fig. 7.
4) Liquid water drag

The liquid water drag (Fig. 11) is weaker than the thermal buoyancy or vertical pressure gradient force, but stronger than the pressure buoyancy. The greatest drag occurs near and just below the location of maximum thermal buoyancy, canceling out about 46% of the thermal buoyancy and is located partly above the significant rainfall region shown in Fig. 3, since the drag is proportional to the weight of the liquid water whether or not precipitation actually approaches the ground. Also, the region of greatest drag is not coincident with the site of the cold downdraft, which was found at and near the left edge of the cloud. It is apparent that at least in this model, the liquid water drag serves much more to curb the updraft than to initiate the downdraft.

5) Friction

The vertical friction force $F_v$ in this model proved to be comparable to the horizontal friction force $F_h$. This indicates that the vertical friction is of minor importance, being over an order of magnitude smaller than any of the other individual vertical forces.

6) Net vertical acceleration

Finally, the net vertical acceleration $dw/dt$ is shown in Fig. 12. Note that $dw/dt$ was computed directly from the identity

$$dw/dt = \partial w/\partial t + u\partial w/\partial x + w\partial w/\partial z,$$

(4)

calculating the local time derivative over the last time increment and the advective terms from current values of $u$ and $w$, and using upstream differencing for calculating the space derivatives. This was done to avoid gross errors which might result from directly adding the individual forces. Since the individual vertical forces (apart from friction, which is quite small) show strong partial cancellation, errors which may be tolerable for the larger individual forces might be unacceptable for the considerably weaker resultant force field. A glance at Fig. 12 shows that the maximum net acceleration is only about one-fifth the maximum thermal buoyancy and is only comparable to the greatest pressure buoyancy. Not surprisingly, parcels are accelerated most strongly somewhat beneath the site of fastest upward motion, then decelerated into the anvil on both sides of the updraft core. Parcels are decelerated into the leading lobe of
the anvil about twice as strongly as into the trailing lobe, also paralleling the asymmetry of the horizontal pressure gradient force in Fig. 7. The cold downdraft is situated in a zone of net deceleration extending over the depth of the domain, even though its initiation site is mainly at 4-6 km.

3. Variations upon the prototype

a. Main experiments

We now present some highlights of the results of the four variations on run M2 referred to in the Introduction, treating each experiment separately. Each was run out to 80 min, with $z-t$ diagrams constructed much as for the comparative experiments in Part I. The four variations were designated as cases M2-A (liquid water drag suppressed), M2-B (precipitation suppressed), M2-C (pressure buoyancy suppressed), and M2-D (upper jet maximum in wind profile).

Height diagrams of vertical velocity for the prototype and these four variations are shown in Fig. 13. Diagrams like those in Part I have been included (Figs. 16-19) to show the significant rainfall zones
and the airflow relative to the moving storm core at one particular time. This time is 60 min in variations A, B and C, and 70 min in variation D. The storm core velocity was represented by the 30-min average of the horizontal displacement rate beginning 20 min before the time shown. Each such diagram should be compared to Fig. 3, which shows the prototype storm at 60 min.

1) Case M2-A: Liquid Water Drag Suppressed

Without liquid water drag, the updraft velocity is 30–50% greater than in run M2 (Fig. 13b). As would be expected, then, the drag is essential as a brake on the updraft intensity. Although the drag is being ignored, the precipitation mechanism is retained just as in the comparative experiments, and liquid water is most heavily concentrated in the lower parts of the cloud. The maximum concentration is about 20% larger than in the prototype run, reflecting the greater condensation in the more vigorous updraft. Although the mode of convection is rather similar to that for the prototype, the drag-free storm goes through the various stages sooner.

The greater severity of the storm without drag is also evident from Fig. 16. The anvil is about one-third longer than in the prototype at the same time. The significant rainfall extends higher, so that a greater portion of the precipitation falls out of the jet downshear of the updraft than in the prototype case. Partly because the air in the jet is moving rightward much faster than the updraft core, the heaviest rainfall lies almost entirely downshear of the dividing streamline. In addition to the weak downdraft near the left cloud edge, a downdraft is forming in the heavy rain near the right edge. At 60 min this downdraft is quite weak, but at 80 min it is about three times as strong as in the prototype storm.

These results indicate that liquid water drag is not essential for initiating a downdraft. This is in line with the recurrent tendency in the comparative experiments for downdrafts to appear near cloud edges rather than in the far interior of the cloud in the comparative runs. In case M2-A, the heavy precipitation falling through the lower right part of the cloud maintains a large supply of liquid water for evaporation into the incoming unsaturated air. The downdraft later becomes moist adiabatic as the broadening region of precipitation (which is saturated according to the manner in which evaporation has been treated in this model) transforms the cold region into part of the cloud interior.

2) Case M2-B: Precipitation Suppressed

The outstanding difference between runs M2 and M2-B is apparent in Figs. 14a and 14b, the z-t charts for liquid water content. When all water drops are simply carried along with the air (but may nevertheless exert a drag), the largest liquid water concentration is located around 8 km, much higher than in any of the comparative experiments (all of which allow precipitation). The maximum is only about 60% as great as in the prototype case, indicating in retrospect that the precipitation process not only greatly lowers the altitude of heaviest liquid water concentration but also leads to accumulations not otherwise possible.

Fig. 13c shows that the precipitation mechanism is important to updraft dissipation, since the updraft profile takes on a steady state at about 55 min with no sign of decay as of 80 min. Until about 50 min, there is little difference in updraft intensity between the prototype and variation B. It is apparent that the temporary decrease in the updraft velocity for the prototype between 34 and 50 min is not mainly caused by precipitation, since a comparable decrease occurs in case M2-B. However, the reinforcement of the updraft between 50 and 65 min in the prototype may be partly due to the redistribution of liquid water by precipitation, since much less regeneration occurs without precipitation.

Fig. 17 reveals that although the overall circulation at 60 min is in some ways quite similar to that of the prototype at the same time, there are also some important differences. The stub-like protuberance in the cloud base never reaches the ground, since precipitation is suppressed. The cloud stub is the counterpart of the precipitating trunk in the prototype storm, but does not broaden appreciably after 40 min. The slight downdraft at and under its left edge is less than half as strong as in the prototype. This minimal downdraft does not intensify significantly later, and the convergence rearward of it is insufficient for a secondary cloud to form. The lack of downdraft development results from the inability of large liquid water accumulations to occur in the lower parts of the cloud; since much less liquid water is available for evaporation than in the prototype, there is much less cooling (only about 1°C at most) and therefore less negative thermal buoyancy. The temperature never changes at the lower boundary, since the static phase adjustment described in Part I is not activated there.

3) Case M2-C: Pressure Buoyancy Suppressed

In variation C, the pressure buoyancy \((-\frac{g\hat{p}}{\rho_0})\) has been ignored and only the thermal buoyancy \((\frac{g\Delta T}{\rho_0})\) retained in the total gravitational buoyancy \((-\frac{g\Delta \rho}{\rho_0})\).

Comparing Fig. 13d with Fig. 13a, note that at 30 min the altitudes of both the cloud top and of the maximum updraft are appreciably greater in case M2-C. As implied by this difference, the updraft through the first 30 min is somewhat stronger in
variation C than in the prototype case, although the initial momentum perturbations are identical. Recall from Fig. 9 that the prototype storm showed positive pressure buoyancy in the lower front part of the updraft, and pronounced negative pressure buoyancy in the upper part of the updraft. Also, preliminary runs showed that before 30 min the prototype storm exhibited pressure falls below the updraft core and pressure rises above it. Thus, in variation C, without the downward acceleration provided by the upper level pressure excess, air parcels in the developing updraft acquire extra vertical momentum not because they are accelerated more at low levels but because they are decelerated less in the upper half of the cloud.

After 35 min, the convection in run M2-C becomes appreciably weaker than in the prototype run, apparently because the low pressure in front of and within the lower part of the updraft in the prototype case dominates the dynamics more than the pressure excess at higher levels does. In case M2-C, the absence of the extra upward push provided by the positive pressure buoyancy at low levels becomes an important factor in limiting the intensity of the convection at maturity. Correspondingly, the maximum liquid water content after 40 min is about 25% smaller than in the prototype storm, due to slower condensation.

The updraft orientation and the location of significant rainfall at 60 min (Fig. 18) are much as in the prototype storm, but the dividing streamline of the updraft approaches the left cloud edge more closely than in case M2; the increased vulnerability of the updraft to mixing with evaporatively cooled air may contribute to its weaker intensity. The region of significant rainfall in case M2-C is smaller than in case M2 although the location within the cloud is similar. The importance of the low pressure ahead of the storm is seen indirectly in the smaller mass flux of air toward the storm (there is only one uninterrupted channel, rather than two, to the right of the cloud).

4) Case M2-D: Upper Jet Maximum in Wind Profile

In variation D, the range of $u$ for the undisturbed initial wind was the same, from 6 m sec$^{-1}$ at the surface to 24 m sec$^{-1}$ at upper levels, but the shape of the vertical wind profile was altered as shown in Fig. 15, with a jet nose between 7 and 11.2 km. The 24 m sec$^{-1}$ maximum wind is at 9.1 km. The profile shape is based on the projection of the environmental winds along the direction of movement of a Geary, Okla., thunderstorm that occurred in May 1961 (Browning and Donaldson, 1963).

Fig. 13e shows that neither the mode nor the intensity of the convection is greatly changed by replacing the initial wind profile of run M2 with that of variation D. However, the corresponding stages occur somewhat later than in the prototype case. The two peaks in updraft velocity occur near 40 and 75 min, compared with 34 and 64 min. This suggests a somewhat longer quasi-steady phase in run M2-D than in run M2, as does the fact that the updraft
above 7 km is still increasing slightly at 80 min whereas the upper half of the prototype cloud showed diminishing upward motion after about 65 min.

Since the convection in variation D lags roughly 10 min behind that in run M2, Fig. 19 shows variation D at 70 min to provide the most direct comparison with the prototype at 60 min. Most key features strongly resemble those of the prototype, but two dissimilarities are evident. The lower fourth of the cloud in Fig. 19 shows a greater rightward slope with height, as might be anticipated from the greater low-level ambient wind shear. Although the asymmetry of the cloud in variation D is determined mainly by the positive shear below the jet, the reversal of shear above it results in more noticeable leftward outflow from the updraft around 11 km. This more nearly symmetric outflow is reflected by the somewhat less pronounced asymmetry of the anvil.

b. Secondary experiments

In addition to the four variations just described, three further variations upon the prototype experiment were performed in order to investigate two secondary but nevertheless interesting points:

(i) Especially in the southern Great Plains (Fawbush and Miller, 1953), severe thunderstorms are often observed in situations featuring a deep layer of very dry air above a moist layer only 150–200 m deep. This suggests that the strong convective instability in the sharp transition zone between the layers may be more important than a large total supply of latent heat. In this connection, the transition zone was deepened in one experiment (case M2-E) and the moist layer was made thermally more unstable but shallower in another run (case M2-F).

(ii) In numerical modeling, the choice of initial perturbation is somewhat arbitrary since little detailed knowledge exists regarding triggering mechanisms for severe storms. One might wonder whether the intensity and persistence of convection are determined mainly by the initial environment, or if two or more mature configurations may be possible depending upon the initial perturbation. To investigate this question, the initial impulse was changed without altering the initial base state (case M2-G).

1) Case M2-E: Deeper transition layer

In variation E, the convective instability (rate of decrease of \( \theta_e \) with height) between 2.8 and 8.4 km was reduced by making the transition layer between the moist and dry air 4.9 km deep rather than 1.4 km.

The increased supply of latent heat energy both intensified and prolonged the storm slightly, despite the reduced convective instability between 2.8 and 4.2 km. The storm weakened slightly by 80 min, but had not yet exhibited the rapid decay apparent in the prototype storm after 74 min. However, the pattern of rainfall and relative airflow at maturity was quite similar to that for the prototype case.

It is interesting that the downdraft intensity was not diminished by deepening the transition layer. In the comparative experiments of Part I, the downdraft tended to weaken as the initial moisture supply below the transition layer was decreased. At least in this model, the initial presence of very dry air around 4 km is not of prime importance to downdraft development. In order to understand this, recall that vertical velocities and hence the departures of other physical variables from their initial base values are overestimated in the two-dimensional model. Where the sinking parcels upshear of the cloud reach their lowest altitudes, the relative humidity decreased almost as greatly in variation E as in the prototype. The much greater depth of the initial transition layer in variation E loses its importance because of the strong drying which takes place in the layer initially occupied only by moist air (to say nothing of the drying in the initial transition layer). While this model indicates that a very sharp initial stratification of dry air over moist is not conducive to either a more

Fig. 16. Same as Fig. 3, but for case M2-A at 60 min, and without inclusion of precipitation streamlines or tagged air parcels.
intense or longer lasting storm, this conclusion must be interpreted with ample caution.

2) Case M2-F: Shallower and more unstable moist layer

In variation F, the moist layer extended from the surface to 1.4 km instead of 2.8 km, again with a 1.4-km transition layer above it. The surface temperature was again 28°C, as in all other experiments, but the constant lapse rate from the surface to the middle of the transition layer was approximately 8.6°C km⁻¹, compared to 7°C km⁻¹ in the prototype, in anticipation that a storm would require greater low-level instability in compensation for the smaller water vapor supply in order to attain about the same intensity as the prototype storm. In the dry air, the lapse rate was the same as in case M2. The shear above 3.5 km was the same as in the prototype run, although u itself above this level was uniformly 4 m sec⁻¹ greater because the wind in the middle of the transition layer was again assumed to be 10 m sec⁻¹.

The strength attained by the updraft was indeed much the same as in the prototype, but the updraft dissipated completely between 60 and 80 min. Also, the downshear leaning of the updraft in variation F resulted in a concentration of heavy rainfall downshear of the updraft core, isolating it from its low-level source and hastening its dissipation. The region of cloud and precipitation spread downshear more rapidly than the location of greatest evaporative cooling, which was downshear of the updraft, so that the main downdraft became protected from the unsaturated surroundings and subsequently developed moist adiabatically. At 80 min, the downdraft was about three times as strong as in the prototype case.

3) Case M2-G: Initial perturbation altered

In run M2-G, the initial base state was the same but the initial perturbation was changed in order to gauge the sensitivity of the convection, in terms of the mature or final state and the evolution toward that state, to the initial perturbation. Rather than a shallow cloud with a weak convective circulation already superimposed on the basic flow, a thermally buoyant impulse with higher relative humidity than the surroundings but without any liquid water or momentum perturbation was assumed. Its dimensions were the same as those of the initial updraft in the prototype run. The positive perturbations of temperature and moisture were greatest at the center

---

Fig. 17. Same as Fig. 16, but for case M2-B at 60 min.

Fig. 18. Same as Fig. 16, but for case M2-C at 60 min.
(1C and 20%, respectively) and decreased outward. The center of the impulse was just barely saturated, enabling condensation to commence at once.

This experiment was first run out to 80 min, as were all others. There was a prolonged interval of very slow cloud development until about 70 min. The maximum updraft velocity was near 1 m sec\(^{-1}\) from 5 through 30 min. Downwind of the cloud, gravity waves about 20 km long propagated nearly horizontally in the upper levels at the same horizontal velocity as the cloud, with vertical air velocities up to 3.5 m sec\(^{-1}\). By 80 min, however, the cloud growth was accelerating sufficiently to justify running this experiment out considerably further in time. Since its development at 80 min was similar to that of the prototype cloud at 15–20 min, it was decided to rerun variation G for 140 min of simulated time.

Between 80 and 100 min, the convection developed rapidly, and gravity waves soon became no more dominant than in the prototype case. Once the cloud became vigorous, it was similar in mode and intensity to the prototype although the mature stages occurred 60–70 min later. Between 115 and 140 min, time variations in each variable in the upper two-thirds of the updraft core were slow enough so that one might term the storm core “quasi-steady” as in the prototype. The first updraft peak at 34 min in run M2 (Fig. 13a) was apparently due to the upward push provided by the initial convective circulation; instead of a peak, Case M2-G showed a plateau in the spatial maximum of \(w\) from about 94–108 min.

The cloud shape and size in case M2-G at 130 min were not significantly different from case M2 at 60 min. The maximum updraft velocity was virtually the same, and the updraft nearly erect. The downdraft was again at the lower left edge of the cloud. Also, the regions of significant rainfall were closely similar in size, shape and position, with the heavy rain mostly to the left of the dividing streamline.

These results suggest that at least for weak buoyant initial perturbations of particular dimensions, the evolution toward the final or quasi-steady state may be strongly influenced by the nature of the impulse but that the size, strength and configuration of the mature circulation is determined almost entirely by the initial environment. The results described for this particular model indicate essentially a unique quasi-steady state, rather than two or more.

4. Comparison with previous investigations

In Part I, some similarities and dissimilarities between the model and observed patterns for large thunderstorms have been noted for the comparative experiments, including the prototype. It is now instructive to compare the results presented in the preceding sections with those of a few previous investigations, especially the recent two-dimensional sheared cumulonimbus model of Takeda (1971).

Takeda considered basic ambient wind profiles with shear of one sign, somewhat as in the prototype case, as well as profiles with well-defined jets at various heights. Takeda’s case B3 corresponded most closely in wind profile to the prototype for this model, while his case C10 with a high jet used a profile similar to that in variation D but with left and right interchanged.

Certain similarities between the prototype and Takeda’s case B3 should be noted:

Vertical wind shear is intensified downshear of the updraft.

Downdrafts are located near cloud edges, rather than in the far interior of the cloud.

The greatest rainfall content is located below the level of strongest upward motion.

However, some prominent dissimilarities were also found:

Takeda’s updraft and maximum rainwater axis in his case B3 are strongly inclined downshear. In the prototype experiment, the updraft core and maximum precipitation content axis (not shown) are nearly erect.

Takeda’s clouds in both cases B3 and C10 are short-lived. The downdraft is downshear of the main updraft, and the secondary updraft is down-
shear of the downdraft, in case B3; for the jet case C10, the same is true with respect to the shear below the jet. The opposite is true in the current model. As noted in Part I, this discrepancy is associated with certain phenomena not taken into account in the hypothesis presented by Takeda to explain his results; most notably, the updraft core propagates downshear faster than (rather than with) the vertically averaged low-level wind, and outflow from the downdraft tends to perpetuate the main updraft while helping to induce a secondary updraft upshear of the main one.

The axes of maximum updraft and maximum thermal buoyancy do not coincide in Takeda’s case B3, but are nearly coincidental in the prototype experiment, indicating less efficient upward heat transport in the former instance. Such a phase difference was also noted in the shallow cumulus model of Asai (1964), who found weaker convective development in uniform ambient wind shear than in unheated surroundings.

Although both models assume shallow initial clouds of similar depths, and neglect the ice phase, precipitation reaches the ground much earlier in Takeda’s runs (before 15 min) than in the prototype case (~40 min). Also, in Takeda’s model, maximum rainwater contents much the same as in the prototype case (~4.5 gm m⁻²) occur with peak updraft velocities about 40% smaller.

In connection with the last of these four dissimilarities, an important difference between the treatments of precipitation in the models should be noted. In the present model, precipitation reaches the ground much earlier in Takeda’s runs than in the prototype case (~40 min). Also, in Takeda’s model, maximum rainwater contents much the same as in the prototype case (~4.5 gm m⁻²) occur with peak updraft velocities about 40% smaller.

In connection with the last of these four dissimilarities, an important difference between the treatments of precipitation in the models should be noted. In the present model, precipitation reaches the ground much earlier in Takeda’s runs than in the prototype case (~40 min). Also, in Takeda’s model, maximum rainwater contents much the same as in the prototype case (~4.5 gm m⁻²) occur with peak updraft velocities about 40% smaller.

In connection with the last of these four dissimilarities, an important difference between the treatments of precipitation in the models should be noted. In the present model, precipitation reaches the ground much earlier in Takeda’s runs than in the prototype case (~40 min). Also, in Takeda’s model, maximum rainwater contents much the same as in the prototype case (~4.5 gm m⁻²) occur with peak updraft velocities about 40% smaller.

5. Summary of principal conclusions

1) During the mature stage of the prototype model storm, with moderate mid-tropospheric ambient wind shear, a quasi-steady updraft in the cloud core is paralled by the streamline configuration for air parcels and the velocity fields for precipitation particles. Streamlines and trajectories for air parcels nearly coincide, and the velocity fields for precipitation change little with time. Relative to the moving storm, potentially warm air from low levels enters the updraft from downshear and ascends through the cloud core into the anvil; potentially cool air above the moist layer descends into the storm from upshear. Light precipitation is suspended aloft in the upper parts of the updraft, and spreads into the anvil, while much heavier rainfall occurs underneath the updraft core.

Large-scale entrainment of potentially cool mid-level air into the sides of the cloud eventually contributes to dissipation of the storm. The updraft then weakens rapidly; rainfall at low levels diminishes, but more slowly, as precipitation formerly suspended in the upper portions of the updraft approaches the ground.

2) Thermal buoyancy is the dominant vertical force, but the perturbed vertical pressure gradient force is comparable, showing a strong tendency to oppose the thermal buoyancy both within and outside of the storm core. Liquid water drag is significant, up to about 40% of the maximum thermal buoyancy. The buoyancy due to pressure perturbations, a force often ignored in convection models, is relatively small but nevertheless appreciable, up to about one-fourth the maximum thermal buoyancy. Due to strong partial cancellation of individual vertical forces, the net
vertical acceleration is only comparable to the pressure buoyancy; since the thermal buoyancy and perturbed vertical pressure gradient forces are dominant and tend to oppose, significant parts of the pressure perturbations are hydrostatic despite the importance of net vertical accelerations to the convection.

3) The horizontal pressure gradient force, comparable in magnitude to the net vertical acceleration, is important to both the dissipation of the updraft and the maintenance of the downdraft. Potentially cool middle-level air is accelerated horizontally into the cloud from both sides, ultimately reducing the buoyancy of the updraft. The downdraft outflow, initiated by continuity requirements as the downdraft decelerates vertically toward the ground, is then dynamically sustained by the pressure gradient force which causes air in the "thunderstorm high" to accelerate outward.

4) Liquid water drag is an important brake on the maximum strength attained by the updraft, but is not essential to the initiation of a downdraft. The negative thermal buoyancy produced by evaporation near cloud edges, where liquid water drag is small, is sufficient to initiate a downdraft which may even become vigorous. In view of the model results, the contentions of Byers and Braham (1949), Das (1964) and Srivastava (1967) that liquid water drag is important to downdraft formation are open to question.

5) The precipitation process, specifically the fall of liquid water relative to the air, causes liquid water to accumulate in the lower part of the cloud in greater concentrations than would otherwise be possible. If all the liquid water were cloud droplets, with negligible fall relative to the air, the greatest liquid water content would be in the middle and upper parts of the cloud. The accumulation in the lower levels of the cloud provides a larger supply for evaporative cooling than in the absence of precipitation, and the more efficient cooling leads, in turn, to a considerably stronger downdraft. The precipitation process appears essential to the dissipation of the updraft, which evolves to a steady state without precipitation but eventually decays rapidly in its presence. The low-level cooling due to precipitation ultimately cuts off the updraft from the potentially warmest air, its main source of buoyancy.

6) While considerably smaller than the vertical perturbed pressure gradient force, the pressure contribution to buoyancy has two main effects in the model. Under the leading edge of the cloud and in the lower part of the main updraft, positive buoyancy due to relatively low pressure maintains the updraft and enhances the mass flux of low-level air into the updraft, while negative buoyancy due to relatively high pressure in the upper levels of the updraft significantly decelerates the updraft at lower altitudes than would be predicted by parcel theory or other theories which neglect pressure perturbations. In the building stage of the storm, the second of these effects overrides the first, making the updraft slightly less intense and shallower than would otherwise be the case. In the mature stage, however, the first of these two effects predominates. As a result, the pressure buoyancy helps to keep the mature updraft significantly stronger than if the buoyancy were merely thermal.

7) In this model, a sharp upper-level jet does not result in a more intense storm than a broad wind maximum throughout the upper levels, although the growth stage and mature stage are prolonged. However, this conclusion should be regarded cautiously since the severity of actual thunderstorms may be increased through synoptic-scale destabilizing mechanisms not incorporated into the model. In particular, large-scale mass convergence has not been included in the horizontally homogeneous base state initially assumed.

Acknowledgments. I am indebted to Prof. John A. Young for his valuable and constructive guidance during the writing of this paper and during the development of the numerical model described therein. Special thanks are also extended to Profs. David D. Houghton, Heinz H. Lettau and Frank S. Sechrist.

The numerical computations were performed on the CDC 6600 at the National Center for Atmospheric Research and on the UNIVAC 1108 at the University of Wisconsin. I would like to thank Daniel Anderson at the National Center for Atmospheric Research, and Peter Guetter at the University of Wisconsin, for their aid in programming.

The research was supported by NOAA Grant E-230-68-G, with partial funding by NSF Grant GA-30676.

APPENDIX

Air Trajectory Calculations

The air parcel trajectories discussed for the prototype experiment were obtained by forward integration in time, estimating velocity components by double linear extrapolation within appropriate grid boxes, as described below.

Let \( X' \) and \( Z' \) denote the coordinates of a given parcel at the end of the \( n \)th time increment, with the origin taken at the lower left corner of the grid (\( i = j = 1 \)). The lower left corner of the grid box containing the parcel has indices \( I \) and \( J \), where

\[
I = 1 + \lceil X' / \Delta x \rceil, \quad (A1a)
\]

\[
J = 1 + \lceil Z' / \Delta z \rceil, \quad (A1b)
\]

and \( \lceil Q \rceil \) denotes the greatest integer not exceeding an arbitrary quantity \( Q \). The parcel velocity components \( U' \) and \( W' \) at the \( n \)th time level are estimated by the following double linear extrapolation formula using the corner points \((x_i, z_j), (x_{i+1}, z_j), (x_i, z_{j+1})\) and
(x_{t+1}, z_{t+1}) of the grid box:
\begin{align}
U'' &= a_1 + a_2 (X' - x_t) + a_3 (Z' - z_t) \\
&\quad + a_4 (X' - x_t) (Z' - z_t), \quad (A2a)
\end{align}
\begin{align}
W'' &= b_1 + b_2 (X' - x_t) + b_3 (Z' - z_t) \\
&\quad + b_4 (X' - x_t) (Z' - z_t), \quad (A2b)
\end{align}
where
\begin{align}
a_1 &= n_1^t, \quad (A3a)
\end{align}
\begin{align}
a_2 &= (n_1^{t+1} - n_1^t) / \Delta x, \quad (A3b)
\end{align}
\begin{align}
a_3 &= (n_2^{t+1} - n_2^t) / \Delta z, \quad (A3c)
\end{align}
\begin{align}
a_4 &= (n_1^{t+1} - n_1^t - n_2^{t+1} + n_2^t + n_1^t - n_1^t) / (\Delta x \Delta z), \quad (A3d)
\end{align}
and the coefficients b_1-b_4 are defined analogously with u replaced by v.

Preliminary estimates of the parcel coordinates at the (n+1)st time level are then given by
\begin{align}
X''' &= X' + U' \Delta t' + U'' \Delta t'' / 2, \quad (A4a)
\end{align}
\begin{align}
Z''' &= Z' + W' \Delta t' + W'' \Delta t'' / 2, \quad (A4b)
\end{align}
where \( \Delta t'' \) is the time increment between the nth and (n+1)st time levels. If any of the four conditions \( X'' < 0, X''' > (M-1) \Delta x, Z'' < 0 \) or \( Z''' > (N-1) \Delta z \) hold, where \( M \) and \( N \) denote the numbers of grid points in the x- and z-directions, respectively, then the parcel has been carried outside the grid domain and is no longer tracked. Otherwise, the preliminary location of the parcel at the (n+1)st time level is now in the grid box whose lower left-hand corner has indices \( II \) and \( JJ \) given by
\begin{align}
II &= 1 + \lceil X''' / \Delta x \rceil, \quad (A5a)
\end{align}
\begin{align}
JJ &= 1 + \lceil Z''' / \Delta z \rceil. \quad (A5b)
\end{align}

Parcel velocity components \( U'' \) and \( W'' \) at the (n+1)st time level are estimated by the same double interpolation scheme used for obtaining \( U' \) and \( W' \).

Finally, a revised estimate \( (X''', Z''') \) of the parcel location at the (n+1)st time level is made by using the average velocity components during the (n+1)st time increment instead of the velocity components at the nth time level. Mathematically,
\begin{align}
X'''' &= X' + (1 + l'' + l'''(\Delta u_{11} t)) / 2, \quad (A6a)
\end{align}
\begin{align}
Z'''' &= Z' + (1 + l'' + l'''(\Delta u_{11} t)) / 2, \quad (A6b)
\end{align}

In order to keep the model cloud within the domain of computation longer than otherwise possible, the grid is bodily shifted toward the right in systematic fashion, \( \Delta x \) at a time, as described previously (Schlesinger, 1973). If the grid is shifted at the (n+1)st time level following the foregoing computations, \( X''' \) is then decreased by \( \Delta x \) while \( Z'''' \) is unchanged. If the parcel is removed from the grid as a result of the shift [in case \( X''' \) as calculated from \( (A6a) \) is less than \( \Delta x \)], tracking is discontinued. Otherwise, \( X'''' \) and \( Z'''' \) play the same role as did \( X' \) and \( Z' \) at the end of the nth time increment, and the parcel is then followed through the (n+2)nd time increment.

REFERENCES


