Thermodynamics of Cloud Glaciation[1]

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(Manuscript received 1 May 1973, in revised form 2 July 1973)

ABSTRACT

A detailed thermodynamical analysis of the glaciation of a supercooled cloud parcel has been carried out, applying a linear approximation to the vapor pressure curves for ice and water. The process can lead to three different final states: one in which partial glaciation of the supercooled cloud water brings the parcel temperature to 0C, one in which sublimation of water vapor takes place from the frozen phase, and one in which deposition occurs to it. It is pointed out that a treatment which considers only the release of latent heat of fusion, without taking into account the subsequent change in water vapor density, is a special case and is generally incorrect.

1. Introduction

Glaciation of a supercooled cloud, once triggered by the appearance of the ice phase, proceeds with the help of the thermodynamic driving force for the phase change involved. The process therefore plays a powerful role in cloud phenomena such as precipitation formation. The process is qualitatively well understood. It is the quantitative aspect of the process that has created occasional confusion and deserves detailed investigation.

Applying thermodynamics, Saunders (1957) outlined the isobaric freezing of supercooled clouds; however, he did not go into a detailed analysis of it. MacCready and Skutt (1967) analyzed it later in some detail, employing a graphical method. However, expressions for describing a clear picture of the process are useful in directly illustrating relationships among thermodynamic variables and in computations involving the process. Orville and Hubbard (1973) have briefly covered this subject during their recent discussion on cloud processes employing Saunder's technique. It is the purpose of this paper to clarify this thermodynamic picture of phase changes and to analyze the obtained expression in more detail in terms of cloud-physical factors such as heating, buoyancy, and masses of condensed phases in a cloud parcel.

2. Thermodynamics of cloud glaciation

a. *Parcel warming due to glaciation*

Suppose a supercooled cloud parcel of volume V_1 and mass m under pressure P_1 and temperature T_1 glaciates isobarically; m_1 grams of supercooled water in the parcel with vapor pressure p_1 change into $m_2=m_1+\Delta m_c$ grams of ice with vapor pressure p_2 (Δm_c; the increased mass of the condensate), and the heat liberated during the freezing of the supercooled droplets and additional deposition of the water vapor on the frozen droplets is spent to heat the system up to temperature T_2. For adiabatic cloud processes at constant pressure, the total enthalphy change of the system is zero (Guggenheim, 1950), i.e.,

$$L_f m_1+L_d\Delta m_c-c_p m\Delta T=0, \tag{1}$$

where L_f and L_d are respectively the latent heats of fusion and of deposition per gram of water, c_p the specific heat of the system at constant pressure (system including dry air, water vapor and condensed phase), and $\Delta T=T_2-T_1$. Eq. (1) is identical to that of Saunders.

Now, the problem is to determine p_2, which is a function of the undetermined variable T_2. The Clausius-Clapeyron equation is applicable for the description of p_1 and p_2, but use of its expanded form does not lead to a simple analytic expression for the process. Therefore, we apply an approximation as shown in Fig. 1, where the treatment relies on the approximate linearity of the vapor pressure curves for both supercooled water and ice within the temperature range T_2 to T_1.

When water vapor saturated with respect to supercooled water deposits on ice at the same temperature, the vapor change is b (Fig. 1a) and the deposited vapor heats up the system by a. The system must move along the line AB, and the final point must be on the ice vapor pressure line, i.e., it must be at point D. Then,

$$\frac{\delta p}{b}=\frac{\delta T}{a}. \tag{2}$$

Inserting

$$\delta T\tan\alpha=b-\delta p \tag{3}$$

[1] Presented at the Third Conference on Weather Modification, 26–29 June 1972, Rapid City, S. D.

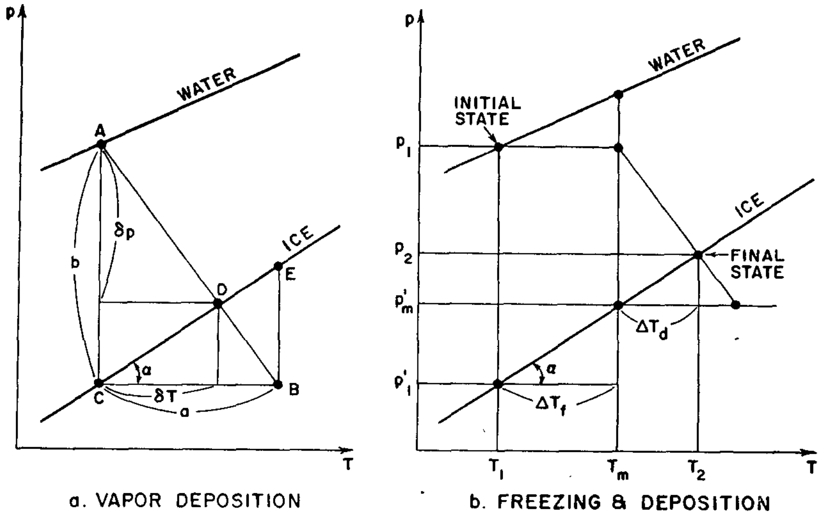

FIG. 1. Thermodynamic processes in the glaciation of supercooled cloud parcels.

into (2), we have

$$\delta T=\frac{b}{\tan\alpha+\dfrac{b}{a}}. \tag{4}$$

Then, from (2) and (4)

$$\delta p=\frac{b}{\dfrac{a}{b}\tan\alpha+1}. \tag{5}$$

When supercooled droplets exist, the released latent heat of fusion due to glaciation warms the system up by $\Delta T_f=T_m-T_1$ (Fig. 1b). Therefore, point C in Fig. 1a now corresponds to point (p_m', T_m) in Fig. 1b. Then

$$\Delta T_f=\frac{L_f m_1}{c_p m}, \tag{6}$$

where m_1/m is the mixing ratio of liquid water in grams of water per gram of air. The differential form of the Clausius-Clapeyron equation is

$$\tan\alpha=\frac{dp}{d_T}\approx\frac{p_1' L_d M_w}{RT_1^2}, \tag{7}$$

where p_1' is the vapor pressure of ice at T_1, M_w the molecular weight of water, and R the universal gas constant. Using Fig. 1b, we can write

$$b=p_1-p_m'=(p_1-p_1')-\Delta T_f \tan\alpha. \tag{8}$$

Now, the problem is to obtain an expression for a in terms of known quantities. Applying the ideal gas law, the state of the air parcel is given as

$$p_1V_1=n_1RT_1, \tag{9}$$

where n_1 is the number of moles of the gas molecules in the parcel. Since the mixing ratios of the water vapor and droplets in air are small, some of their extensive quantities such as volume and heat capacity can be neglected. Therefore, n_1 may be taken as the number

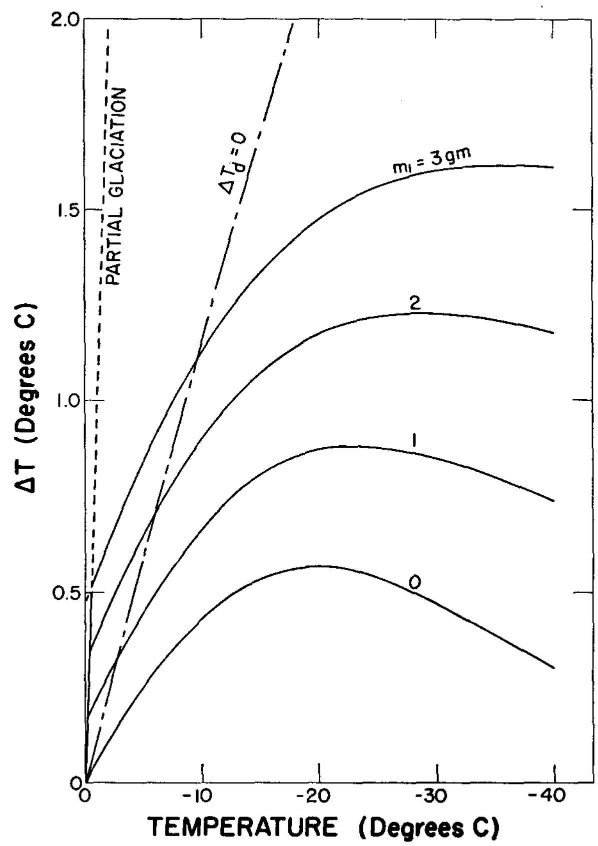

FIG. 2. Warming of supercooled cloud parcels after glaciation in the standard atmosphere ($V_1=1$ m³).

of moles of air molecules in the parcel. Using the ideal gas law again to the water vapor, we have

$$(p_1-p_2)V_1=\Delta nRT_1, \qquad (10)$$

where Δn is the number of moles of water vapor corresponding to the pressure change (p_1-p_2) under V_1 and T_1. Inserting (9) into (10), and remembering that $n_1=m/M_a$, where M_a is the average molecular weight of air, and $p_1-p_2=\delta p$, we obtain

$$\Delta n=\frac{m}{P_1M_a}p. \qquad (11)$$

We then multiply (11) by M_w on both sides of the equation, compare the product with (1) under $m_1=0$ (i.e., deposition of the saturated vapor with respect to water), and set $b=p_1-p_m'$ and $\Delta m_c=\Delta nM_w$, to obtain

$$a=\frac{L_dM_w}{c_pM_a}\frac{p_1-p_m'}{P_1}. \qquad (12)$$

Inserting (6), (7), (8) and (12) into (4) and setting $\delta T=\Delta T_d$, we have

$$\Delta T_d=\frac{\left[\dfrac{p_1-p_1'}{p_1'}\left(\dfrac{RT_1^2}{L_dM_w}-\dfrac{L_f}{c_p}\right)\dfrac{m_1}{m}\right]}{\left[1+\dfrac{P_1c_pM_aRT_1^2}{p_1'L_d^2M_w^2}\right]}. \qquad (13)$$

Now, all the variables needed to give the temperature increase are functions of the initial conditions and are hence readily available. Then, using (6) and (13), the total warming

$$\Delta T=T_2-T_1=\Delta T_f+\Delta T_d, \qquad (14)$$

may be obtained. The computed ΔT for the standard atmosphere is given in Fig. 2.

When the cloud parcel is only slightly supercooled, there exists a condition in which glaciation is sufficient to warm the parcel to 0C. In this case

$$\Delta T=T_0-T_1, \qquad (15)$$

where T_0 is the melting point of ice or 273.2K. Inserting (15) into (14) and using (6) and (13), this condition can be sought between T_1 and m_l/m (trial-and-error method). The result is shown in Figs. 2 and 3.

Since ΔT_d can take either positive or negative values, the condition where $\Delta T_d=0$ is found by setting the numerator of (13) to be zero:

$$\frac{m_1}{m}=\frac{c_pRT_1^2}{L_fL_dM_w}\frac{p_1-p_1'}{p_1'} \qquad (16)$$

(see Figs. 2 and 3). When the left side of (16) is larger than the right side, corresponding to the zone to the left of $\Delta T_d=0$ line in Figs. 2 and 3, droplets lose mass by sublimation after freezing. In the zone on the right of the $\Delta T_d=0$ line, the frozen droplets become heavier because of the additional deposition of vapor.

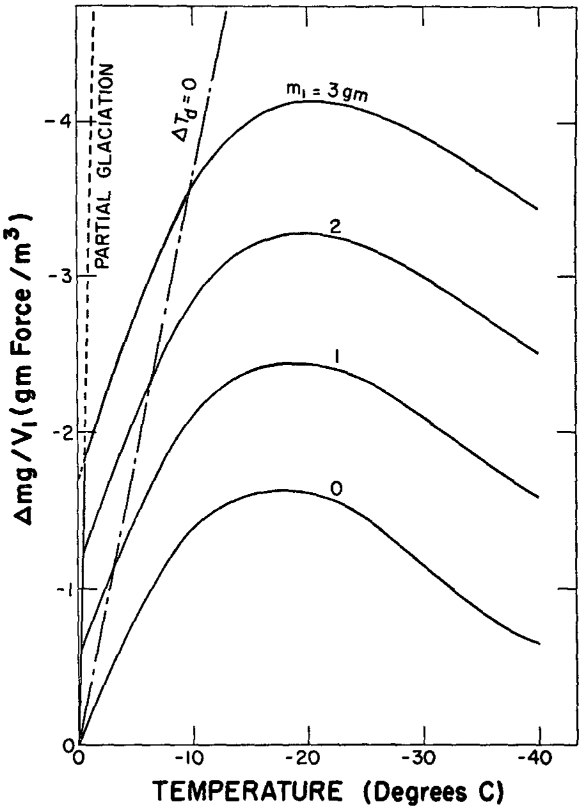

Fig. 3. Buoyancy increase in supercooled cloud parcels due to glaciation in the standard atmosphere (V_1=1 m^3).

b. Change of condensate weight due to glaciation

The amount of additional vapor deposited onto the formed ice or sublimed from it, $\Delta m_1(=\Delta m_c)$, in the air parcel of volume V_1 is given, after consideration of the ideal gas law, as

$$\Delta m_1=\frac{M_wV_1}{RT_1}(p_1-p_2)$$

$$=\frac{M_wV_1}{RT_1}\left[(p_1-p_1')-\frac{p_1'L_dM_w}{RT_1^2}(T_2-T_1)\right]. \qquad (17)$$

The computed values of Δm_1 for the standard atmosphere are shown in Fig. 4.

When the liquid water content is zero but the air is still saturated with respect to water, vapor deposition occurs throughout the temperature range in question. The maximum amount of vapor deposition is about 0.15 gm m^{-3} at around -15C. As the liquid water content increases in a slightly undercooled cloud,

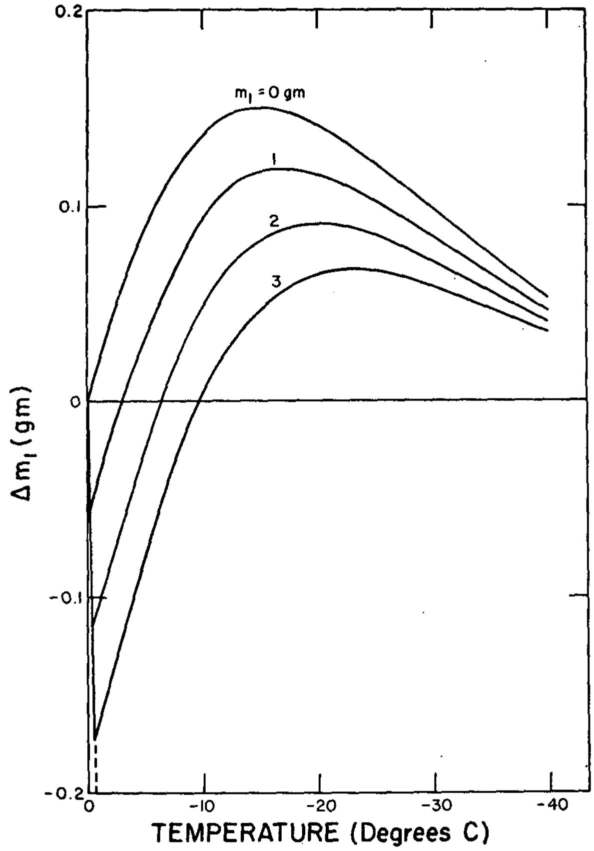

FIG. 4. Change of condensate masses during the course of supercooled cloud glaciation in the standard atmosphere ($V_1 = 1$ m³).

deposition switches over to sublimation from the frozen droplets.

c. Cloud buoyancy increase due to glaciation

Using (6), (13) and (14), the buoyancy increase per unit volume,

$$\frac{\Delta mg}{V_1} = \frac{mg}{T_1 V_1} \Delta T, \qquad (18)$$

where g is the gravitational acceleration, can also be obtained. The computed values of the buoyancy increase for the standard atmosphere are shown in Fig. 3.

It may be seen from Fig. 3 that the buoyancy increase due to glaciation of a supercooled cloud parcel is greater if the liquid water content is larger. As the temperature decreases, the magnitude of the buoyancy increase in a parcel of fixed liquid water content becomes larger. It reaches a maximum at around −18 to −20C, depending on the liquid water content, and thereafter decreases. At this temperature, the buoyancy increase in a cloud parcel with $m_1 = 2$ gm m^{-3} is approximately double that in the vapor saturated with respect to supercooled water.

For a slightly supercooled cloud, the warming or buoyancy increase is mostly controlled by the liquid water content.

3. Discussions and concluding remarks

We have discussed above the thermodynamical problem of cloud parcel glaciation, applying a linear approximation method. A more rigorous treatment results in a transcendental equation, and its evaluation with the help of an electronic computer will give slightly better numerical accuracy in the treatment.

It is evident from the present analysis that glaciation of a supercooled cloud parcel falls into the following three categories: partial glaciation of the supercooled water bringing the final temperature of the parcel to 0C; sublimation of water vapor from the frozen phase after glaciation; and deposition of the vapor to the frozen after phase change. Correspondingly, there are two bordering cases, i.e., glaciation of the parcel resulting in the 0C temperature with completely frozen liquid phase (the partial glaciation line in Figs. 2 and 3), and parcel glaciation which merely alters the phase of the supercooled liquid into ice without changing the vapor density.

A treatment which considers only the release of the latent heat of fusion during the glaciation of the parcel, therefore, is rigorous only in this latter bordering case, and as conditions shift away from this case, it rapidly becomes invalid. If the liquid water content is larger and the temperature is higher than in this, the bordering case, vapor sublimes out of the frozen droplets so as to achieve the final equilibrium, and vice versa. The relatively large amount of sublimation at slightly undercooled conditions is attributed to the smaller distance between saturated vapor pressure curves of ice and water. The freezing of a given amount of supercooled water therein shifts the temporary state of the system well into the undersaturated area with respect to ice, so that a system readjustment takes place by subliming vapor from the frozen particles.

The partial freezing occurs only when the initial temperature of the cloud plume is between 0 and −1C, if the liquid water content is less than 3 gm m^{-3}. Since the rate of ice crystal growth is very slow in this temperature zone, this process is of little importance in normal practice.

As has been shown above, the buoyancy increase for a cloud parcel of a given liquid water content goes through a maximum while the density of the air is a monotonically decreasing function of altitude (or temperature) in the standard atmosphere. Therefore, an altitude (or a temperature) exists where the acceleration due to the phase change becomes a maximum for a parcel containing a given amount of supercooled water in unit volume.

Acknowledgment. This study was supported by the Atmospheric Water Resources Research Program, Bureau of Reclamation, under Contract 14-06-D-7048.

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