

Snow Crystal Habit at Small Excesses of Vapor Density over Ice Saturation

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ABSTRACT

Artificial snow crystals were grown in a diffusion chamber of parallel-plate design which afforded accurate control of the crystal growth conditions. Observations were made at small excesses of vapor density over ice equilibrium ($\Delta\rho=0.015\text{--}0.169\text{ gm m}^{-3}$) within the temperature range -8 to -24C . Crystal type was found to be primarily determined by the magnitude of the vapor density excess, in apparent contrast with the situation at supersaturations with respect to water where growth habit is a function of temperature. Transitions between crystal types were noted to be gradual rather than abrupt.

1. Introduction

The striking variations that snow crystal forms exhibit in the atmosphere have motivated a great deal of observation and research. The systematic nature of the variations of crystal habit with environmental conditions was recognized early, but many details still remain to be solved. In this paper we address some questions connected with the growth of ice crystals from the vapor.

The physical factors now recognized to be contributing to the ice crystal growth process fall into two interacting areas: the transport of vapor to the crystal and the incorporation of molecules into the crystal. Vapor transport is governed by the diffusion field near the growing crystal and this has to be in concert with the thermal transport field. At the crystal's surface, the factors of importance are adsorption and surface migration of the molecules on the surface, nucleation of growth steps, the possible existence of a quasi-liquid film on the ice surface, and the bonding of molecules into the lattice. Several of these growth factors are influenced by temperature and humidity conditions; of lesser influence are variables such as total pressure, and the impurity content of the environment and of the crystal. In different descriptions of ice crystal growth, different factors have received emphasis; some of the factors listed above are, in fact, incompatible with one another. An excellent summary of the literature up to 1970 was given by Lamb (1970). More recent contributions are those of Lamb and Hobbs (1971), Michaeli (1971), Knight and Knight (1972) and Lacmann and Stranski (1972).

In gross terms, crystal habits are determined by the

values of the external variables of temperature and water vapor pressure. For atmospheric application, definitions of growth habits are important for modeling of cloud glaciation and precipitation processes. The observations of Gold and Power (1954), Nakaya (1954) and Magono and Lee (1966) helped delineate the temperature and supersaturation dependence of crystal forms. It is difficult, however, to accurately determine the temperature and supersaturation for the region of a crystal's origin. Furthermore, for natural ice crystals the vapor pressure is always between ice saturation and a slight excess over water saturation. It was hoped that laboratory experiments would overcome both difficulties. A succession of experiments by aufm Kampe *et al.* (1951), Nakaya (1954), Hallett and Mason (1958) and Kobayashi (1957, 1960, 1961) led to increasing refinement of the functional dependence of crystal form on growth conditions. The pattern that emerged is a cyclic change of habit with decreasing temperature, plate-column-plate-column, at least as long as the vapor pressure exceeds water saturation. For conditions between ice saturation and water saturation, temperature was found to be insufficient to define growth habit; vapor pressure appeared to have a significant influence. The relative roles of temperature and of vapor density were extensively discussed by Marshall and Langleben (1954), Kobayashi (1957, 1960, 1961) and Hallett and Mason (1958). This question is of importance in testing various growth hypotheses and for predicting natural crystal habits.

In this paper we report on observations of crystals grown in the laboratory at saturation ratios less than 1 with respect to water. This regime appeared worthy of renewed investigation because of the relatively small numbers of previous measurements. Kobayashi's (1961) observations provided the most comprehensive data; these were obtained in an externally humidified

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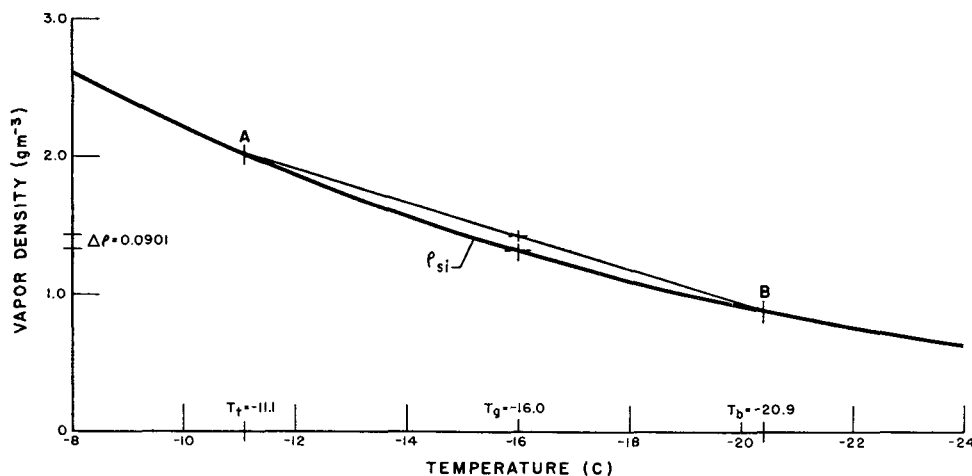


FIG. 1. The actual vapor density distribution within the chamber, line AB represents an excess with respect to the saturation vapor density (ρ_{si}).

chamber which was forcibly stirred. The influence of ventilation in crystal growth is not well understood and thus a different experimental approach was needed to confirm his results.

2. Experimental concept

a. Diffusion chamber

The environments of controlled supersaturations, which were needed to evaluate crystal habits at low vapor density excesses ($\Delta\rho$), were produced by means of a parallel plate thermal diffusion chamber similar to diffusion cloud chambers used for cloud condensation nucleus (CCN) counters. In these chambers a temperature difference is maintained between two horizontal plates such that the top plate is warmer than the bottom plate. The top is usually a wet porous surface and serves as the vapor source. The bottom plate is also wet and serves as the vapor sink. At both plates, the saturation ratio is unity or, in other words, the vapor density excess is zero. The temperature inversion in the chamber suppresses convection and hence the transport of vapor downward from the top is assumed to occur solely by diffusion. Since the air in the chamber is stable, the gradients of temperature and of water vapor pressure can be assumed to be linear.² However, due to the fact that the saturation vapor pressure for water or for ice does not vary linearly with temperature, a supersaturation exists at all points between the plates. At the vertical center the vapor density excess becomes a maximum. The vertical center was used in this study as the growth region and ice coatings were used on both plates.

² The linearity assumption implies that the thermal conductivity K and diffusivity D are constant. Comparison calculations, using the known dependences of K and D on temperature, gave results that were negligibly different from the linear values. This agrees with the finding of Hallett and Mason (1958).

A graphical example of the manner in which supersaturations arise in the chamber may be seen in Fig. 1. The curve represents the saturated vapor density (ρ_{si}) with respect to ice as a function of temperature. Since temperature varies linearly with vertical distance in the chamber, the abscissa can also be thought of as a distance scale. The actual vapor pressure between top and bottom plates can thus be represented by the straight line AB, where A and B denote the saturated vapor density at the plates. Since this straight line is above the saturated vapor density curve at all points between A and B, a supersaturation exists everywhere within the chamber with a maximum near the vertical center. The supersaturation ratio (actual to saturated vapor density) may be found by reading the two corresponding vapor densities on the ordinate at any given temperature. The maximum supersaturation in the chamber may be changed by varying the temperature difference between top and bottom plates. Smaller temperature differences correspond to lower supersaturations and larger temperature differences to higher supersaturations. Fig. 2 shows the variations of temperature, vapor density excess, and saturation ratios within the thermal diffusion chamber for conditions typical of those used in the crystal growth experiments. The vapor density excess curve was calculated as a "true" vapor density excess by using conduction-diffusion equilibrium.

b. Crystal growth

As a crystal grows, vapor is deposited at its surface and is converted to ice. This is accompanied by the release of latent heat at the surface of the crystal. In a steady-state growth condition this heat must be carried away at the same rate as it is released, resulting in an equilibrium between deposition rate and heat conduction. Thus, a growing crystal is warmer than its surroundings and the vapor density excess (environ-

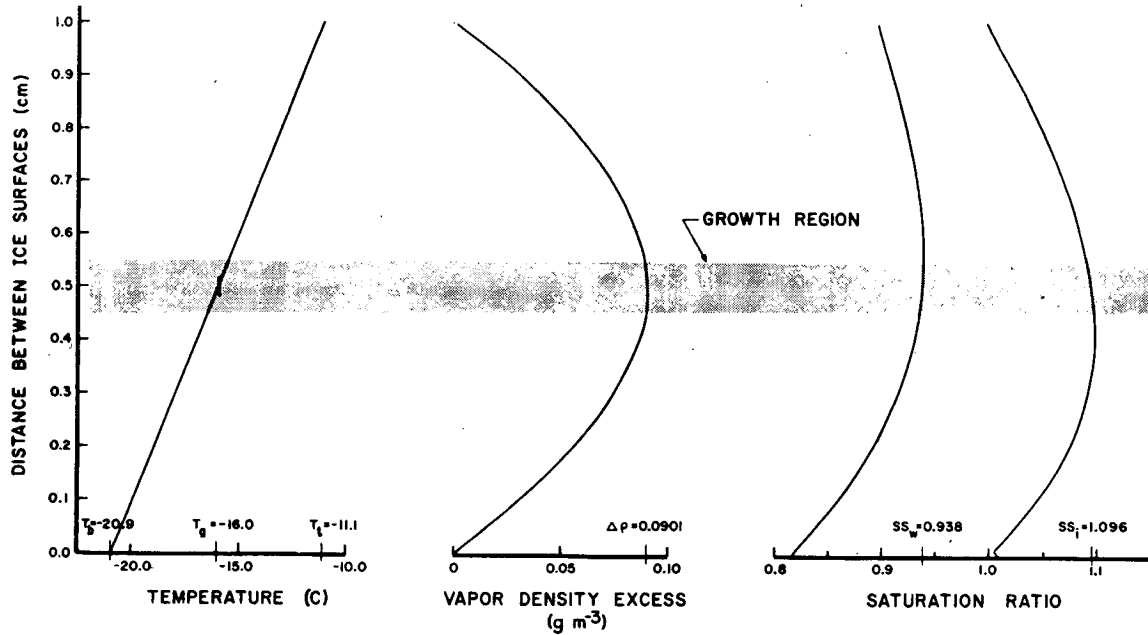


FIG. 2. Calculated profiles of temperature, vapor density excess and saturation ratio within the diffusion chamber.

mental over the crystal surface vapor density) with respect to the growing ice surface is less than would be anticipated if latent heat release were not considered. The use of relative humidity to describe growth or sublimation is therefore fictitious because the conventional expression of relative humidity is with respect to a plane surface at equilibrium. A better alternative is the use of vapor density excess defined as the true vapor density at the surface of the growing crystal (considering a thermal conduction-vapor diffusion equilibrium) subtracted from the environmental vapor density (Marshall and Langleben, 1954).

In the free atmosphere, convection as well as conduction can play a part in the removal of heat from a crystal's surface especially for larger crystals having appreciable fall velocities. The increased heat transfer leads to the maintenance of a larger vapor density excess and increased rates of growth and possibly to differences in habit. Convection was excluded from the experiments by the nature of the apparatus; thus, the results are of primary relevance to the growth habits of small crystals.

c. Equations

Consideration of the thermal conduction-vapor diffusion equilibrium of a growing crystal involves the use of vapor diffusion and thermal conduction equations in conjunction with latent heat, ideal gas law, and water vapor pressure relations.

The basic equation for diffusional growth of ice can be written (e.g., Byers, 1965) as³

$$\frac{dM}{dt} = -AD \frac{d\rho}{dr} \tag{1}$$

The latent heat released at the surface of a crystal which is growing at a rate dM/dt is

$$\frac{dQ}{dt} = L \frac{dM}{dt} \tag{2}$$

For steady-state growth the left side of (2) must be balanced by the rate of heat removal by conduction. The thermal conduction equation for spherical symmetry can be written as

$$\frac{dQ}{dt} = AK \frac{dT}{dr} \tag{3}$$

By combining (1), (2) and (3), the equilibrium relationship between ρ and T can be given in the form

$$\frac{d\rho}{dT} = -\frac{K}{DL} \tag{4}$$

where, in general, K , D and L are functions of T . This differential equation defines the variations of ρ with T between ambient conditions and the boundary conditions at the surface of the growing crystal.

In a plot of vapor density vs temperature, Eq. (4) corresponds to a line such as that shown in Fig. 3 along with the saturation vapor density curve for ice. Point B in Fig. 3 corresponds to conditions at the crystal surface; the ambient conditions correspond to some point A which is at a higher ρ and lower T . In general,

³ A list of symbols is given in an appendix.

point A is defined by the experimental conditions and it is necessary to calculate point B and thereby the true vapor density excess ($\Delta\rho$). This can be done by finding a simultaneous solution of the saturated vapor density function and of (4). Of the several relations proposed in the literature for the saturated vapor density function the Goff-Gratch (1946) formulation was adopted.

Marshall and Langleben (1954) used a finite-difference form of (4) with K , D and L assumed constant for the interval spanned by points A and B. In the present work calculations were made with both constant and temperature-dependent values of K , D and L . The differences in the resulting $\Delta\rho$ values were found to be negligible.

Since the temperature between the top plate (T_t) and the bottom plate (T_b) is assumed to vary linearly, and the crystal is always grown at the vertical center, the average of the two plate temperatures is the growth temperature (T_g). The ambient vapor density at the crystal was obtained as the average of the vapor densities at the two ice plates. When the true temperature of the growing crystal was found from (4), the Goff-Gratch formulation was again used to determine the actual vapor density over the (warmer) growing crystal. This value was then subtracted from the ambient vapor density to yield the true vapor density excess ($\Delta\rho$).

3. Experimental technique

Problems can be encountered with diffusion chambers due to heating or cooling of the chamber walls and convection may result from too great a vertical depth within the chamber (Byers, 1965). It may be noted that Hallett and Mason (1958) were forced to vary the distance between the parallel plates in order to vary

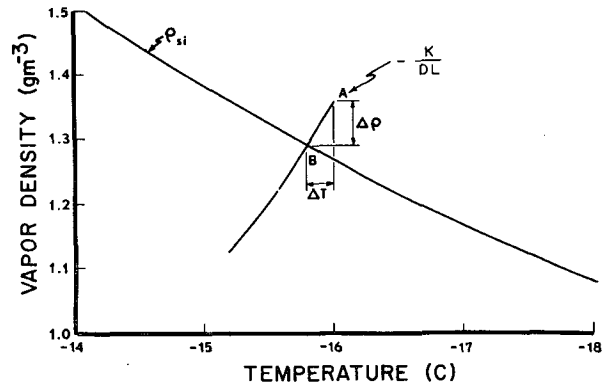


FIG. 3. Temperature-vapor density conditions at the surface (B) and in the environment (A) of a growing crystal.

the temperatures of the plates. The present investigation used thermoelectric coolers and electronic control of the temperatures; therefore, the distance between the plates could be selected to suit other requirements. Twomey (1963) found that a height to diameter ratio of 1:4 was the maximum allowable for a saturation ratio of 1.01 with respect to water before wall effects became significant. For lower saturations he recommended a shallower chamber. The chamber for this investigation had a height to diameter ratio of 1:5.4 with a 1-cm vertical separation of the plates. The walls were constructed of expanded polystyrene. A recent study by Elliott (1971) recommended a maximum ratio of 1:5 and preferably 1:6 or 1:7 or even smaller. The 1-cm separation was therefore still a compromise, but had to be retained in order to have a 1-mm crystal in the chamber cause no great inhomogeneities. Fig. 4 shows a cross section of the thermal diffusion chamber to scale.

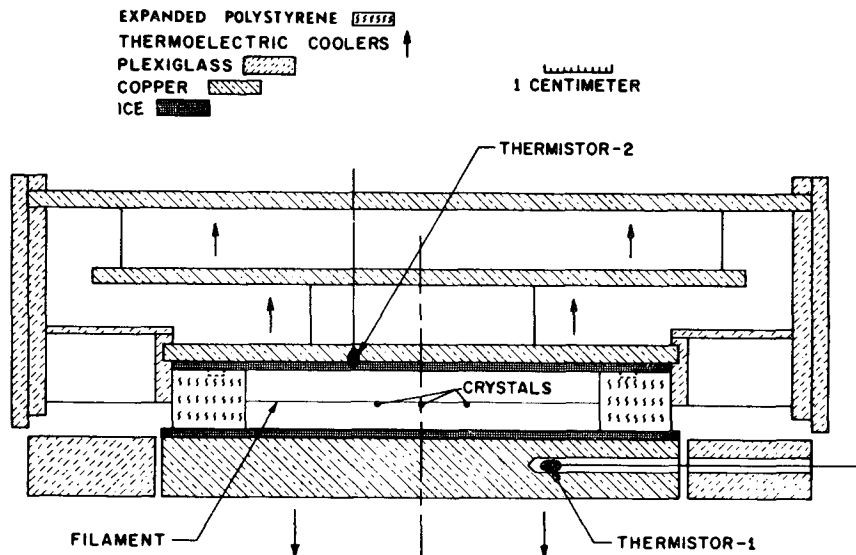


FIG. 4. Cross section of the thermal diffusion chamber to scale.

Calibrations of the temperatures of the two plates in relation to the indicated temperatures and their respective controls were accomplished with sensitive thermocouples mounted on the plate surfaces.

Crystals were grown at the vertical center of the chamber on filaments stretched over expanded polystyrene frames. The frames were 1 cm high and 7.3 cm square with a 5.4-cm diameter hole bored through them. The filaments used were fine human hair of $\sim 50 \mu\text{m}$ diameter.

The ice surfaces were prepared from distilled water. Three small distilled water droplets ≤ 1 mm diameter were placed on the filament at the center and approximately 1 cm to each side. The filament support with the droplets was then placed in a cold box containing dry ice to freeze the droplets and then quickly transferred to the thermal diffusion chamber. The frozen droplets served as centers for the subsequent crystal growth. It has been shown by Hallett and Mason (1958) that ice crystal habit is dependent only upon the immediate environment at a given time and not upon the form or habit the crystal may have achieved previously. The crystal structure of the frozen droplets was, for this reason, taken to be of no relevance. Frequently, at the colder temperatures, crystal growth originated from points on the fiber itself. This was advantageous in producing well-defined, isolated crystals. For good definition of habit, a crystal growing from a frozen droplet had to be sufficiently large to reduce vapor-density gradient effects caused by the droplet. Independence of the observed habits from the position of the crystals on the droplet and from the surrounding growth, as well as reproducibility in repeated experiments, were taken as justification for the validity of the observations.

For subsaturations with respect to water, crystal growth times averaged 5 hr per experiment, but ranged from 3 to 10 hr. Longer times were allowed for growth at smaller supersaturations while supersaturations with respect to water required the least time. After the period of growth the crystals were observed by one of two methods. They were either replicated (with 0.5% Formvar in dichloromethane) in a cold box containing Dry Ice or immersed in a cold hexane bath. Subsequently, the crystals were examined by microscope and classified.

To test the validity of the composite of temperature calibrations, of the assumptions of linearity of temperature and vapor density, and of the negligibility of wall effects, the following experiment was devised. The chamber was set to saturation with respect to water at its vertical center and a supercooled liquid droplet was placed at the center. The change of mass was then determined over a long period of time. The saturation ratio of unity with respect to water was chosen in order to create an equilibrium condition within the chamber and, therefore, remove the necessity of a conduction-diffusion solution of the growth

equations. The liquid state of the droplet allowed for easy geometric determination of mass. If the chamber calibrations and assumptions were correct, no evaporation or growth of the supercooled droplet would be expected. The conditions in the growth region of the chamber were as follows:

$$\left. \begin{aligned} T_g &= -12.0\text{C} \\ SS_w &= 0.9985 \\ SS_i &= 1.1224 \\ \rho &= 2.0221 \text{ gm m}^{-3} \end{aligned} \right\}$$

The initial diameter of the droplet was 8.15×10^{-4} m, the residence time in the chamber was 4.75 hr, and the observed change in the radius of the droplet (Δr_0) was -7×10^{-5} m. The classical growth equation (Byers, 1965) was then applied to obtain the vapor density excess $\Delta \rho_w$ with respect to water:

$$\rho L \frac{dA}{dt} = 8\pi D \Delta \rho_w \quad (5)$$

The value of dA was calculated from Δr_0 and r_0 assuming sphericity for the droplet. The value of $\Delta \rho_w$ was found from (5) to be $-0.0053 \text{ gm m}^{-3}$. The saturated vapor density with respect to water (ρ_{sw}) at -12C is 2.026 gm m^{-3} . The expected value of $\Delta \rho_w$ was -0.0039 ; the difference between the actual and the expected value is equivalent to an error of 0.07%. Based on this result it is reasonable to assume that the accuracies of the crystal growth conditions were on the order of 0.1%. This accuracy was more than adequate for definition of crystal growth regimes for this investigation.

TABLE 1. Temperature and supersaturation values for the 25 experimental conditions.

T_g (°C)	$\Delta \rho$ (gm m ⁻³)	T_b (°C)	T_i (°C)	SS_i	SS_w
- 8.0	0.0152	- 9.7	- 6.3	1.010	0.934
	0.0331	-10.5	- 5.5	1.022	0.945
	0.0615	-11.4	- 4.6	1.041	0.963
	0.0894	-12.1	- 3.9	1.059	0.980
	0.1672	-13.6	- 2.4	1.111	1.028
-12.0	0.0146	-13.8	-10.2	1.012	0.900
	0.0329	-14.7	- 9.3	1.027	0.914
	0.0619	-15.7	- 8.3	1.051	0.935
	0.0917	-16.5	- 7.5	1.076	0.957
	0.1692	-18.1	- 5.9	1.140	1.015
-16.0	0.0148	-18.0	-14.0	1.016	0.869
	0.0336	-19.0	-13.0	1.036	0.886
	0.0630	-20.1	-11.9	1.067	0.913
	0.0901	-20.9	-11.1	1.096	0.983
	0.1696	-22.7	- 9.3	1.181	1.011
-20.0	0.0144	-22.2	-17.8	1.021	0.840
	0.0326	-23.3	-16.7	1.046	0.861
	0.0636	-24.6	-15.4	1.090	0.897
	0.0913	-25.5	-14.5	1.130	0.929
	0.1713	-27.5	-12.5	1.244	1.024
-24.0	0.0145	-26.5	-21.5	1.028	0.814
	0.0338	-27.8	-20.2	1.066	0.843
	0.0638	-29.2	-18.8	1.124	0.889
	0.0911	-30.2	-17.8	1.177	0.931
	0.1696	-32.4	-15.6	1.330	1.052

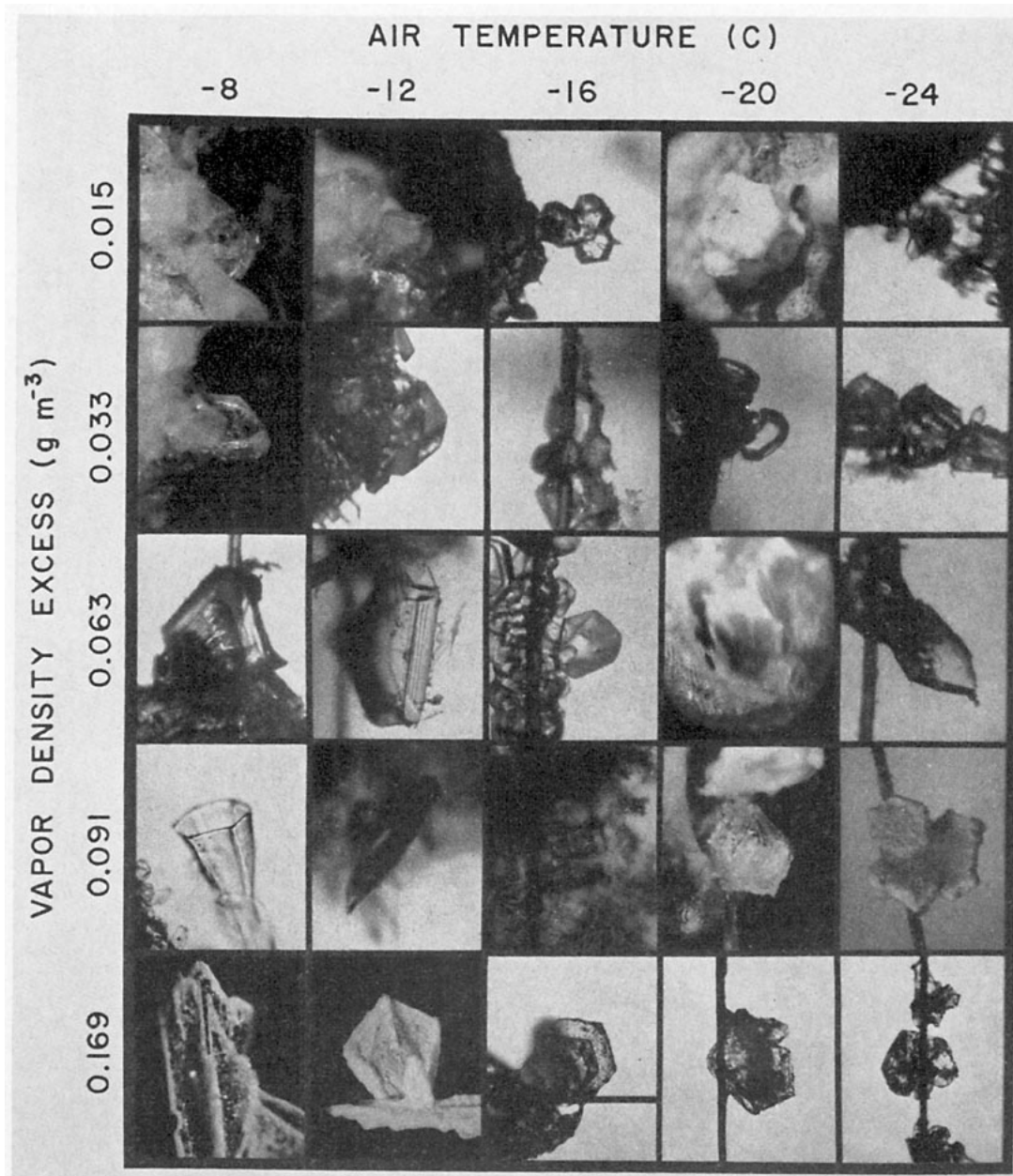


FIG. 5. Selected crystals for each of the temperature-vapor density excess conditions of the experiment. The vertical extents of the frames correspond to viewed dimensions of 1 mm for the top four rows and 1.7 mm for the bottom row.

During one of the experiments, crystals were grown along the entire span of the filament. No noticeable differences in growth were noted along the filament to within 2 mm of the walls. It was therefore concluded that convective and wall effects were negligible. As an added measure, all data points were obtained from crystals which were at least 1 cm away from the walls.

4. Results

Five growth temperatures (T_g) were chosen and at each of these temperatures experiments were performed

at five different vapor density excesses ($\Delta\rho$). The five growth temperatures were -8 , -12 , -16 , -20 and -24 C; the five vapor density excesses were approximately 0.015, 0.033, 0.063, 0.091 and 0.169 gm m^{-3} . The first four $\Delta\rho$ values correspond to subsaturated conditions with respect to water, the last value to a slight supersaturation. Small variations in actual vapor density excesses from one temperature to the other occurred due to the fact that plate temperatures were calculated only to the nearest tenth of a degree Celcius.

Conditions for the matrix of 25 temperature-vapor

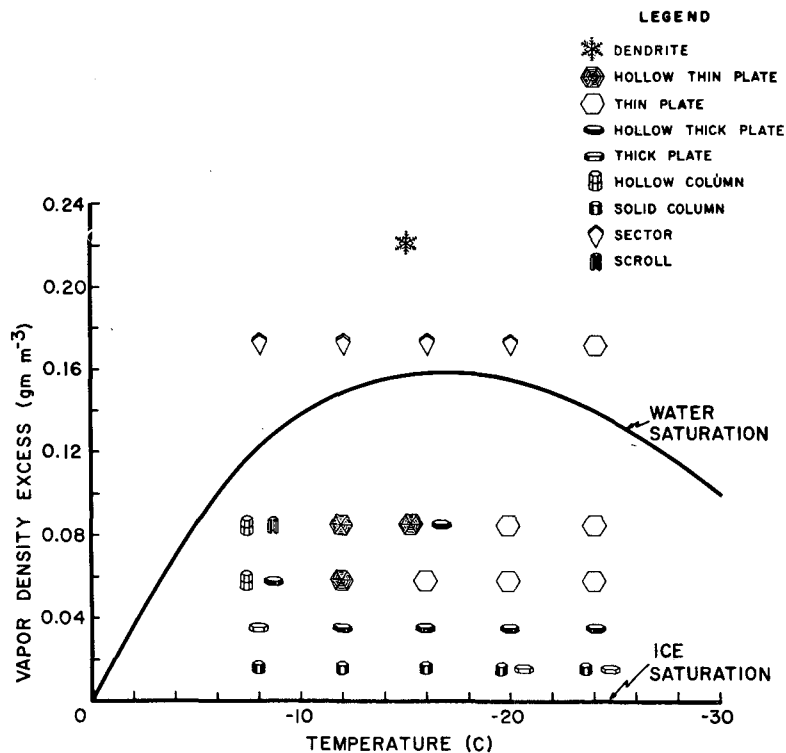


FIG. 6. Classification of crystal types grown at different conditions of temperature and vapor density excess. Where two types of crystal were observed for the same conditions both types are shown even though only one may be illustrated in Fig. 5.

density excess combinations are listed in Table 1. More than one experiment was performed for the majority of points in order to establish repeatability.

Fig. 5 shows typical photomicrographs of the crystals which were obtained for the 25 different growth conditions. Crystals were chosen as typical if and only if they were the predominant species. If two types of crystals were observed for a given point then the most frequent type was chosen for display.

Crystals were classified according to the dimensional criteria

$$\left. \begin{array}{l} \text{Thin plates: } c/a < 0.2 \\ \text{Thick plates: } 1.0 > c/a > 0.2 \\ \text{Columns: } c/a > 1.0 \end{array} \right\},$$

where c and a denote the dimensions of a crystal along the prism axis and along the basal plane, respectively.

Fig. 6 shows a summary of the crystal types observed in this investigation for different growth temperatures and vapor density excesses. At points where more than one type of crystal was observed, both types were indicated in Fig. 6. Kobayashi's (1961) findings are shown for comparison in Fig. 7.

Dendritic crystals were grown at relatively high supersaturations to complement the observations at low saturation ratios. Fig. 8 shows a crystal grown at a temperature of -15.0°C and ice supersaturation of 23% ($\Delta\rho=0.22 \text{ gm m}^{-3}$). Growth time was 1.5 hr.

The pattern of variations in Fig. 6 is not easy to summarize. Changes of habit are apparent with respect to variations of both temperature and vapor density excess. There appears to be a somewhat greater uniformity among the crystals which were grown at specific vapor density excesses than those belonging to a given temperature. Changes of habit tend to be gradual, that is, the c/a ratio varies monotonously with changes in either $\Delta\rho$ or T_g . This is very reassuring, lending credence to the observations and indicates that the existence of undetected additional crystal types between observation points is unlikely. Some ambiguities in the results, where two different habits were observed, remain unresolved; these could have arisen from fluctuations in experimental conditions or from minute factors which have not yet been recognized.

The frequency of occurrence of hollow crystals deserves mention. While the Formvar replication technique obscured hollow features, direct observation in hexane revealed almost all crystals to be skeleton-like. This feature is of importance in considering descriptions of the growth mechanism of the crystals and in influencing the density and the fall velocities of the crystals.

5. Discussion

Nakaya's (1954) laboratory observations do not agree completely with this study. In particular, the

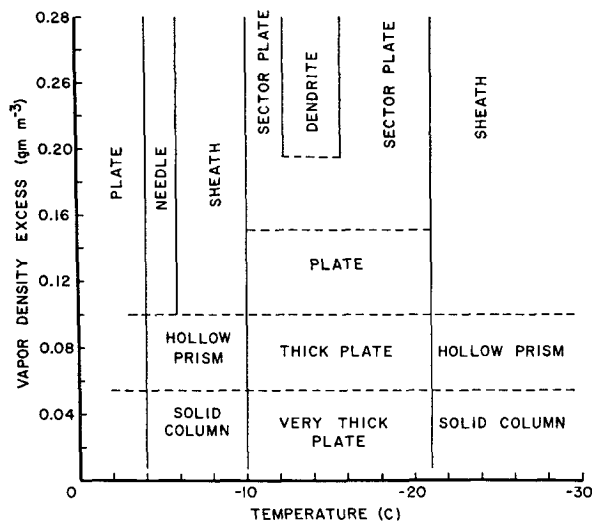


FIG. 7. Classification of crystal types according to Kobayashi (1961). The original data were converted to the units and scales of Fig. 6.

dendritic growth that he noted at saturation ratios less than unity with respect to water was not observed by us. Since neither this study nor the observations of Hallett and Mason (1958) and of Kobayashi (1957, 1960, 1961) have shown dendritic growth at such low saturations, one may conclude that Nakaya had an error in his method of measurement of supersaturations within his chamber.

Comparison of the results of this investigation with those of Hallett and Mason (1958) is limited to the dendritic growth and the relatively few of their observations which were at low supersaturations with respect to ice. The points that can be compared show favorable agreement. The plates observed at $T_g = -24\text{C}$ and $\Delta\rho \approx 0.169\text{ gm m}^{-3}$ agree with Hallett and Mason's observations. However, the sectors grown at $T_g = -12, -16$ and -20C and $\Delta\rho \approx 0.169\text{ gm m}^{-3}$ seem to imply that their boundary for sector growth should be widened to include a greater temperature range.

Extensive comparisons are possible with the results of Kobayashi (1961). Fig. 7 is a translation of his results to the units of Fig. 6. From the water saturation curve given by Kobayashi it appears that his experiments were performed at or near the environmental pressure of 1000 mb. Our experiments were performed at environmental pressures near 775 mb. Thus, due to the thermal conduction-vapor diffusion equilibrium for growing crystals, water saturation at 775 mb corresponds to slightly lower vapor density excesses ($\Delta\rho$) and the maximum is reached at a temperature $\sim 0.8\text{C}$ colder than at 1000 mb. Therefore, an exact translation of Kobayashi's temperature boundaries would require new conduction-diffusion solutions which would yield temperature boundaries beginning at the temperatures given by Kobayashi at the ice saturation line (abscissa)

and which would slope slightly (less than 1C in the region of the water saturation curve) toward colder temperatures. This conversion was not performed since the density of data points was relatively small, and as can be seen from Figs. 6 and 7, the points fall well within the boundaries given by Kobayashi.

The results of this investigation compare favorably with those of Kobayashi and also provide some refinements. The following discrepancies can be noted: for vapor density excesses between 0.05 and 0.10 gm m^{-3} and temperatures colder than -21.5C Kobayashi observed hollow columns, while in this study thin plates were noted. In the region above water saturation and between -6 and -8C he observed sheaths, while our observations showed very long or stretched sector growth. Finally, in the region above water saturation and below -21.5C Kobayashi observed sheaths, while this study yielded thin plate growth.

Between -10 and -21.5C Kobayashi observed very thick plates ($c/a \approx 0.8$) from 0 to 0.05 gm m^{-3} vapor density excess, thick plates from 0.05 to 0.10 gm m^{-3} vapor density excess, and plates from 0.10 to 0.15 gm m^{-3} vapor density excess. We found that the transition from columns ($c/a > 1.0$) to plates ($c/a < 0.5$) occurred gradually over the span from 0 to 0.10 gm m^{-3} vapor density excess. The regions of solid columns given by Kobayashi do not disagree with the present results although thick plates as well as columns were found in these regions. The solid columns occurred at the lower $\Delta\rho$ values; the experiments of Kobayashi apparently did not reveal this transition.

It should be pointed out that Kobayashi utilized a fan to circulate the moist air within his chamber. Some convection was therefore certain to be present and this could be expected to influence the temperature and hence the vapor density excess for the growing crystal. It cannot be ascertained whether this was taken into account or not and this leads to some further latitude in the strictness of comparison between

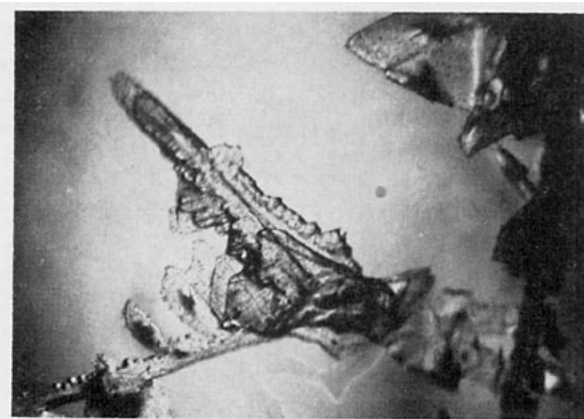


FIG. 8. Dendritic crystals grown at -15C and ice supersaturation of 23% . The horizontal extent of the frame corresponds to 2.5 mm .

Kobayashi's and our results. The relatively good agreement between Figs. 6 and 7 would indicate that the ventilation effects may have been small or were adequately accounted for in the calculations.

The observations of Magono and Lee (1966) were of natural snow. Their observations were quantitative with respect to temperature, but qualitative with respect to moisture; thus it is difficult to make direct comparisons. A cursory comparison reveals several points of good agreement and some possible variances.

In summary, no major discrepancies exist between the different sets of observations. Some ambiguities remain but the dominant features are clear: Solid columns grow at very small vapor density excesses at all temperatures and these gradually change to plate-like growth with increasing vapor supply. Above water saturation, the growth habits become temperature-dependent, as shown by Hallett and Mason (1958) and Kobayashi (1960).

There is a difficulty in precisely evaluating our experiments for conditions exceeding water saturation. The photomicrographs show the presence of water droplets on the crystals. If these droplets were present in the diffusion chamber, and were not artifacts due to the transfer of crystals out from the chamber, then the saturation ratio could not have been significantly above that of water. This may explain the growth observed at all temperatures for $\Delta\rho=0.17$ gm m⁻³. On the other hand, the growth of dendritic crystals at $\Delta\rho=0.22$ gm m⁻³, at -15C, shows that the chamber was not strictly limited to water saturation.

The saturation ratios achieved in a natural cloud are difficult to estimate, especially because ventilation leads to an "effective supersaturation" which is different from the static value. The incorporation of droplets into a growing crystal, as discussed by Nakaya *et al.* (1958), is an additional factor whose role in natural clouds is unknown. These factors will probably resolve why dendritic crystals readily develop in natural clouds even though laboratory experiments indicate that supersaturations of 5-10% are needed for dendritic growth.

The rates of growth of crystals in our experiments were much slower than what would be predicted from calculations for diffusional growth. Jayaweera (1971) shows, for example, that 500- μ m diameter plates, at -15C and at water saturation, should develop in about 10 min, even if ventilation is not considered. In our chamber, growth to that size required as much as 4 hr. Competition among crystals on the fiber during the initial phases of growth, until a large dominant crystal emerged, could have been responsible for this slower growth in the experiment.

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APPENDIX

List of Symbols

A	area through which the vapor flux passes (m ²)
D	the diffusivity of water vapor in air (m ² sec ⁻¹)
K	thermal conductivity in air [cal m ⁻¹ sec ⁻¹ (°K ⁻¹)]
L	latent heat of sublimation (cal gm ⁻¹)
M	mass of vapor (gm)
Q	heat (cal)
r	radius from crystal center (m)
r_0	droplet radius (m)
Δr_0	change in droplet radius (m)
ρ	vapor density (gm m ⁻³)
ρ_{si}	saturation vapor density with respect to ice (gm m ⁻³)
ρ_{sw}	saturation vapor density with respect to water (gm m ⁻³)
$\Delta\rho$	vapor density excess with respect to ice (gm m ⁻³)
$\Delta\rho_w$	vapor density excess with respect to water (gm m ⁻³)
SS_i	saturation ratio with respect to ice
SS_w	saturation ratio with respect to water
t	time (sec)
T	temperature (°K)
T_b	bottom plate temperature (°C)
T_θ	environmental growth temperature (°C)
T_t	top plate temperature (°C)

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