NOTES AND CORRESPONDENCE

On Streamline Patterns and Momentum Transports

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ABSTRACT

The relationship between streamline pattern and momentum transport has been examined. It is shown, for divergent flow, that it is possible for a streamline pattern with troughs and ridges oriented northeast–southwest to have zonally averaged momentum transports associated with it which are directed equatorward in the Northern Hemisphere.

This brief discussion is designed to illustrate that it is possible for a streamline pattern with troughs and ridges oriented northeast–southwest to have zonally averaged momentum transports associated with it which are directed equatorward. This is contrary to the generally accepted view that northeast–southwest tilted troughs are indicative of poleward momentum transports in the Northern Hemisphere. In fact, this viewpoint has led, on occasion, to the inference of a northeast–southwest tilted wave pattern from the positive correlation between the zonal and meridional winds at a single station (e.g., Nitta, 1970). It will be evident from the following analysis that under some circumstances such inferences are at best inconclusive.

Consider a wind field defined such that

$$\left.\begin{aligned} u &= \cos(kx-\alpha y) \\ v &= \cos(kx-\alpha y+\phi) \end{aligned}\right\}, \tag{1}$$

where u and v are deviations from the zonal mean, ϕ the phase difference between them, x and y the eastward and northward coordinates, respectively, and k and α the zonal and meridional wavenumbers. Unit

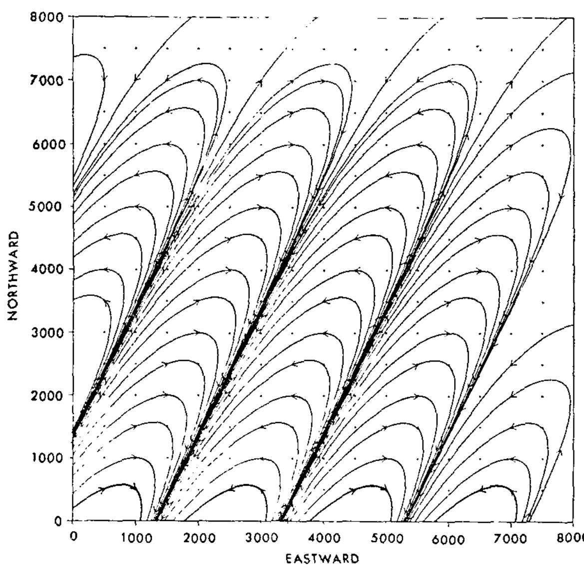

FIG. 1a. Streamlines of the pertubation meridional and zonal wind field for a phase difference of $\pi/4$, zonal wavenumber $2\pi/4000$, and meridional wavenumber $2\pi/8000$.

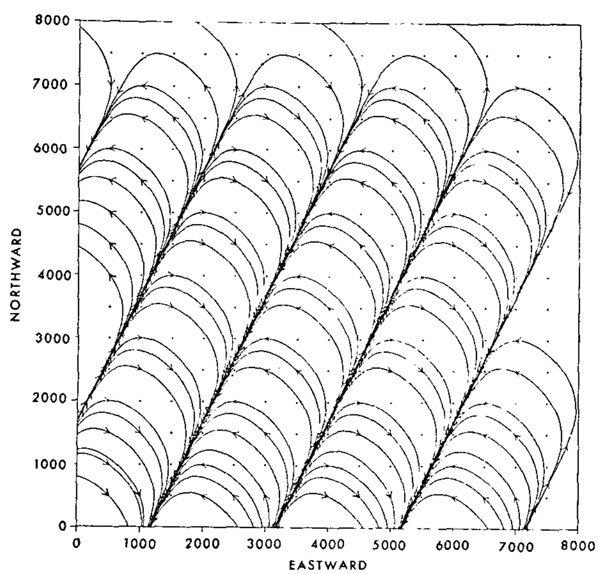

FIG. 1b. As in Fig. 1a except for a phase difference of $3\pi/4$.

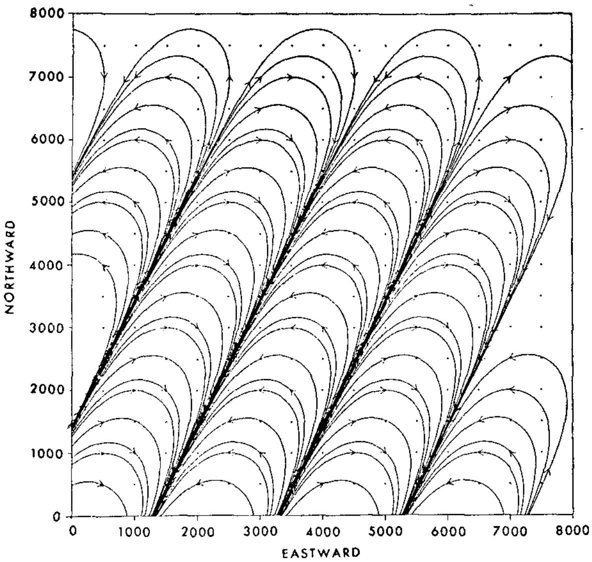

FIG. 1c. As in Fig. 1a except for a phase difference of $\pi/2$.

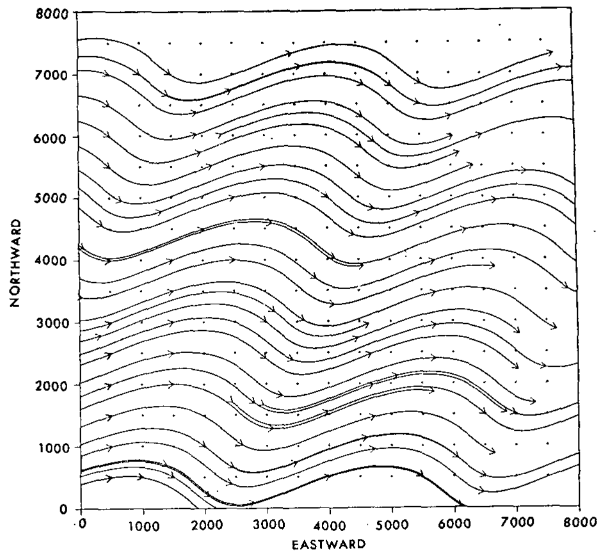

FIG. 1d. As in Fig. 1a except that a basic zonal flow of 2 units has been added.

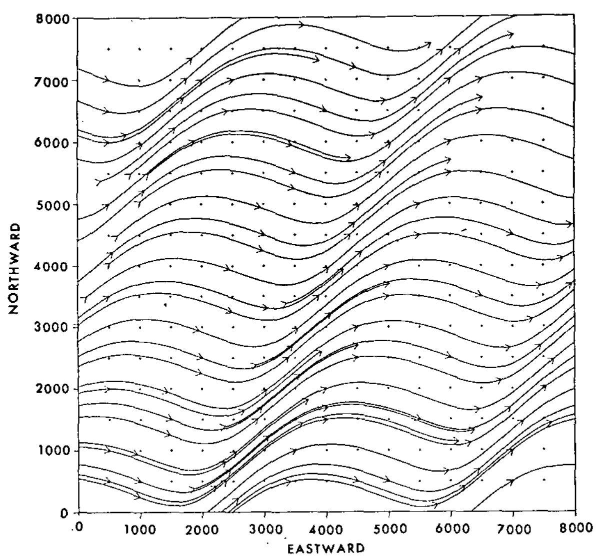

FIG. 1e. As in Fig. 1b except that a basic zonal flow of 2 units has been added.

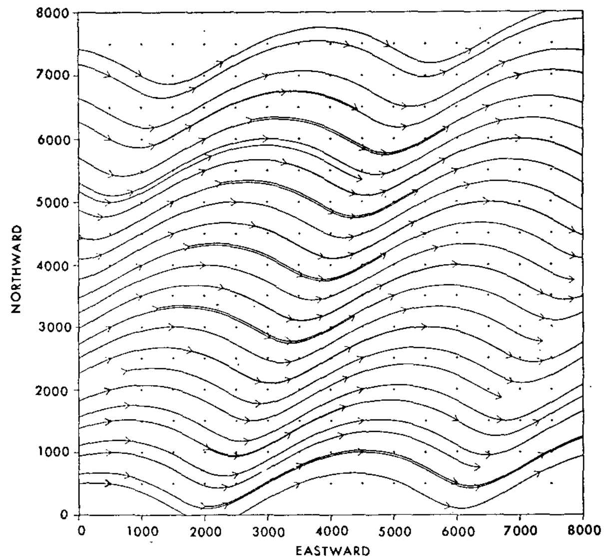

FIG. 1f. As in Fig. 1c except that a basic zonal flow of 2 units has been added.

amplitude has been chosen for both u and v. The slope of the line joining the values of u when $v=0$, i.e., the line through the crests or troughs of the wave pattern, is given by k/α.

The momentum transport at a given latitude over a wavelength L is given by

$$\overline{uv}=\frac{1}{L}\int_0^L \cos(kx-\alpha y)\cos(kx-\alpha y+\phi)dx=\frac{\cos\phi}{2}. \quad (2)$$

The momentum transport around an entire latitude circle is given by the wavenumber times the above expression. And the direction of momentum transport depends on $\cos\phi$.

If

$$\left.\begin{array}{ll}\pi/2>\phi>3\pi/2, & \overline{uv}>0\\ \pi/2<\phi<3\pi/2, & \overline{uv}<0\\ \phi=\pm\pi/2, & \overline{uv}=0\end{array}\right\}.$$

Examples of three cases with and without a zonal flow are shown in Figs. 1a–1f. The zonal flow was taken as a constant value of 2 units and the values of ϕ are $\pi/4$, $3\pi/4$ and $\pi/2$, respectively. The zonal wavelength is $2\pi/4000$ and the meridional wavelength $2\pi/8000$.

It can also be shown that when the flow is nondivergent the direction of momentum transport depends only on the slope of the stream pattern (k/α). Consider the same field as before. We can obtain the nondivergent part of the wind by calculating the vorticity and setting it equal to the Laplacian of the streamfunction. The result after some trigonometric manipulation is given by

$$\nabla^2\psi = -\sin(kx-\alpha y)(k\cos\phi+\alpha) - k\cos(kx-\alpha y)\sin\phi. \quad (3)$$

A solution is of the form

$$\psi_s = A\sin(kx-\alpha y)+B\cos(kx-\alpha y), \quad (4)$$

and

$$\nabla^2\psi_s = -A(k^2+\alpha^2)\sin(kx-\alpha y)-B(\alpha^2+k^2)\times\cos(kx-\alpha y). \quad (5)$$

Thus, ψ_s is a solution if

$$A=(k\cos\phi+\alpha)/(k^2+\alpha^2), \quad (6)$$

$$B=k\sin\phi/(k^2+\alpha^2). \quad (7)$$

The non-divergent zonal and meridional wind components are given as

$$U_s=-\partial\psi_s/\partial y=\frac{\alpha(k\cos\phi+\alpha)}{D^2}\cos j-\frac{\alpha k\sin\phi}{D^2}\sin j, \quad (8)$$

$$V_s=\partial\psi_s/\partial x=\frac{k(k\cos\phi+\alpha)}{D^2}\cos j-\frac{k^2\sin\phi}{D^2}\sin j, \quad (9)$$

where $j=(kx-\alpha y)$ and $D^2=k^2+\alpha^2$. The momentum transport by the nondivergent part of the wind is given by

$$\overline{u_s v_s}=\frac{1}{L}\int_0^L u_s v_s dx=\frac{\alpha k}{2D^4}(k\cos\phi+\alpha)^2+\frac{\alpha k^2\sin^2\phi}{2D^4}$$
$$=\frac{k}{\alpha}\frac{\alpha^2}{2D^4}[(k\cos\phi+\alpha)^2+k^2\sin^2\phi], \quad (10)$$

and we see that the direction of momentum transport depends on the sign of k/α which is the slope as previously defined.

Thus, we might generalize these results to indicate that when we have divergent flow it is possible for stream patterns with a NE–SW slope to exist and have momentum transports directed equatorward in the Northern Hemisphere.

In the mid-latitudes, where the large-scale flow is nearly in geostrophic balance and thus quasi nondivergent, NE–SW tilted troughs and ridges almost always indicate momentum transport northward. This case has been studied by Machta (1949) and Lorenz (1954).

In the tropics, however, the situation is somewhat more complicated. In the absence of precipitation the synoptic-scale divergences are about one order of magnitude smaller than in the mid-latitudes. There are, however, two classes of disturbances in which horizontal divergences are relatively large and cannot be ignored.

One class is precipitating synoptic disturbances. Holton (1972) shows that the large amounts of condensation heating, generally concentrated in the upper troposphere, results in large horizontal divergences. This appears to have been the type of disturbance studied by Nitta. The other class of disturbances are the vertically propagating planetary-scale gravity waves, with vertical scales which are generally small compared to the scale height of the atmosphere, resulting in horizontal divergences which are large (Holton, 1969, 1972). Thus, in the tropics it is possible to have wave patterns tilted northeast–southwest in the Northern Hemisphere and have equatorward directed momentum transports. Clearly, the reliable determination of wave slope in the tropics will be by examination of phase differences with latitude rather than through covariance of zonal and meridional winds at a single station.

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