Determining the Drag Coefficient from Vorticity, Momentum, and Mass Budget Analyses

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ABSTRACT

Data from Period III (19 June–2 July, 1969) of the Barbados Oceanographic and Meteorological Experiment (BOMEX) are used to compute the vorticity budget in the planetary boundary layer. The computed residual, assumed to be the curl of the stress, is then used to obtain an estimate of the drag coefficient. This estimate compares well with results obtained by other BOMEX investigators and in other independent experiments. The top of the planetary boundary layer is assumed to be located at the level where both the stress and stress gradient, computed from the momentum conservation equations, vanish. This is at approximately 1300 m, but tests indicate that the results are rather insensitive to assumed values in the range of 1300 to approximately 1600 m, the base of the trade-wind inversion. Computations for a relatively undisturbed period show a near-balance between anticyclonic vorticity generation by boundary layer divergence and vorticity destruction by friction. However, during a mildly disturbed period the advection and local change terms no longer appear to be negligible.

1. Introduction

a. Purpose and objectives

The purpose of this study is to compute the bulk aerodynamic drag coefficient using the vorticity equation. By incorporating results from the Barbados Oceanographic and Meteorological Experiment (BOMEX) mass and momentum budget analyses (Holland and Rasmusson, 1973, 1974; Rasmusson, 1974) and using surface winds measured on booms extending from the bows of the BOMEX fixed array ships, the various terms in the vorticity equation are evaluated for the planetary boundary layer (PBL). Subsequent calculations using the bulk aerodynamic equation yield the drag coefficient, which is then compared with independently derived estimates of this parameter.

b. Background

Several efforts have been made to include the effects of the PBL in numerical models of the atmosphere. Charney and Eliassen (1949) identify a mass flux or a vertical velocity at the top of the friction layer due to convergence in the PBL. In their study and in others (Cressman, 1960; Panofsky *et al.*, 1957), the vertical velocity at the top of the PBL was assumed to be proportional to the curl of the surface stress. Panofsky *et al.* (1955) compared and discussed the results of the vertical velocities obtained from kinematic, adiabatic and omega equation computations with those calculated from the vorticity equation, excluding friction. They concluded that friction cannot be neglected in estimating the vertical motion from the vorticity equation applied to data in the lower atmosphere, and suggested that the inclusion of the curl of the surface stress is essential for proper computation of vertical velocity from the vorticity equation. It is, however, impractical to obtain direct measurements of stress from a network of stations in the field. Therefore, in typical schemes the flux is parameterized with the square of either the wind speed at some height within the surface layer or the surface geostrophic wind through either a height-dependent drag coefficient C_D (Panofsky *et al.*, 1955, 1957), or a geostrophic transfer coefficient C_g (Charney and Eliassen, 1949; Cressman, 1960):

$$C_D \equiv \tau_0/\rho S^2, \tag{1}$$

$$C_g \equiv \tau_0/\rho G^2, \tag{2}$$

where τ_0 is the surface stress, ρ the air density, and S and G the surface and geostrophic wind speed, respectively.

Panofsky *et al.* (1957) derived a mean drag coefficient, which they termed the resistance coefficient, for the eastern half of the United States by assuming that the difference between vertical velocities at the

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top of the PBL computed by the kinematic method and that computed by the frictionless vorticity equation technique is the vertical velocity due to friction. They then computed the drag coefficient by dividing this difference by the product of the vorticity and the wind speed. Holopainen (1967) also stated that:

> "If calculations of the surface stress curl from the vorticity budget of the atmosphere were made on a daily basis, it might be possible . . . to produce a map of some kind of empirical stress coefficient which then could be used, for example, in the numerical weather prediction models."

Studies of frictionally induced vertical velocity based on actual observations have been sparse. This state of affairs is understandable because accurate measurements in the PBL are very difficult, especially over the oceans. The data set now available over water from BOMEX makes it possible to study the low-level divergence field. BOMEX was primarily designed for study for 1) the vertical fluxes of moisture, momentum and heat, 2) the vertical and horizontal divergences of these fluxes, and 3) the feasibility of parameterizing these fluxes on the BOMEX three-dimensional grid (Kuettner and Holland, 1969). The observational programs of BOMEX, and their objectives, are described in detail in BOMEX Field Observations and Basic Data Inventory (BOMAP Office, 1971).

Here, we will be concerned with the period from 22–29 June 1969, part of BOMEX Observation Period III. Four ships, the *Oceanographer*, *Rainier*, *Mt. Mitchell* and *Discoverer* were positioned at the corners of the 500 km square array, with a fifth ship, the *Rockaway*, placed in the center of the array. Surface wind speed and direction were measured at each ship on a boom which extended about 10 m forward of the bow and approximately 10 m above the sea surface. Instrumentation consisted of a standard U. S. Weather Bureau cup anemometer with accuracy to within 0.5 m s^{-1} and a split-tail windvane, F420 series. Hourly averaged winds centered on the half-hour and corrected for ship motion are computed from two samples per second measurements.

2. Calculation of C_D

a. *Method of solution*

A good approximation to the vorticity equation applicable to BOMEX data is

$$\frac{d\eta}{dt}-\eta\frac{\partial w}{\partial z}\approx\alpha\mathbf{k}\cdot\frac{\partial}{\partial z}\,\mathrm{curl}\,\boldsymbol{\tau},\tag{3}$$

where

$$\eta=\zeta+f,$$

$$\frac{d\eta}{dt}=\frac{d\zeta}{dt}+\frac{df}{dt},$$

$$\frac{df}{dt}=\beta v,$$

$$\mathbf{k}\cdot\mathrm{curl}\,\boldsymbol{\tau}=\frac{\partial\tau_y}{\partial x}-\frac{\partial\tau_x}{\partial y},$$

and all other terms have their usual meanings. Further, it is assumed that the surface stress $\boldsymbol{\tau}_0=\rho C_D\mathbf{V}S$, where S is the speed, $\mathbf{V}$ the vector wind, and C_D a function only of the height of measurement of S. Integrating the vorticity equation from $z=0$ to H, the level where $\mathbf{k}\cdot\mathrm{curl}\,\boldsymbol{\tau}=0$, and solving for C_D at 10 m gives

$$C_D(10)=\frac{-\int_0^H(d\zeta/dt+\beta\bar{v})dz+\eta w(H)}{\mathbf{k}\cdot\mathrm{curl}\,(\mathbf{V}S)_{z=10}}.\tag{4}$$

A much simpler but approximate relationship results if $d\eta/dt$ is neglected, implying that the vertical velocity $w(H)$ is entirely forced by friction. Thus,

$$C_D(10)=\frac{\eta w(H)}{\mathbf{k}\cdot\mathrm{curl}\,(\mathbf{V}S)_{z=10}}.\tag{5}$$

Eqs. (4) and (5) will be tested subsequently. In order to solve (4) or (5), the level H must be determined. Under conditions when $d\eta/dt$ can be neglected, as in "undisturbed" steady flow and where $\rho\eta$ is non-zero and assumed to be constant with height in the PBL, this is accomplished if both curl $\boldsymbol{\tau}$ and $\partial/\partial z$ (curl $\boldsymbol{\tau}$) vanish at $z=H$. Under these conditions H is then collocated at a level of non-divergence, and the profile of vertical velocity and curl $\boldsymbol{\tau}$ are similar.

Usually, however, the term $d\eta/dt$ is at least of second order, and at times of decreased subsidence and "disturbed" unsteady flow this term can be of a magnitude comparable to that of the divergence and/or the frictional terms. Under these more general conditions, the level H deviates from the level of $|w|$ maximum. Since the computed rate of change of vorticity during BOMEX was never zero on a daily basis, it was necessary to turn to an independent measure of H rather than use the level of maximum $|w|$. One possibility is to assume that this level is equivalent to the top of the PBL, the level where the frictional influences of the underlying surface can be neglected. In analyzing the momentum budget for BOMEX Period III, Rasmusson (1974) required a boundary condition on the shear stresses at the top of the PBL. He assumed H_1 to be the level where both the shear stress and the frictional force or stress gradient are negligible. This level was determined to be about 1300 m above the sea surface. Riehl and Soltwisch (1974), using data from both the Atlantic and the Pacific northeast trade-wind regions, also arrived at 1300 to 1500 m as the level where the frictional force vanishes. Therefore for purposes of this research $H_1 = 1300$ m will be designated as the level where $\mathbf{k}\cdot\mathrm{curl}\,\boldsymbol{\tau}=0$. The sensitivity of the vorticity derived C_D to a range of H values is discussed further in Section 3.

The vertical velocities were computed by vertical integration of the horizontal divergence calculated from mass budgets using rawinsonde data (Holland and Rasmusson, 1973). The daily averaged profiles are shown in Fig. 1.

We can now evaluate C_D if $d\zeta/dt$, defined as

$$\frac{d\zeta}{dt}=\frac{\partial\zeta}{\partial t}+\mathbf{V}_h\cdot\nabla\zeta+w\frac{\partial\zeta}{\partial z},$$

can be computed. Here,

$$\mathbf{V}_h\cdot\nabla\zeta=u\left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x\partial y}\right)+v\left(\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2}\right), \quad (6)$$

and u, v are the east–west and north–south wind components, respectively. Rough calculations show that the term $w\partial\zeta/\partial z$ is at least one order of magnitude smaller than the horizontal advection term during Period III and contributes at most about 5% un-

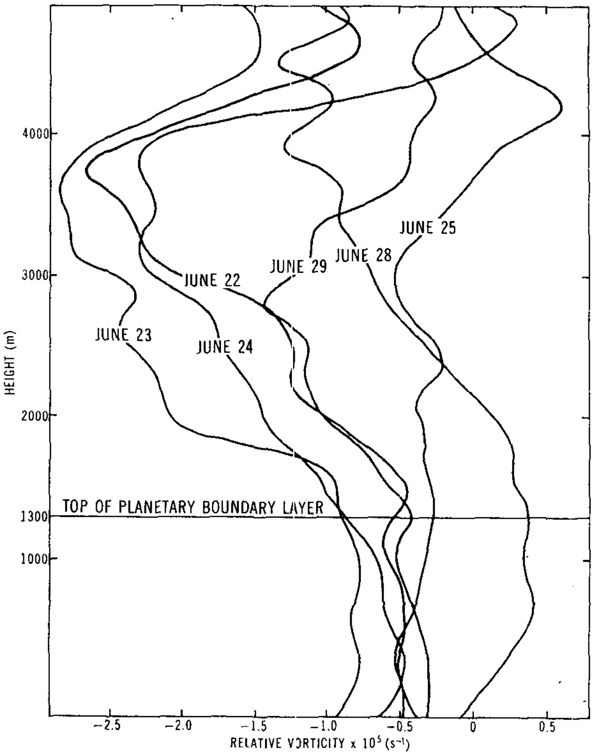

FIG. 2. Daily profiles of relative vorticity ζ, BOMEX Period III.

certainty to C_D. This uncertainty is judged small and the term is therefore neglected. At least six observing stations are required to calculate these second derivatives in the wind field. However, there were only five stations during BOMEX. Moreover, the upper air data from the *Rockaway* were not suitable for this calculation. Two conditions are therefore required for computing the advection of vorticity. First, the changes in relative vorticity will be assumed to be satisfactorily estimated based on the surface measurements alone. This requires the relative vorticity to be independent of height in the PBL, an approximation for which there is some support in the computed relative vorticity profiles (Fig. 2). These profiles are seen to be reasonably constant with height through the PBL. For the other condition it was decided to adopt the constraint of maximum smoothing of the curvature of the wind field, i.e., minimum curvature of the quadratic surface. This technique has been used by Yanai *et al.* (1973) in their work with Marshall Island data and is adopted here in modified form. It is shown in the Appendix that

$$\mathbf{V}_n\cdot\nabla\zeta=\frac{2}{l^2}\left[u_{00}\left(\bar{v}-v_{00}-\frac{u_1+u_3-u_2-u_4}{2}\right)\right.$$
$$\left.+v_{00}\left(\frac{v_1+v_3-v_2-v_4}{2}-\bar{u}+u_{00}\right)\right], \quad (7)$$

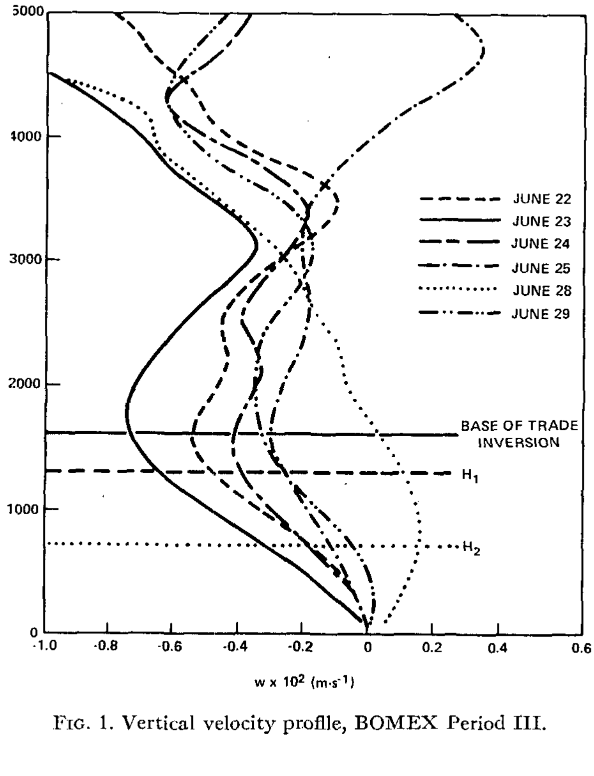

FIG. 1. Vertical velocity profile, BOMEX Period III.

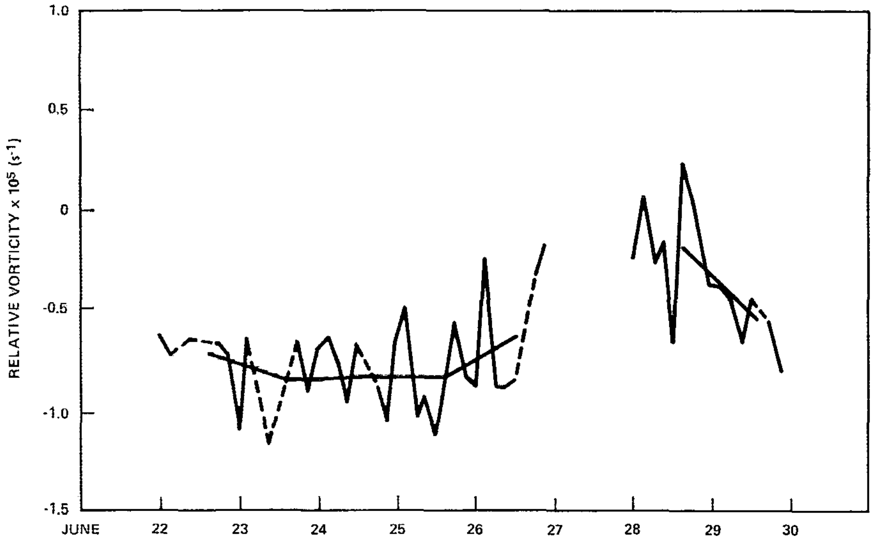

Fig. 3. Time series of surface vorticity. Also indicated are the daily averaged vorticity centered at midday.

where l is the length of a side of the BOMEX square array and where the subscript 00 refers to the central position, and 1, 2, 3, 4 are, respectively, the NE, NW, SW, SE fixed-ship positions. To simplify the computations, the wind components from each of the ships were rotated counterclockwise 10° to correspond to the axes of the BOMEX array. The overbars refer to four-corner ship averages.

The values of the parameters used in the vorticity computations are:

Latitude	15°N
f	3.764×10^{-5} s^{-1}
β	2.703×10^{-11} (m s)$^{-1}$
H	1.3×10^{3} m
ρ	1.17×10^{-6} kg m^{-3}
l	5×10^{5} m

b. Discussion of individual terms in the vorticity equation

Fig. 3 shows the time series of the surface vorticity based on 3 h averaged boom winds corrected for ship motion. The low-level flow was generally anticyclonic but diminished in magnitude throughout the period, with cyclonic flow occurring for a short time on 28 June. A disturbance passed through the BOMEX region on 28 and 29 June as reported by Frank (1970), which accounts for the decrease in the anticyclonic flow at the surface. Missing data are indicated by dashed lines. The curl of the $\mathbf{V}S$ time series is qualitatively similar to that of the relative vorticity.

The daily average values of relative and absolute vorticity, curl of $\mathbf{V}S$, and $\beta\bar{v}$ are listed in Table 1. The $\beta\bar{v}$ term is a vertical average from the surface to 1300 m for the four corner ships. Also included are the vertical velocity at 1300 m from the mass budget computations of Holland and Rasmusson (1973).

Table 2 lists the results of the vorticity budget computations. Finite differences using vorticity estimates for averaging periods of 3–12 h were found to be quite noisy. In order to reduce the uncertainty, the local rate of change of relative vorticity was computed by taking the difference between two successive

Table 1. Daily averages of terms in vorticity equation.

June 1969	$\zeta\times10^5$ (s^{-1})	$\mathbf{k}\cdot$curl $(\mathbf{V}S)\times10^4$ (m s^{-2})	$\eta\times10^5$ (s^{-1})	w(1300 m)$\times10^2$ mass budget (m s^{-1})	$\beta\bar{v}\times10^{11}$ (s^{-2})	$C_D=\dfrac{\eta w(H)\times10^3}{\mathbf{k}\cdot\text{curl }\mathbf{V}S}$
22	−0.7238	−0.7832	3.0402	−0.519	2.798	2.02
23	−0.8543	−1.0849	2.9097	−0.675	0.110	1.81
24	−0.8362	−0.9058	2.9278	−0.407	0.617	1.32
25	−0.8480	−0.8778	2.9160	−0.278	−0.617	0.92
26*	−0.6401	−0.5707	3.1239	−0.711	−0.727	3.89
28	−0.1824	−0.1470	3.5816	0.084	2.247	2.05
29	−0.5477	−0.5443	3.2163	−0.291	2.203	1.72

* Data limited to 18 h (0300–2100 GMT).

TABLE 2. Average vorticity and vorticity changes.

June 1969	$\frac{\partial\zeta}{\partial t}\times 10^{10}$ (s^{-2})	$\mathbf{V}\cdot\nabla\zeta\times 10^{10}$ (s^{-2})	$\frac{d\zeta}{dt}\times 10^{10}$ (s^{-2})	$\beta v\times 10^{10}$ (s^{-2})	$\frac{d\eta}{dt}\times 10^{10}$ (s^{-2})	$\eta\times 10^{5}$ (s^{-1})	$w(H)\times 10^{2}$ ($m\ s^{-1}$)	$\frac{\mathbf{k}\cdot\text{curl}\,\boldsymbol{\tau}_0\times 10^4}{\rho C_D}$ ($m\ s^{-2}$)
22–23	−0.1510	−0.3587	−0.5097	0.1454	−0.3643	2.9750	−0.597	−0.9341
23–24	0.0210	−0.3293	−0.3083	0.0364	−0.2719	2.9188	−0.541	−0.9954
24–25	−0.0134	−0.1935	−0.2069	0.0000	−0.2069	2.9219	−0.343	−0.8918
25–26*	0.2750	−0.0474	0.2276	−0.0672	+0.1604	3.0051	−0.464	−0.7462
28–29	−0.4228	0.6367	0.2139	0.2250	+0.4364	3.3990	−0.104	−0.3457

* Data for June 26 limited to 18 h (0300–2100 GMT).

daily averages. The other terms of Eq. (4) are 2-day means. The time series of $\partial\zeta/\partial t$ is presented in Fig. 4. At the beginning of Period III the local change is negative, indicating increasing anticyclonic vorticity. The flow then remains unchanged until 26 June when it becomes less anticyclonic. This change is associated with a disturbance (Frank, 1970) that passed Barbados on 27 June. The negative value of $\partial\zeta/\partial t$ computed between 28 and 29 June is consistent with another more intense disturbance passing westward out of the BOMEX area on 29 June. The time series of the computed vorticity advection is also shown in Fig. 4. The computed advection, $-\mathbf{V}\cdot\nabla\zeta$, is positive from 22 to 26 June, implying that the vorticity to the east of the BOMEX array was relatively more cyclonic. This result is credible since, in fact, a westward-moving disturbance passed the area on 26 June as indicated by radar data (Hudlow and Scherer, 1975) and by Frank's study. In contrast, $-\mathbf{V}\cdot\nabla\zeta$ is negative during 28 and 29 June, which implies that the vorticity to the east of the BOMEX area was more anticyclonic.

The PBL vertically integrated terms in the vorticity equation are compared to the $\bar{\eta}w(H)$ in Fig. 5. During the undisturbed period 22 to 26 June, the magnitudes of $Hd\zeta/dt$, $H\beta\bar{v}$ and $Hd\eta/dt$ are small relative to $\bar{\eta}w(H)$: 0.28, 0.05 and 0.23, respectively. This is in contrast to the period of disturbed weather, 28–29 June, when the fractions are 0.79, 0.82 and 1.60, respectively, for the same three terms. It is clear that during the undisturbed period the primary balance is between the surface friction term C_D curl ($\mathbf{V}S$) and the production term $\eta\bar{w}(H)$ (see Fig. 5). During the disturbed situation, changes in vorticity arising from the local, advective and beta terms become as prominent as the friction term.

3. Results

The drag coefficients C_D computed from the vorticity equation are listed in Table 3 and illustrated in Fig. 6. The results of extrapolating these values to conditions of neutral stability by Deardorff's (1968) method, as seen in the table, reduces C_D by about 5%. The non-neutral value of C_D computed from the complete vorticity expression is found to be 1.4×10^{-3} for the undisturbed period and 2.7×10^{-3} for the disturbed period, with a period mean of 1.65×10^{-3}. The unbiased estimate of the standard deviation of the daily averages is 0.66×10^{-3} for the entire period and 0.56

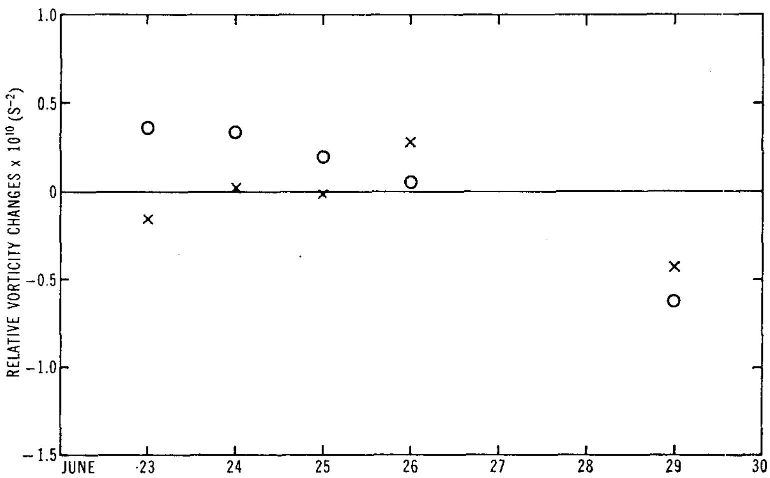

FIG. 4. Time series of estimated surface $\partial\zeta/\partial t$ (crosses), and advection of vorticity $-\mathbf{V}\cdot\nabla\zeta$ (circles).

$\times 10^{-3}$ for the undisturbed period. The non-neutral values of the drag coefficient calculated under the assumption that the mass flux at the top H of the friction layer is entirely forced by convergence due to frictional torque [see Eq. (5)] are 1.6×10^{-3} for the undisturbed and 1.0×10^{-3} for the disturbed period, as shown in Table 3. The day-to-day variation from (5) is somewhat smaller than those obtained from the complete expression. The unbiased estimate of the standard deviation varies between 0.4×10^{-3} for the entire period and 0.36×10^{-3} for the undisturbed period. The difference between results from Eq. (5) and Eq. (4) during the undisturbed period is small. In contrast, the difference is large during the disturbed period partly because the approximate relationship (5) may be inaccurate, and also because uncertainties in mass budget computations tend to be larger during disturbed conditions.

As shown in Fig. 7, no discernible dependence on speed is noted in either the simple or the more complex technique for obtaining C_D.

The data can also be viewed in a different perspective. By introducing a value for C_D, one can compute the vertical velocity at H. Rearranging the terms in Eq. (4) gives

$$w(H)=\frac{1}{\bar{\eta}}\int_0^H \frac{d\eta}{dt}dz+\frac{C_D}{\bar{\eta}}\mathbf{k}\cdot\mathrm{curl}\,(\mathbf{V}S), \tag{8}$$

while Eq. (5) becomes

$$w(H)=\frac{C_D}{\bar{\eta}}\mathbf{k}\cdot\mathrm{curl}\,(\mathbf{V}S). \tag{9}$$

Fig. 8 shows the comparison between the vertical velocity derived from the mass budget computations and the value calculated from the vorticity equation, under the assumption that $C_D=1.5\times 10^{-3}$, the value reported by Pond *et al.* (1971) and Roll (1965). The day-to-day differences are of the order of 2×10^{-3} m s^{-1},

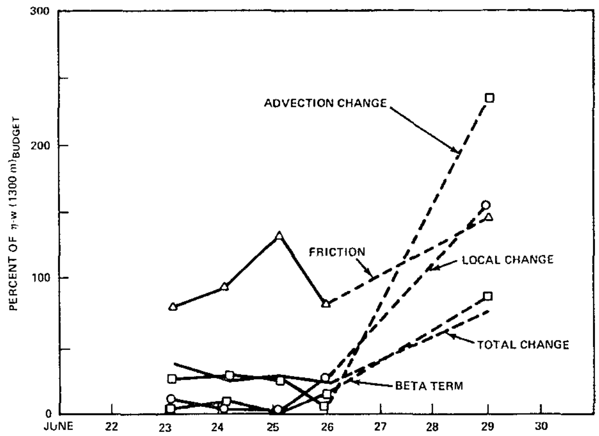

Fig. 5. Percent of various terms in the vorticity equation making up the PBL integrated mass divergence.

and except for 26 June the results for the two computations are in close agreement. The increased subsidence in the PBL on 26 June obtained from the mass budget calculations are not consistent with the vorticity calculation, but, as previously noted, the mass budget results are considered to be less reliable for disturbed than for undisturbed periods.

4. Discussion

To determine the sensitivity of the C_D computations using Eq. (4) to the choice of H, the ratio $R=C_{DH}/C_{DH_1}$ was computed, where $H_1=1300$ m and the resulting values are listed in Table 4 for the following choices of H: (i) 1600 m, the height H_I of the trade inversion, (ii) the height where $|w|$ is maximum, and (iii) 730 m, the height H_2 where the wind is maximum in the PBL. Values of R near unity imply invariance of C_D to PBL heights between H_1 and H, while a significant difference from unity suggests some inconsistency in that choice of H. Average ratios for the undisturbed period are nearly unity based on

Table 3. PBL integrated vorticity and the transfer coefficient ($H=1300$ m).

June 1969	$H\frac{d\zeta}{dt}\times 10^7$ (m s^{-2})	$\eta w(H)\times 10^7$ (m s^{-2})	$\beta v H\times 10^7$ (m s^{-2})	$H\frac{d\eta}{dt}\times 10^7$ (m s^{-2})	ϕ	$C_d=\frac{-H\frac{d\eta}{dt}+\eta w(H)\times 10^3}{\mathbf{k}\cdot\mathrm{curl}\,\mathbf{V}S}$ Non-neutral	Neutral	$C_D=\frac{\eta w(H)\times 10^3}{\mathbf{k}\cdot\mathrm{curl}\,\mathbf{V}S}$ Non-neutral	Neutral
22–23	−0.6626	−1.7761	0.1890	−0.4736	0.97	1.39	1.35	1.90	1.84
23–24	−0.4008	−1.5791	0.0473	−0.3535	0.96	1.23	1.18	1.59	1.53
24–25	−0.2690	−1.0022	0.0000	−0.2695	0.94	0.82	0.77	1.12	1.03
25–26*	0.2959	−1.3929	−0.0874	0.2085	0.94	2.15	2.02	1.87	1.76
				Means					
22–26	−0.2764	−1.4406	0.0435	−0.2309	0.95	1.40	1.33	1.61	1.54
28–29	0.2781	−0.3535	0.2893	0.5673	0.95	2.66	2.53	1.02	0.97
22–29	0.1517	−1.2230	0.0876	0.0640	0.95	1.65	1.57	1.49	1.43

* Data limited to 18 h (0300–2100 GMT).

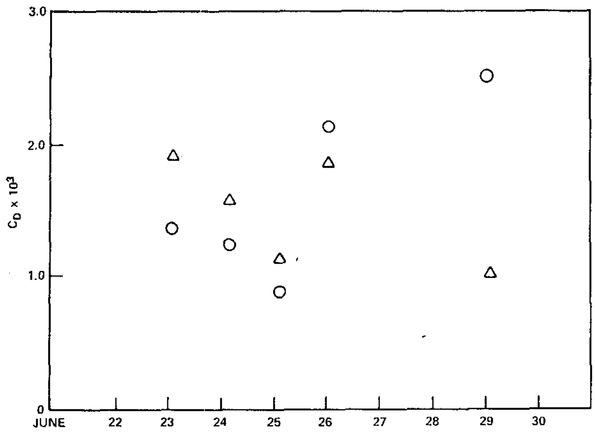

FIG. 6. Computed values of bulk aerodynamic drag coefficient C_D using vorticity relationships. Triangles are values from Eq. (5), circles from Eq. (4).

values of H from the reference level H_1 up to the base of the trade inversion and also to the level where $|w|$ is maximum. In contrast, if the curl of the stress is assumed to vanish at H_2, the level of maximum wind where the stress component in the direction of the surface wind is usually assumed to vanish in ageostrophic departure computations, then the ratio is less than $\frac{1}{2}$. It appears that this height is unsatisfactory for purposes of computing the drag coefficient. Supporting evidence can be found in the momentum budget analyses of Holland and Rasmusson (1974). They found that when the stress is assumed zero at $H_2=730$ m the vertically integrated frictional force in the layer between that level and 4200 m vanishes but that large deviations of the frictional force exist within that layer. From their results and the results of this study, it appears that the height of the PBL is not located at level H_2.

It has been hypothesized that the depth of the PBL for steady, neutral and barotropic conditions is $H=pu_*/f$, where $u_* \approx (\tau_0/\rho)^{\frac{1}{2}}$ and p is some constant (Blackadar and Tennekes, 1968; Csanady, 1967; Clarke and Hess, 1973). A value of $p=0.25$ was suggested by Blackadar and Tennekes and 0.3 by Clarke and Hess. For the u_* value from the BOMEX momentum budget computation, H would be 1860 m for $p=0.25$ and 2230 m for $p=0.3$. Using $H=2000$ m as an intermediate between the two gives a ratio $R<1$ varying between 0.6 and 0.8 (Table 4). However, the flow during BOMEX was to some extent non-barotropic, unsteady, and operated under non-neutral thermodynamic conditions. It is not clear that the height H can be determined by pu_*/f under unstable conditions (Deardorff, 1972). The comparison between theory and experiment must therefore be judged qualitatively. From this discussion, it appears that the PBL extends only up to the height where small-scale turbulence can be maintained, and that this height is greater than the mixed layer height ($\sim$600 m) because of cumulus convection in an otherwise stable environment. The radar results (Hudlow and Scherer, 1975) indicate about 1–2% of the BOMEX area was covered by radar echoes during the undisturbed period (6–7% during the disturbed period). Of the total number of echoes, about 70% during the undisturbed and 60% during the disturbed period had summit values less than or equal to 3000 m.

The measurement of wind aboard ship is notoriously difficult (see, for example, Augstein *et al.*, 1974; Seguin

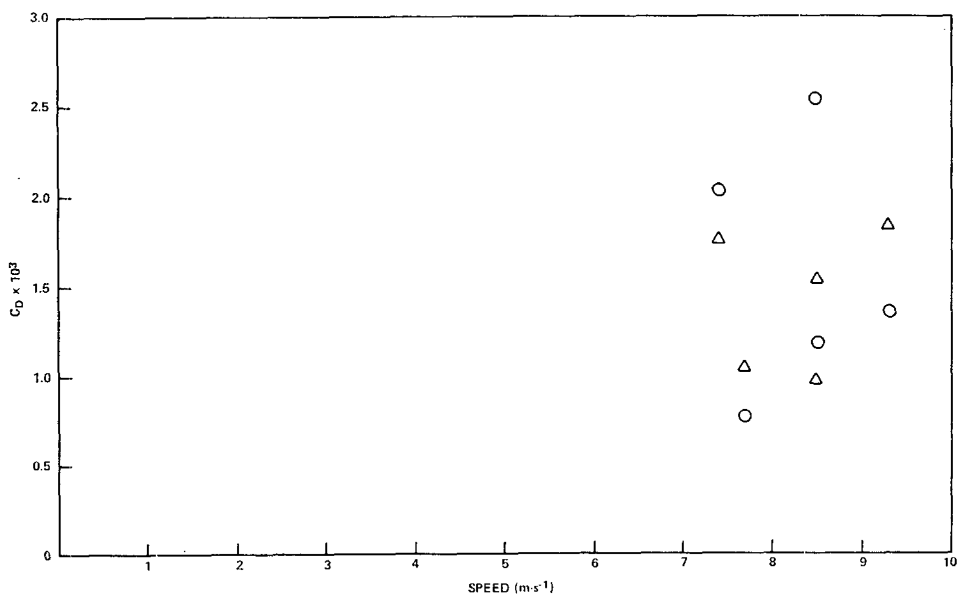

FIG. 7. Vorticity-derived C_D vs wind speed S. Symbols are as in Fig. 6.

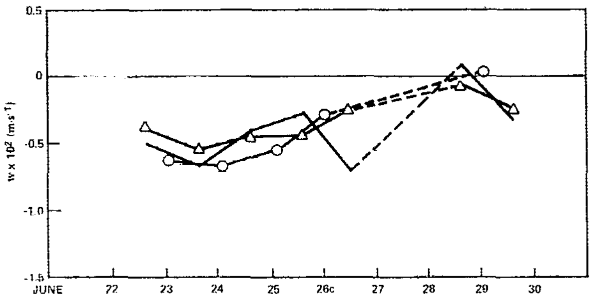

FIG. 8. Comparison of the vertical velocity at $H=1300$ m, the top of the planetary boundary layer, computed from the mass-budget (solid line) and the vorticity-derived estimates using $C_D=0.0015$. Symbols are as in Fig. 6.

and Garstang, 1971; Ching, 1974). Uncertainties in wind measurements affect the drag coefficients computed from the vorticity method. In a separate analysis, this author (Ching, 1974) found a 2% error in C_D associated with a 10% error in the difference between the wind speed at the center and the four corners of the fixed array. On the other hand, there is a 9% error in the computation for every degree of error in terms of the relative wind direction difference.

Table 5 lists the values of C_D obtained by various BOMEX investigators. All coefficients were extrapolated to a height of 10 m and neutral stability. It can be seen that the vorticity results from the present study agree favorably with the other studies. A comparison between BOMEX findings and those obtained in other field experiments is presented in Table 6. As this table shows, the results of the vorticity technique are consistent with the Atlantic Tradewind Experiment (ATEX) results (Augstein *et al.*, 1973), and are close to the mean C_D of 1.5×10^{-3} reported by Roll (1965) who used the "democratic" averaging technique. However, the scatter in the various experiments compiled in Roll's summary is larger (0.8×10^{-3}) than the difference between the results

TABLE 4. Ratios (R) of vorticity-computed drag coefficients at various levels to C_D using $H_1=1300$ m.

June 1969	R(1600 m)	$R(H_{w\max})$	R(730 m)	R(2000 m)
22 to 23	1.01	1.02 (1600 m)	0.48	0.75
23 to 24	1.01	1.03 (1575 m)	0.52	0.80
24 to 25	0.96	1.01 (1500 m)	0.47	0.61

for the complete (0.5×10^{-3}) and the approximate (0.3×10^{-3}) formulations.

5. Conclusions

Based on BOMEX data, it was found that:

1) The value of C_D computed from the vorticity equation compares well with values derived by other BOMEX investigators. The simplified drag relationship which is derived assuming a balance between the divergence and friction terms appears to be a fairly good approximation under undisturbed conditions. This simplification is unsatisfactory when the flow becomes more disturbed. It has been shown that during these periods, the rates of change of vorticity become important.

2) The value of $H=1300$ m determined from momentum budget computations is judged of sufficient accuracy for computing C_D from the vorticity equation during the undisturbed period. In contrast, tests showed $H=730$ m, the height where the low-level wind component in the direction of the surface wind and stress first becomes maximum, to be unsatisfactory for the purpose of computing C_D. In ageostrophic computations, this is the level at which the stress in the direction of the surface wind is assumed to vanish. Under conditions where the total rate of change of vorticity is negligible, i.e., undisturbed conditions, the height H of the PBL is located at the level where $|w|$ reaches its maximum value. Also,

TABLE 5. Bulk aerodynamic transfer coefficients derived by various BOMEX investigators (normalized to 10 m).

Investigator	Description	$C_D^{10}(R_i\neq0)$	$\sigma(R_i\neq0)$	$C_D^{10}(R_i=0)$	$\sigma(R_i=0)$
Donelan (1970)	covariance	1.69	0.21	1.60	0.20
Ching (1974)	ζ (approximate)	1.61	0.36	1.54	0.36
Cain (1971)	structure function	1.51	0.42	1.47	0.41
Paquin (1972)	dissipation	1.49	0.31	1.38	0.29
McBean (1970)	covariance	1.46	0.12	1.33	0.11
Ching (1974)	ζ (complete)	1.40	0.56	1.33	0.52
Phelps (1971)	covariance	1.37	0.23	1.27	0.21
Rasmusson (1974)	atmospheric budget ($H=H_1=1300$ m)	1.25	0.11	1.19	0.10
Portman *et al.* (1970)	covariance	1.03	0.42	0.97	0.39
Rasmusson (1974)	atmospheric budget ($H=H_2=730$ m)	0.96	0.05	0.92	0.05
Stegen *et al.* (1973)	dissipation	0.99	—	0.91	—

TABLE 6. Comparison of transfer coefficients obtained in BOMEX and other field programs.

Experiment	Description	$C_D\times10^3$ Neutral	$C_D\times10^3$ Non-neutral
BOMEX	local and budget	1.37	1.46
BOMEX	vorticity (complete)	1.33	1.40
BOMEX	vorticity (approximate)	1.54	1.61
ATEX	direct	1.39	
ATEX	dissipation	1.26	
ATEX	profile	1.50	
Station Papa (Denman, 1972)		1.63	—
Argus Tower (Deleonibus, 1971)		1.14	1.24
Roll (1965)		1.49	

under these conditions, the profile of $\mathbf{k}\cdot\text{curl}\,\boldsymbol{\tau}$ varies exactly as the vertical velocity profile.

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APPENDIX A

Calculating the Advection of Vorticity from BOMEX Data

The horizontal advection of vorticity can be written

$$\mathbf{V}\cdot\nabla\zeta=u\frac{\partial\zeta}{\partial x}+\frac{\partial\zeta}{\partial y}=u\left(\frac{\partial^2v}{\partial x^2}-\frac{\partial^2u}{\partial x\partial y}\right)+v\left(\frac{\partial^2v}{\partial x\partial y}-\frac{\partial^2u}{\partial y^2}\right). \quad \text{(A1)}$$

Assume any field ϕ_i to be at least of second order, such that

$$\phi_i=AX_i^2+BX_iY_i+CY_i^2+DX_i+EY_i+F, \quad \text{(A2)}$$

where $i=1, 2, \ldots, 5$, represents the grid points of the BOMEX array (Fig. 9). With only five ships in the fixed array, and six coefficients, Eq. (A2) is indeterminate. By requiring that the curvature in the field be minimized, (A2) can be closed. This technique of maximizing the smoothing has been applied successfully by Yanai *et al.* (1973). The constraint is stated as follows:

$$\text{Min}\{B^2+(A-C)^2\}. \quad \text{(A3)}$$

Consider A as an independent variable. Then B, C, D, E and F can be solved as a function of A which has its optimum value A^* determined from (A3). Thus,

$$\text{Min}_A\{B(A)^2+[A-C(A)]^2\},$$

or

$$\left[B\frac{\partial B}{\partial A}+(A-C)\left(1-\frac{\partial C}{\partial A}\right)\right]_{A=A^*}=0. \quad \text{(A4)}$$

The method of solution is as follows:

The algebra is facilitated by normalizing the grid so that Δx and Δy are non-dimensional unit distances $2\Delta x'/l$ and $2\Delta y'/l$, respectively, where l is the length of one side of the BOMEX square, and the primes refer to dimensional lengths (Fig. 9).

By multiplying both sides of Eq. (A2) by X_iY_i, summing over i, and referring to the coordinates in Fig. 10, we have

$$\tfrac{1}{4}\sum_{i=1}^{5}\phi_iX_iY_i=B, \quad \text{(A5)}$$

or

$$\frac{\partial B}{\partial A}=0. \quad \text{(A5a)}$$

Similarly, multiplying (A2) by X_i^2, Y_i^2, X_i, Y_i and 1 individually, and summing these five separate equations over i gives:

$$D=\tfrac{1}{4}\sum_{i=1}^{5}\phi_iX_i \quad \text{(A6)}$$

$$\frac{\partial D}{\partial A}=0. \quad \text{(A6a)}$$

$$E=\tfrac{1}{4}\sum_{i=1}^{5}\phi_iY_i \quad \text{(A7)}$$

$$\frac{\partial E}{\partial A}=0 \quad \text{(A7a)}$$

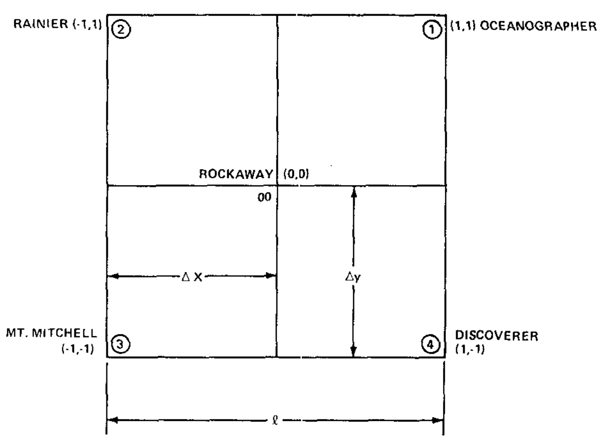

FIG. 9. BOMEX fixed-ship array rotated with x and y axes oriented E–W and N–S, respectively. Grid points are non-dimensional and normalized to unit values.

$$A+C+F=\tfrac{1}{4}\sum_{i=1}^{5}\phi_i Y_i^2=\bar{\phi} \tag{A8}$$

$$A+C+\frac{SF}{4}=\tfrac{1}{4}\sum_{i=1}^{5}\phi_i=\frac{4\bar{\phi}+\bar{\phi}_{00}}{4}. \tag{A9}$$

Subtracting (A9) from (A8) gives

$$F=\phi_{00}. \tag{A10}$$

Thus,

$$A+C=\bar{\phi}-\phi_{00},$$

or

$$C=\bar{\phi}-\phi_{00}-A, \tag{A11}$$

and

$$\frac{\partial C}{\partial A}=-1. \tag{A11a}$$

From (A5a) and (A11a) the minimum curvature constraint (A4) can only be satisfied by the condition that $A=A^*=C$. Thus,

$$C=\frac{\bar{\phi}-\phi_{00}}{2}. \tag{A12}$$

All the coefficients are now determined by some combination of grid values of ϕ. To summarize:

$$A=\frac{\partial^2\phi}{\partial x^2}=\frac{\bar{\phi}-\phi_{00}}{2} \tag{A13}$$

$$B=\frac{\partial^2\phi}{\partial x\partial y}=\frac{\phi_1-\phi_2+\phi_3-\phi_4}{4} \tag{A14}$$

$$C=\frac{\partial^2\phi}{\partial y^2}=\frac{\bar{\phi}-\phi_{00}}{2} \tag{A12}$$

$$D=\frac{\partial\phi}{\partial x}=\frac{\phi_1-\phi_2-\phi_3+\phi_4}{4} \tag{A15}$$

$$E=\frac{\partial\phi}{\partial y}=\frac{\phi_1+\phi_2-\phi_3-\phi_4}{4} \tag{A16}$$

$$F=\phi_{00}. \tag{A10}$$

By substituting these results and collecting terms, Eq. (A1) in dimensional form can be written

$$\mathbf{V}\cdot\nabla\zeta=\frac{2}{l^2}\{u_{00}[\bar{v}-v_{00}-\tfrac{1}{2}(u_1-y_2+u_3-u_4)]$$
$$+v_{00}[u_{00}-\bar{u}+\tfrac{1}{2}(v_1-v_2+v_3-v_4)]\}. \tag{A17}$$

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