A Note on a Theory of Vacillating Baroclinic Waves

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This note concerns the theory of amplitude vacillation given by Pedlosky (1971, 1972) for finite-amplitude baroclinic waves in a two-layer, quasi-geostrophic, zonal flow model. It was shown recently (Smith, 1974) that Pedlosky's calculations are inconclusive owing to the omission of a certain side-wall boundary condition on the mean zonal flow. The neglect of this boundary condition results in an unspecified (and non-physical) energy source at the side-wall boundaries and the problem is not well-posed.

Since writing the above paper I have repeated Pedlosky's analysis with the appropriate side-wall boundary condition included and the present note is a statement of the principal results: the details have been submitted for publication elsewhere.

Despite considerable differences in detail between the two analyses, the new calculations confirm Pedlosky's conclusions for the asymptotic case of vanishingly small (but non-zero) viscous effects. In this parameter regime, the amplitude evolution equations (now an infinite autonomous system of ordinary differential equations) have a stable periodic (limit-cycle) solution, corresponding with a wave-amplitude vacillation, and an equilibrium solution, corresponding with a steady-wave solution. The latter may be stable or unstable according to the zonal and meridional wavenumbers of the wave. If the equilibrium solution is stable, which happens for small wavenumbers, an unstable limit-cycle solution also exists and this delineates in some sense the ranges of attraction in phase space of the steady solution and the stable limit-cycle solution.

The case in which viscous effects, although small on the time scale for the initial growth of an incipient wave, are not vanishingly small has also been studied and parameter regimes in which the possible equilibrium solution are stable or unstable are obtained. However, a series of numerical integrations of the amplitude evolution equations in this viscous regime have failed to yield periodic solutions in concordance with the related calculations of Hart (1973).

Acknowledgment. I am grateful to Prof. Pedlosky for suggesting that I write this note.

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Reply

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I am delighted that Dr. Smith's more elaborate calculations have confirmed the existence of the limit cycles I found in the simplified system I had studied earlier (Pedlosky 1971, 1972). This is an interesting extension of that former work.

It is important to note that this agreement is not a fortuity. The central question Smith raised recently (Smith, 1974) was whether the "side-wall source term" introduced by my introduction of periodic boundary conditions completely vitiated that analysis. Smith (1974) showed that this boundary term was $O(4m^2\pi^2/a^2)$ when compared with the internal generation terms. This parameter is less than 1 in the parameter region in which the limit cycles were found and, indeed, the smaller this parameter the *more* unstable the *steady* wave solutions became. Hence it appears from Smith's analysis that in the parameter region of interest the effect of the boundary term is energetically unimportant compared with the internal conversions. It is important also to note that the problem solved in Pedlosky (1971, 1972) was certainly well-posed, and that what was questioned was its relevance. I am gratified that Dr. Smith's analysis has confirmed that the third-order autonomous system I studied gave results so similar to Dr. Smith's infinite autonomous set of equations.