An Analytical View of Updating Meteorological Variables: Part II. Weighted Assimilation

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ABSTRACT

A simple theoretical approach is formulated, based on part I of this study, in which inadequate modelling of physical processes and the use of numerical algorithms are assumed to introduce random phase errors between model predictions of large-scale atmospheric disturbances and the true state of the atmosphere. An attempt is made to prevent excessive growth of these errors by updating the model predictions with error-contaminated observations, available either periodically or aperiodically. It is demonstrated that the root mean square prediction error can be controlled by updating, when the technique of weighted assimilation is employed. This well-known technique uses both predicted and observed values of an atmospheric variable to form an estimate of the true state. In general, this estimate is a better data source than direct replacement by an observed value. However, the results show that when observation errors are relatively small, weighted assimilation is essentially equivalent to replacement by the observed variable. When the model prediction errors are relatively small, significant improvement over replacement by the observed value is attained. These results are displayed for various model and observation errors and for different length scales of the wave disturbance.

A critique of the present results and inherent difficulties that are met in application to numerical weather prediction are discussed.

1. Introduction

In part I of this study (Blumen, 1975), a simple model of updating meteorological variables was examined. The analysis was based on the premise that phase errors, brought about by the use of numerical algorithms and inadequate modelling of physical processes, make a major contribution to the root mean square prediction error of a meteorological variable. As shown by Williamson's (1973) experiments and the analysis presented in I, updating the model with the *true value* of one atmospheric variable, say pressure, does not reduce the error level by a significant factor. The normalized rms error in Williamson's experiments was order 0.5, which means that the rms error that represents the difference between randomly chosen states of the model was only reduced by a factor of 2. Errors of this magnitude were also found by Blumen, who derived expressions for phase errors, both mean and random, associated with a divergent barotropic model.

In addition to the type of phase errors examined in I, random observation errors, both in phase and amplitude, will be introduced in the present extension. As a consequence, updating the model with *observed*, rather than true, values will introduce another source of error. This additional feature takes the subsequent analysis a step closer to practical assimilation with real data. However, the principal aim of the present study is to reduce the error, found by updating with an observed meteorological variable, by using the statistical best estimate of the true field value to update the model. The present results establish quantitative upper bounds on the error reduction possible by the use of a procedure that is both mathematically sound and, with some modification, a practical assimilation tool.

The basic model and method of analysis presented in I will be followed. The principal past results to be used in the present study are presented in Section 2. Then the prediction error, determined by updating the model with error contaminated observations, is discussed in Section 3. A method of reducing the prediction error, by providing the statistical best estimate of the true field values, is taken up in Section 4. This method, generally referred to as sequential analysis or weighted assimilation, has been discussed by many authors (cf. Morel, 1973; Talagrand and Miyakoda, 1971; Rutherford, 1972). The technique is simply a variant of the method of optimum interpolation proposed earlier by Gandin (1963). Comparisons between predictions made with and without weighted assimilation appear in Section 5. Finally, a critique of the present approach to the problem of updating meteorological variables is presented in Section 6.

2. Model

It is assumed that the true atmospheric state or control state, denoted by subscript c, is a divergent quasi-geostrophic barotropic state that evolves according to

$$\left(\frac{\partial}{\partial\tau}-\frac{\partial\Pi_c}{\partial y}\frac{\partial}{\partial x}+\frac{\partial\Pi_c}{\partial x}\frac{\partial}{\partial y}\right)(\Delta\Pi_c-\lambda_c^{-2}\Pi_c+\beta y)=0, \quad (1)$$

where Π_c denotes the geostrophic pressure field, $\Delta\equiv\partial^2/\partial x^2+\partial^2/\partial y^2$, $\beta=df/dy=$constant, the geometry is Cartesian, and the domain infinite. The constant parameter λ_c denotes the nondimensional radius of deformation defined by

$$\lambda_c^2\equiv g_c'D_0/(f_0^2L^2), \quad (2)$$

where g_c' denotes reduced gravity, D_0 the depth of a lower stable layer capped by an infinitely thick inert upper layer of lesser density, f_0 the constant Coriolis parameter, and L the characteristic length scale of the quasi-geostrophic field (typically $L\approx1000$ km). The particular pressure field, satisfying (1), that defines the true state is

$$\Pi_c(x,y,\tau)=-Uy+\Psi e^{i(kx+\sigma_c\tau)+ily}, \quad (3)$$

where U is a constant zonal flow, Ψ a constant, (k,l) are (x,y) wavenumbers, and σ_c the frequency of the wave.

In I it was assumed that the model state evolved according to (1) with the *essential* difference that $\lambda\neq\lambda_c$ replaces λ_c in this equation. Periodically, or aperiodically, the predicted pressure field Π is updated by simple replacement of the true field Π_c, although the geostrophic wind field $\mathbf{v}=\mathbf{k}\times\nabla\Pi\equiv\mathbf{k}\times\nabla\psi$ ($\mathbf{k}$ is a unit vector and ψ the geostrophic streamfunction) is left untouched. After each update, the imbalance between Π_c and ψ is responsible for the generation of gravity-inertia waves with a corresponding adaptation toward a new state of geostrophic balance. However, repeated updating cannot force the prediction to approach Π_c. Instead, an asymptotic error level is reached that depends on the length scale of the assumed disturbance and the phase error

$$\theta=(\sigma_c-\sigma)\Delta\tau, \quad (4)$$

where $\sigma_c-\sigma$ represents the frequency difference between the true and predicted disturbance fields and $\Delta\tau$ denotes the increment of time between updates.

The analysis of the case when $\theta=$constant was shown to agree qualitatively well with Williamson's experiments. The results indicate that this simple model captures the essence of error growth, associated with truncation error, when *periodic* updating of one meteorological variable is performed. It was further assumed that θ could be represented as the sum of a mean $\langle\theta\rangle$ and a random phase error θ_n'. (The subscript refers to the nth event of an ensemble of random events or realizations.) The introduction of θ_n' may be interpreted as a means of representing truncation error in the finite-difference approximation, random differences between the model and true atmospheric state due to unsatisfactory representation of physical processes, and the lower boundary, etc. All of these features are lumped into the difference $\lambda_c-\lambda$ in the present model. An alternative interpretation is to assume $\Delta\tau$ is a random variable to simulate asynoptic data assimilation. The representation of θ' as a random variable does not distinguish between either assumption, or both in combination. Any of these interpretations is valid, assuming that the statistical description of $\lambda_c-\lambda$, $\Delta\tau$, or their combination is the same in all cases. This approach will be taken in the present paper. In particular, a rectangular probability distribution will be assumed, such that any value θ_n' is equally probable in the interval $|\theta_n'|\leqslant\alpha$. Although a normal distribution would likewise serve our purposes, the rectangular distribution would appear to better represent the assimilation of a more or less uniform density of data received aperiodically from all types of observing systems.

Under the assumption that statistical independence exists between the deviations θ_n' and θ_m' $(n\neq m)$, it was shown in I that the expected asymptotic prediction error approaches

$$E=\left[\left(1+\frac{1-a}{1+a}\right)\left(1-\frac{\sin^2\alpha}{\alpha^2}\right)+r_M^2\frac{\sin^2\alpha}{\alpha^2}\right]^{\frac{1}{2}} \quad (5)$$

as the number of updates $N\rightarrow\infty$. In (5) the parameter a is given by

$$a=\begin{cases}\dfrac{\delta}{1+\delta}, & \text{pressure updating}\\[2ex] \dfrac{1}{1+\delta}, & \text{wind updating}\end{cases} \quad (6)$$

where $\delta=\lambda^2(k^2+l^2)$ and (k^2+l^2) is the square magnitude of the horizontal wavenumber of the disturbance. The asymptotic error due to the presence of a mean phase error $\langle\theta\rangle$ is denoted by r_M, given by Eq. (52) in I. The first expression in (5) represents the "noise" in the prediction due to the addition of a random phase error. For present purposes mean phase errors will not be considered, i.e., $r_M=0$. Rather the purpose here is to analyze the reduction in error of those more or less random effects that are more elusive to control by prediction model or numerical algorithm refinements. For example, a systematic truncation error could presumably be controlled by reduction in grid size.

3. Prediction error

The reader is referred to I (Sections 4 and 6) for most details of the following analysis. (The analysis is unaffected by the presence of a constant zonal flow U superposed on the disturbance field. Consequently, U may be set equal to zero wherever it appears in I.) The essential difference is that updating is not performed with the true pressure field Π_c but with the observed pressure field

$$\Pi^0=(1-\epsilon_n^0)\Pi_c, \tag{7}$$

where $\epsilon_n^0=(\Pi_c-\Pi^0)/\Pi_c$ denotes a complex random observation error with zero mean value. The observation errors are assumed to be statistically independent, i.e., independent of observation errors at previous times. Consider the probability average, denoted by angle brackets, of the following expression that arises in the analysis:

$$\langle \sum_{n=0}^{N-1} a^n(1-\epsilon_n^0)e^{i\theta_n'} \sum_{m=0}^{M-1} a^m(1-\epsilon_m^{0*})e^{-i\theta_m'}\rangle,$$

where the summation occurs over N updating periods, ϵ_m^{0*} denotes the complex conjugate, and θ and a are defined by (4) and (6). Since the observation errors are statistically independent, the above expression reduces to

$$\langle| \sum_{n=0}^{N-1} a^n e^{i\theta_n'}|^2\rangle+\sum_{n=0}^{N-1}\langle|\epsilon^0|^2\rangle a^{2n},$$

where the subscript n may now be dropped from ϵ_n^0. The expected value of the prediction error at the Nth update is denoted by

$$E^{(N)}\equiv\left|\frac{\Pi-\Pi_c}{\Pi_c}\right|^{(N)}. \tag{8}$$

As noted in I, and implied by (6), E refers to the error in the predicted pressure field or in the wind field (geostrophic streamfunction), depending on which variable is updated, when the appropriate expression for a is used.

The expression for $E^{(N)}$ is

$$E^{(N)}=\left\{1-2a^N\frac{\sin\alpha}{\alpha}+a^{2N}\right.$$
$$-2\frac{\sin\alpha}{\alpha}\left(\frac{\sin\alpha}{\alpha}-a^N\right)(1-a^N)+\frac{\sin^2\alpha}{\alpha^2}(1-a^N)^2$$
$$\left.+\frac{(1-a)}{1+a}\left[1-\frac{\sin^2\alpha}{\alpha^2}+\langle|\epsilon^0|^2\rangle\right](1-a^{2N})\right\}^{\frac{1}{2}}, \tag{9}$$

where the probability distribution of θ_n' is again assumed to be rectangular in the range $\pm|\alpha|$. The probability distribution of ϵ^0 is left unspecified. The limiting value of E is given by

$$\lim_{N\to\infty} E^{(N)}=E^{(\infty)}=\left[\left(1+\frac{1-a}{1+a}\right)\left(1-\frac{\sin^2\alpha}{\alpha^2}\right)\right.$$
$$\left.+\langle|\epsilon^0|^2\rangle\frac{1-a}{1+a}\right]^{\frac{1}{2}}. \tag{10}$$

Evaluation of (9) and (10) is deferred to Section 5. However, an examination of (10) shows that if the geostrophic adjustment process is not permitted to occur ($a\equiv 0$), then so-called static initialization yields

$$E_s=\left[2\left(1-\frac{\sin^2\alpha}{\alpha^2}\right)+\langle|\epsilon^0|^2\rangle\right]^{\frac{1}{2}}. \tag{11}$$

This error E_s is the maximum value of E. This result occurs because the error energy due to random phase and observation errors remains in the predicted geostrophic pressure field Π and, as a consequence, the prediction error is a maximum. If the adjustment process is permitted to occur after each update, then part of the error energy is imparted to ageostrophic gravity-inertia waves. As noted from (10) the prediction error decreases with increasing values of a, indicating that an increasing amount of error energy is partitioned to ageostrophic motions. This result is consistent with geostrophic adjustment theory (cf. Blumen, 1972). For example, suppose the pressure field is updated, but the velocity field, which is geostrophically balanced, is left unaltered. For the most part, relatively short waves ($\delta\gg 1$, $a\lesssim 1$) will tend to bring about an adaptation between the pressure and velocity fields. However, since the velocity field was initially geostrophically balanced, it remains essentially in this state. As a consequence, most of the error energy is partitioned to ageostrophic gravity-inertia waves.

Note that when $a\neq 0$, $E_s>E$. Consequently, it appears that when a is not essentially zero the proper approach is to let the geostrophic adjustment process occur so that energy may be partitioned to gravity-inertia waves. Under this circumstance the prediction error would attain its minimum value, for a given value of a, for the updating procedure used here. In this analysis, as discussed in I, it has been assumed that the prediction model contains a mechanism to filter out high-frequency gravity-inertia waves. In practice, it would also be necessary to filter these waves, generated by nonphysical processes, before their presence ultimately contaminates the forecast.

4. Weighted assimilation

We now turn attention to an established method of error reduction by generating a more accurate value of pressure Π^e, than the observed value Π^0, which will then be used to update the model. The basic development may be found in the references cited in Section 1. Only a brief repetition of the essential ingredients will be attempted here.

The basic idea as expressed by Rutherford (1972), for example, is to define

$$\Pi_k^e=\Pi_k^p+\sum_{j=1}^{J}\mu_{kj}(\Pi_j^0-\Pi_j^p), \tag{12}$$

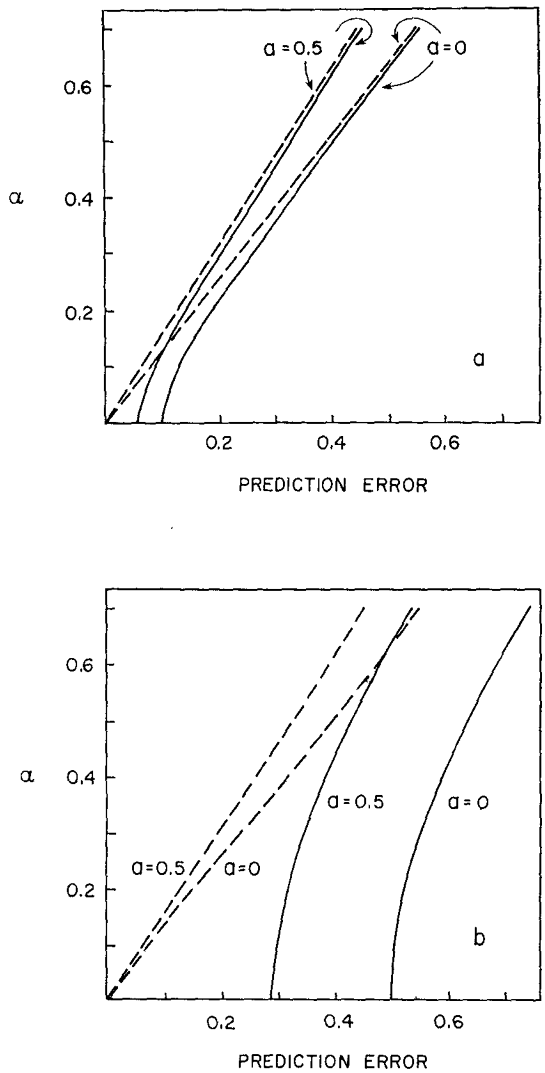

FIG. 1. Asymptotic ($N=\infty$) prediction errors, with (dashed) and without (solid) weighted assimilation, as a function of α, the random phase error. The values of a, defined by (6), are indicated. (a) $p\equiv(\langle|\epsilon^0|^2\rangle)^{\frac{1}{2}}=0.1$; (b) $p=0.5$.

where Π^e and Π^0 are defined above, Π^p is the predicted value of Π [in the notation of I, $\Pi^p\equiv\Pi^{(N)}$], and the μ_{kj} are constants. The estimate at grid point k, say, is determined by means of a linear combination of the apparent forecast errors $(\Pi_j^0-\Pi_j^p)$ at the set of observing stations $j=1, 2, \ldots, J$. Eq. (12) is based upon the assumption that observations are available only at discrete points. The present model is based on the availability of observations continuously in space. Consequently, (12) reduces to a single equation

$$\Pi^e=\Pi^p+\mu(\Pi^0-\Pi^p). \tag{13}$$

(The effect of this simplifying assumption on the present results is discussed in Section 6.) The "weight factor" μ is determined by subjecting Π^e to the least-squares minimization condition that $\langle|(\Pi_c-\Pi^e)/\Pi_c|^2\rangle$ be a minimum, where Π_c is the true value of Π. As a result we obtain

$$\mu=\frac{\langle|\epsilon^p|^2\rangle}{\langle|\epsilon^p|^2\rangle+\langle|\epsilon^0|^2\rangle}, \tag{14}$$

where ϵ^0 is defined by (7) and the prediction error is given by

$$\epsilon^p=\frac{\Pi_c-\Pi^p}{\Pi_c}. \tag{15}$$

In addition to statistical independence between observation errors, statistical independence between observation and prediction errors has also been assumed, i.e., $\langle\epsilon^p\epsilon^0\rangle=0$. Here we use ϵ^p rather than $E^{(N)}$, defined in (8), because $\langle|\epsilon^p|^2\rangle^{\frac{1}{2}}$ will be the expected rms prediction error when *weighted assimilation* is carried out.

If the expression for μ is inserted into (13), the best estimate simply weights the predicted and observed values linearly, according to the relative magnitudes of the prediction and observation errors. Moreover,

$$\langle|(\Pi_c-\Pi^e)/\Pi_c|^2\rangle|_{\min}=\mu\langle|\epsilon^0|^2\rangle=(1-\mu)\langle|\epsilon^p|^2\rangle. \tag{16}$$

Eq. (16) shows that the optimal estimate [Eqs. (13) and (14)] provides a smaller *initial* error than simple replacement by Π^0. The above results have been stated previously (cf. Rutherford, 1972, Sections 2 and 3).

The analysis leading to (9) is again followed, but Π^e, given by (13), is used to update the model. However, Π^0 is still expressed by (7). The prediction error for weighted assimilation may then be expressed as

$$\begin{aligned}
(\langle|\epsilon^p|^2\rangle^{\frac{1}{2}})^{(N)}
&=\Bigg\{1-2a_{N-1}a_{N-2}\cdots a_0\frac{\sin\alpha}{\alpha}+(a_{N-1}a_{N-2}\cdots a_0)^2\\
&\quad-2\frac{\sin\alpha}{\alpha}\left[\frac{\sin\alpha}{\alpha}-a_{N-1}a_{N-2}\cdots a_0\right][1-a_{N-1}a_{N-2}\cdots a_0]\\
&\quad+\frac{\sin^2\alpha}{\alpha^2}[1-a_{N-1}a_{N-2}\cdots a_0]^2+\left[1-\frac{\sin^2\alpha}{\alpha^2}+\langle|\epsilon^0|^2\rangle\right]\\
&\quad\times[(1-a_{N-1})^2+(1-a_{N-2})^2a_{N-1}{}^2\\
&\qquad+\cdots+(1-a_0)^2(a_{N-1}a_{N-2}\cdots a_1)^2]\Bigg\}^{\frac{1}{2}},
\end{aligned} \tag{17}$$

where

$$a_N=1-(1-a)\mu_N, \tag{18}$$

$$\mu_N=\left(\frac{(\langle|\epsilon^p|^2\rangle)^{(N)}}{(\langle|\epsilon^p|^2\rangle)^{(N)}+\langle|\epsilon^0|^2\rangle}\right), \tag{19}$$

and a is given by (6). Note that a_N must be evaluated at each update period N using the prediction error at that time to determine Π^e; but $\langle|\epsilon^0|^2\rangle$ is assumed constant. We see that when $\langle|\epsilon^0|^2\rangle\equiv0$ ($\mu_N\equiv1$), Eq. (17)

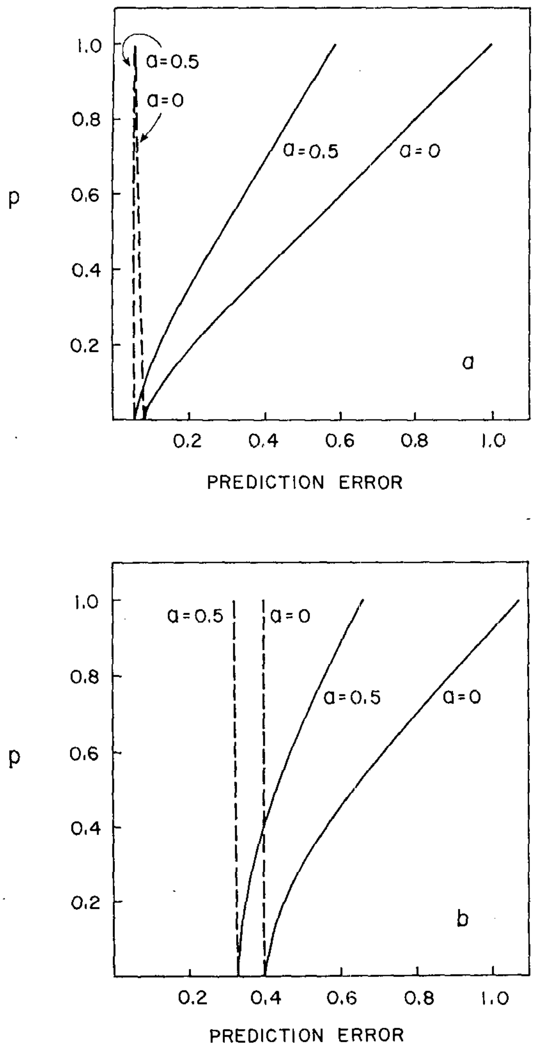

FIG. 2. Asymptotic ($N=\infty$) prediction errors, with (dashed) and without (solid) weighted assimilation, as a function of root mean square observation error p. Values of a are indicated. (a) $\alpha=0.1$; (b) $\alpha=0.5$.

reduces to Eq. (9). When $\mu_N\equiv 1$, the last expression in (17) becomes a geometric series that sums to the comparable term in (9). In general, this latter expression has to be evaluated term by term, so the asymptotic value of (17) cannot be written simply as in (10). However, an important difference between (17) and (9) is evident. First note that $1\geqslant a_N\geqslant a$. Recall that the prediction error $E^{(\infty)}$ given by (10), and similarly $E^{(N)}$ given by (9), decreases with increasing values of a for the reasons stated in Section 3. Then, since $a_N>a$, except when $a_N=1$, the prediction error with weighted assimilation $(\langle|\epsilon^p|^2\rangle^{\frac{1}{2}})^{(N)}$, will be smaller than $E^{(N)}$ *for the same value of a.* The natural consequence is that *error reduction by weighted assimilation is,* in essence, *accomplished by partitioning more energy to ageostrophic motion.* This is carried out by means of increasing the "effective" value of a, as shown in (18). For example, the prediction error for static initialization ($a\equiv 0$) is reduced because the "effective" wavenumber becomes $a_N=1-\mu_N>0$.

5. Predictions with and without weighted assimilation

The prediction errors $(\langle|\epsilon^p|^2\rangle^{\frac{1}{2}})^{(N)}$ and $E^{(N)}$, with and without weighted assimilation, and the "effective" wavenumber a_N have been determined from (17), (9) and (18), respectively. The results are displayed for $a=0$, static initialization, and $a=0.5$ [$\delta=\lambda^2(k^2+l^2)=1$]. In the latter case, according to (6), either pressure or wind updating may be employed to yield equivalent results. Since the prediction errors decrease monotonically with increasing a, if α and p are fixed, the two values of a chosen for display provide sufficient inference to the prediction errors for intermediate values of a.

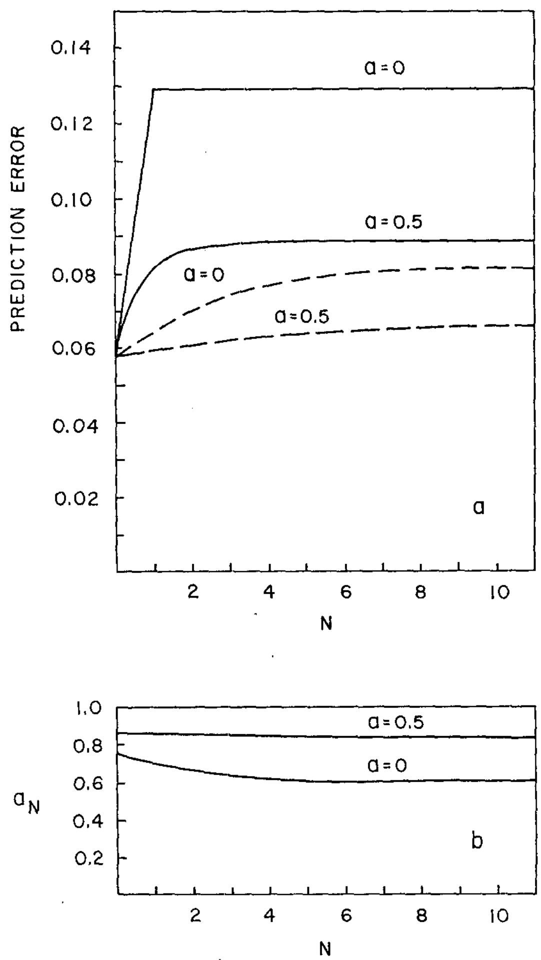

FIG. 3. (a) Prediction errors, with (dashed) and without (solid) weighted assimilation, as a function of the number of updates N for indicated values of a. (b) The "effective" wavenumber a_N, defined by (18), as a function of N. The parameter values are $\alpha=0.1$ and $p=0.1$.

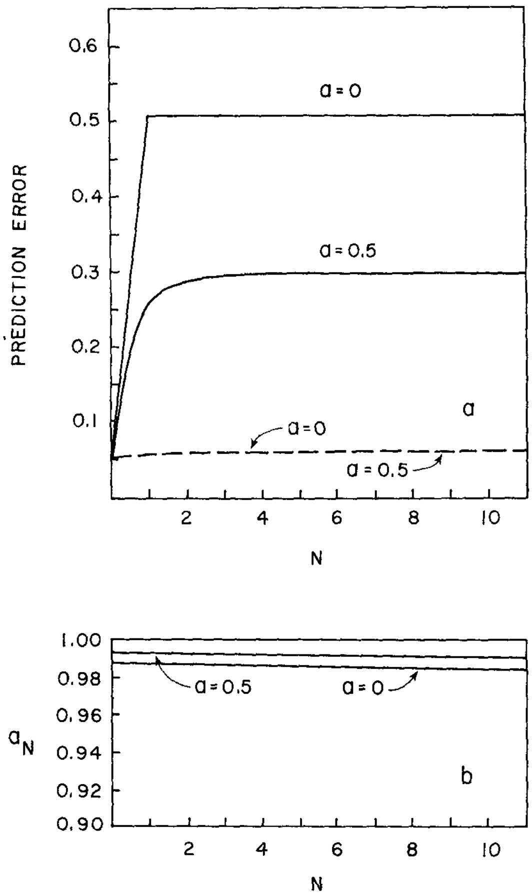

Fig. 4. As in Fig. 3 except for $\alpha=0.1$, $p=0.5$. Note scale change.

Figs. 1 and 2 show the asymptotic errors as functions of α and of $p\equiv(\langle|\epsilon^0|^2\rangle)^{\frac{1}{2}}$. Recall that α is a measure of the inaccuracy of the prediction equation and p is the rms observational error. ($\alpha=0$ implies that the forecast equation would give a perfect prediction of the true atmospheric state if $p\equiv 0$.) We note that the asymptotic prediction error $E^{(\infty)}$ is reduced, for the same α or for the same p, if the geostrophic adjustment process is permitted to occur. Moreover, weighted assimilation reduces the prediction error, with the greatest improvement shown for large observational errors. This latter result is evident from (13) and (14). However, if the prediction error is relatively large compared to the observation error, then the best updating procedure is essentially replacement by the observed value. On the other hand, if the observation error is relatively large, more weight is placed on the predicted value for optimum updating of the model. In this latter case, the weight factor becomes small, and according to (18), the effective wavenumber is close to unity. The asymptotic prediction error (17) approaches $(1-\sin^2\alpha/\alpha^2)^{\frac{1}{2}}$. Consequently, weighted assimilation is a significant improvement over replacement in the latter case.

Figs. 3–6 show the evolution of the prediction errors as functions of the number of updates N. Also plotted is the change in the "effective" wavenumber a_N as a function of N. Since $a_N>a$, not only does weighted assimilation reduce the prediction error, but the number of updates necessary to reach the asymptotic error is increased. If pressure updating is performed, the predicted pressure field will be close to the initial field, for a given N, when the disturbance is a relatively long wave ($a\ll 1$). Consequently, shorter wavelength disturbances need a longer time to attain the final asymptotic state. When the winds are updated the argument is the same, but the spatial scales are reversed. In either case, however, $a\ll 1$ if the appropriate definition of a in (6) is used. These conclusions are consistent with the geostrophic adjustment process described by the present model.

Root mean square observation errors of mid-tropospheric geopotential height and temperature fields from currently employed measuring systems are of order $(\langle|\epsilon^0|^2\rangle)^{\frac{1}{2}}\sim 0.1$, but model performance is more difficult to estimate in terms of the parameter α. However, it is possible to state, on the basis of the present results, that assimilation experiments should be carried out with real data, or data alien to the prediction model, in order to judge the usefulness of assimilation

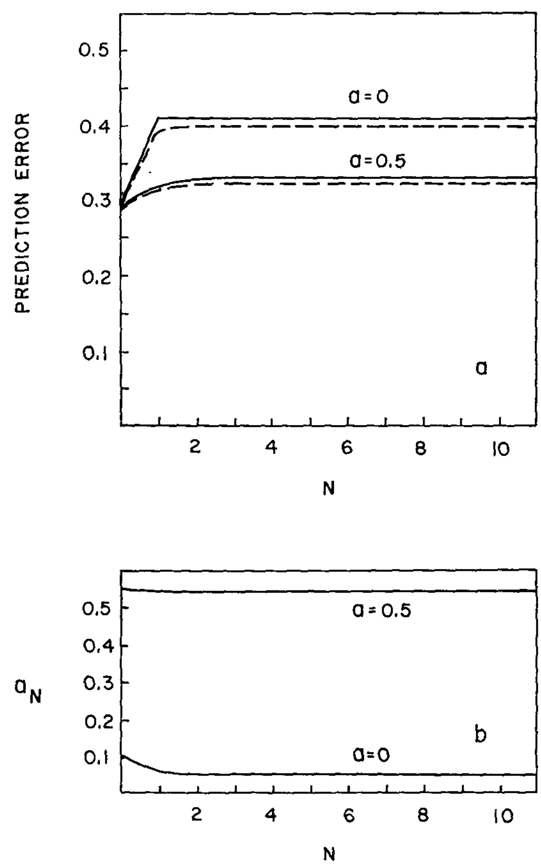

Fig. 5. As in Fig. 3 except for $\alpha=0.5$, $p=0.1$. Note scale change.

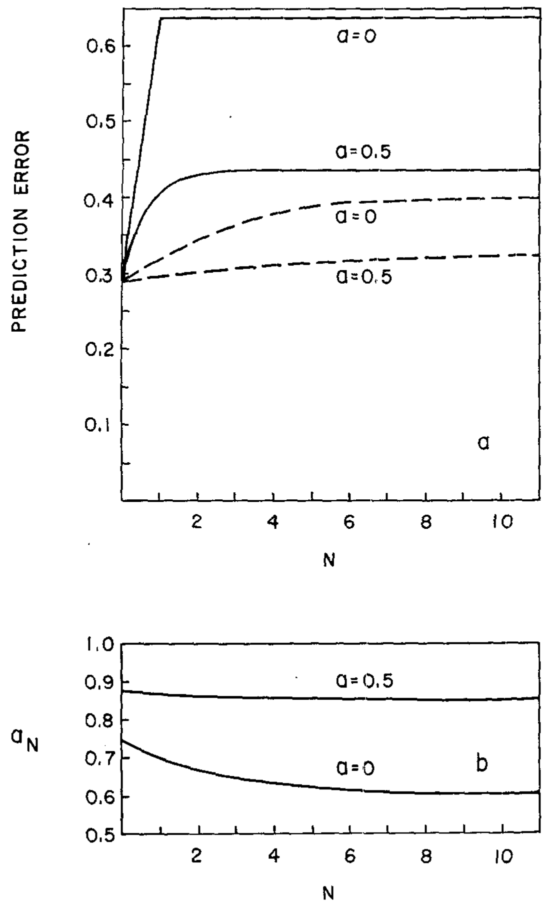

FIG. 6. As in Fig. 3 except for $\alpha=0.5$, $p=0.5$. Note scale change.

procedures. In this regard, we note that the present technique of weighted assimilation cannot eliminate the error associated with random model deficiencies and random observation errors. These are essentially "noise" that contaminate predictions. All that may be expected from the application of (12) is a control, on the "noise" level, which in some cases is a very effective one.

6. Critique

The particular technique of weighted assimilation applied in this investigation appears to offer the promise of improved weather predictions when this technique is used in practice. However, this conclusion is tentative. The present results were attained by assuming that data were available at each point in space. Although (12) is designed to make use of observations in the vicinity of grid points, there will generally be data-void areas where climatological values or some other estimate of the true data will have to be employed. Consequently, the same degree of error reduction, attained under the present ideal circumstances, would not be realized in practice.

As noted by Talagrand and Miyakoda (1971), for example, the present technique will yield optimum results when applied to linear prediction equations. The prediction equation will not yield an optimum forecast when nonlinear quadratic terms are present. In essence, the optimum prediction of $\langle|\epsilon_p|^2\rangle^{\frac{1}{2}}$ from initial data cannot be made unless the nonlinear terms are represented as the product of the optimal estimates of each individual factor. The general validity of this simple closure assumption has not been established. Moreover, it is not possible to specifiy the weight factor μ_N or, more generally, the covariance matrix $(\mu_{kj})_N$ exactly since the true state of the atmosphere is unknown. Talagrand and Miyakoda computed the $(\mu_{kj})_N$ $(N=1,2,3)$ from over 300 statistical samples of forecasts. The factor $(\mu_{kj})_3$ is used when $N\geqslant 3$. This set of weights may then be used for all forecasts with a given prediction model. While this empirical approach provides a practical solution to the above problem, as well as simplifying the computation of the $(\mu_{kj})_N$, the prediction error cannot be reduced by an optimum amount.

In application to hemispheric or global prediction models, the present assimilation scheme may not prove useful in tropical regions. A mass-wind relationship, exemplified by quasi-geostrophic balance in mid-latitudes, is not well established in the tropics and, as a consequence, updating with one flow variable may not reduce the error variance of the other variables. Moreover, defects in prediction models and observation errors are generally a more serious problem in tropical regions. Under this circumstance, a relatively lesser amount of error reduction by the present method could be anticipated.

From a positive viewpoint, updating by simple replacement of one observed variable is generally better than not updating at all; and updating with weighted assimilation is better than replacement. The improvement noted in the latter case is most impressive when observation errors are relatively large compared to errors produced by an imperfect prediction model. However, it would appear on the basis of Williamson's (1973) experiments, that model imperfections are generally responsible for greater prediction errors than imperfect observations. If model deficiencies and/or numerical errors are sufficiently large, so that $\mu_{kj}\lesssim 1$, then weighted assimilation will not prove to be a viable approach. Nonetheless, the present results provide quantitative information on the expected error reduction from model improvements and/or more accurate observations. This information should be helpful in making value judgments in regard to the effort that should be expended in making model improvements versus upgrading observing systems.

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