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Trapeze Instability Modified by a Mean Shear Flow

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ABSTRACT

The influence of weak mean vertical wind shear upon the trapeze instability of Orlanski (1973) is investigated. It is found that the shear limits the growth of unstable waves unless they are propagating at nearly right angles to the mean wind vector, or in other words, the equi-phase lines are parallel to the mean wind direction.

1. Introduction

Orlanski (1973) suggested that the diurnal variations of the Brunt–Väisälä frequency N may parametrically excite internal gravity waves. In mid-latitudes the low-frequency cutoff for these unstable waves is the inertial frequency. Since there are many other mechanisms which excite internal gravity waves in the same frequency band, positive identification of the unstable waves discussed by Orlanski will be difficult. However, in equatorial regions the low-frequency cutoff is half that of the diurnal frequency. It is felt that the trapeze instability will be manifested most clearly in the equatorial region because of the absence of other clear-cut physical mechanisms which produce waves with 2-day periods. Therefore, in this note attention will be confined to the equatorial region. The equatorial region is here defined as the area which lies within 1000 km of the equator. The averaged value of the Coriolis parameter f (over this region) is 1.21×10^{-5} s^{-1}. The effects of the earth's rotation are negligible when $(f^2/N^2)(\lambda_H{}^2/\lambda_V{}^2)\ll1$. The Brunt–Väisälä frequency N is typically 0.01 s^{-1}. For internal gravity waves of 2-day periods the ratio of horizontal scale to vertical scale $\lambda_H/\lambda_V\sim300$. Using these numbers, one finds that $(f^2/N^2)(\lambda_H{}^2/\lambda_V{}^2)=0.13$ which is small compared to unity. Hence, internal gravity waves of horizontal scale perhaps as large as 200 km in an equatorially centered canal of width 2000 km may propagate as though the earth were not rotating.

In general, the vertical shear of the mean horizontal wind influences the propagation and stability properties of internal gravity waves. Near the equator the vertical shear of the mean longitudinal wind is quite weak. A typical value computed from the tables of Oort and Rasmussen (1971) is approximately 5 m s^{-1} (10 km)$^{-1}$. Although this value is small, it will be shown that the mean wind shear can greatly influence the trapeze instability.

Phillips (1966) solved the problem of determining the behavior of internal gravity waves propagating in a mean shear flow. He assumed the mean flow to depend linearly on the vertical coordinate and adopted a coordinate system which moves with the mean flow. Having N^2 as a function of time still allows this procedure to be taken. The present analysis differs with that of Phillips at the point when the final approximate solutions are required due to the time variability of N^2 in the present case.

A synthesis of the respective analyses of Orlanski (1973) and Phillips (1966) reveals that the unstable waves must propagate at nearly right angles to the mean wind vector if they are to avoid being absorbed by the mean wind before they have time to grow appreciably.

2. Analysis

The flow considered is assumed to be inviscid and adiabatic. The perturbations to the basic state are such that the Boussinesq approximation is valid. The governing equation for small perturbations to a stratified shear flow under the above conditions is then

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2\nabla^2w-U_{zz}\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\frac{\partial w}{\partial x}+N^2\nabla_H{}^2w=0,\quad(1)$$

where

$$N^2 \equiv \frac{g}{\theta}\frac{\partial\theta}{\partial z} = N^2(z,t)$$

and $U=U(z)$ is the mean horizontal wind in the x direction; w is related to the actual vertical velocity w' by the transformation $w=[\rho(z)/\rho_0]^{\frac{1}{2}}w'$; and N^2 will be considered to be independent of altitude. Eq. (1) is simplified further by assuming a linear variation of U with z, i.e., $U=\Gamma z$ so that Eq. (1) becomes

$$\left(\frac{\partial}{\partial t}+\Gamma z\frac{\partial}{\partial x}\right)^2 \nabla^2 w + N^2(t)\nabla_H{}^2 w = 0. \tag{2}$$

In the shear-free analysis of Orlanski, inclusion of boundary effects leads to no analytical difficulties. However, when shear effects are included with a time varying stratification, the treatment of the boundary conditions severely complicates the analysis. In the following analysis, the effect of shear is investigated by considering the projection of the vector wavenumber on the mean vector to be slightly nonzero. If it were exactly zero, the analysis would be identical with Orlanski's, only then the wave propagation would be restricted to the meridional direction. Also, from the analysis of Orlanski it is evident that the most unstable waves will exhibit 2-day periods. Now in any wave system the effects of boundaries may be neglected if the velocity of energy propagation (usually the group velocity) is so small that energy initially released from a source returns in a time which is greater than the time scale of interest. The vertical group speed for low-frequency internal gravity waves is $|\omega^2/Nk|$. For the analysis to follow, waves of horizontal scale of $\sim$150 km are considered. The vertical group speed is then 250 m day^{-1}. Since the e-folding time for the unstable (trapeze) waves is $\sim$5 days, one can see that the wave packet center moves 1 km which is a fraction of the deep (3 km) equatorial boundary layer. Hence, for the remainder of this analysis, only freely-propagating waves are considered. Of course, there is no physical restriction on the horizontal scale for these unstable waves (indeed the scale is arbitrary). The horizontal scale of 150 km defines the upper limits for the applicability of the analysis to follow. The solutions, which have a very clear-cut physical interpretation, do suggest that the conclusions obtained here may be extended to the longer wavelength situations.

When the shear Γ is zero, Eq. (2) reduces to

$$\nabla^2 w_{tt} + (N_0{}^2 + N_1{}^2\cos\Omega t)\nabla_H{}^2 w = 0, \tag{4}$$

which is Orlanski's (1973) Eq. (2.9) with f equal zero. The solution to (4) may be found by first letting

$$w = W(t)\exp[i(kx+ly+mz)]. \tag{5}$$

The resulting equation for $W(t)$ is Mathieu's equation, the solutions of which may be unstable. Of interest is the fact that the solution can be unstable even when the system is always gravitationally stable. Orlanski found that, for the case $f=0$, the most unstable wave exhibits a period of 2 days. This seemingly peculiar behavior can be explained as follows: In the absence of dissipative processes in a uniformly stratified fluid, the vertical motion of a fluid parcel will be determined by buoyancy forces. Therefore, if the stratification, which determines the strength of these forces, changes in time in such a way that the buoyancy force is small when the parcel is leaving its equilibrium position and is strengthened as the parcel is returning to it, then there will be a net gain of energy by the parcel. The stratification is weaker than its daily mean N_0 for half of the day and stronger for the other half. Hence, if the fluid parcel rises (or falls) from its equilibrium position during that half-day period, then it has completed only one-quarter of its cycle of wave motion. Thus, changes in the diurnal stratification yield a bi-diurnal wave motion.

When $N_1{}^2$ is zero, Eq. (2) is

$$\left(\frac{\partial}{\partial t}+\Gamma z\frac{\partial}{\partial x}\right)^2 \nabla^2 w + N_0{}^2\nabla_H{}^2 w = 0. \tag{6}$$

The solution of (6) as given by Phillips (1966) is briefly outlined. With the shift of coordinates $\xi=x-Ut=x-\Gamma zt$, $\eta=y$, $\zeta=z$, $\tau=t$, the equation for w becomes

$$\frac{\partial^2}{\partial\tau^2}\left[\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}+\left(\frac{\partial}{\partial\zeta}-\Gamma\tau\frac{\partial}{\partial\xi}\right)^2\right] + N_0{}^2\left[\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right]w = 0. \tag{7}$$

We now assume a solution of the form

$$w = f(\tau)\exp[i(k\xi+l\eta+m\zeta)] \tag{8}$$

and define

$$\cos\phi = \frac{k}{(k^2+l^2)^{\frac{1}{2}}}, \quad \tan\theta = \frac{m}{(k^2+l^2)^{\frac{1}{2}}}. \tag{9a,b}$$

Upon substituting (8) and then (9a, b) into (7) one recovers

$$\frac{d^2}{d\tau^2}[1+(\tan\theta-\Gamma\cos\phi\tau)^2]f + N_0{}^2 f = 0. \tag{10}$$

Then if $T=\tan\theta-\Gamma\cos\phi\tau$ and $F(T)=[1+T^2]f$, Eq. (10) becomes

$$\frac{d^2F}{dT^2} + \frac{N_0{}^2/(\Gamma^2\cos^2\phi)}{1+T^2}F = 0. \tag{11}$$

The asymptotic solution for large $N_0{}^2/(\Gamma^2\cos^2\phi)$ was given as

$$w\approx\frac{G\exp(i\Theta)}{[1+(\tan-\Gamma\cos\phi\tau)^2]^{\frac{1}{4}}},\qquad(12)$$

where

$$\Theta\equiv kx+ly+mz-\Gamma kzt\pm\frac{N_0}{\Gamma\cos\phi}\int^{\Gamma\cos\phi t-\tan\theta}\frac{dT}{(1+T^2)^{\frac{1}{2}}}.$$

The frequency is given by

$$\omega=-\frac{\partial\Theta}{\partial t}=Uk\pm\frac{N_0(k^2+l^2)^{\frac{1}{2}}}{[k^2+l^2+(m-\Gamma kt)^2]^{\frac{1}{2}}}.\qquad(13)$$

The solution represents a wave motion whose amplitude, vertical wavenumber and frequency continually change due to the effects of shear. The wave first propagates in a direction determined by the initial wavenumber vector. However, since the fluid parcels are advected at speeds that change with depth, there will be a progressive tilting of the lines of constant phase as time advances. This tilting continues until the motion is reduced to a steady horizontal sliding. Note that the rate of tilting is determining by the parameter $\Gamma\cos\phi$. Hence for weak shear (Γ small) or for a wave propagating at nearly a right angle to the direction of the mean flow ($\phi\approx\pi/2$), the process of wave absorption takes place very slowly.

Upon returning to (2), one finds that the same steps as taken by Phillips may be used for $N^2=N^2(t)$ with the result that (2) becomes

$$\frac{d^2F}{dT^2}+\frac{N_0^2+N_1^2\cos\Omega\tau}{1+(\tan\theta-\Gamma\cos\phi\tau)^2}F=0.\qquad(14)$$

The crucial question is: How does the wind shear term, which is proportional to $\Gamma\cos\phi$, modify the behavior of the parametric instability without wind shear as studied by Orlanski?

For the sake of convenience the transformation $\hat{\tau}=\Omega\tau/2$ and the definition $\epsilon=\Gamma\cos\phi/(\Omega/2)$ will be used so that equation (14) becomes

$$\frac{d^2F}{d\hat{\tau}^2}+\frac{\left(\frac{2N_0}{\Omega}\right)^2+\left(\frac{2N_1}{\Omega}\right)^2\cos2\hat{\tau}}{1+(\tan\theta-\epsilon\hat{\tau})^2}F=0.\qquad(15)$$

There are two distinct time scales in this problem: one associated with the diurnal period $[\sim\Omega^{-1}]$ and the other with the modified shear $[\sim1/(\Gamma\cos\phi)]$. The quantity ϵ is a measure of the ratio of these two time scales. Hence, for small ϵ the time scale for the shear effects is much longer than the diurnal period. The solution of (15) is expected to depend upon a "fast" time and a "slow" time as if each was an independent variable. The slow time will be denoted by $\hat{\xi}=\epsilon\hat{\tau}$ and the fast time by $\hat{\eta}=g(\hat{\xi})/\epsilon$.[1] The dependent variable F is assumed to have an expansion of the form

$$F=F_0(\hat{\xi},\hat{\eta})+\epsilon F_1(\hat{\xi},\hat{\eta})+\cdots.\qquad(16)$$

The function $g(\hat{\xi})$ is to be determined in the course of the analysis. To a good approximation one may replace $\hat{\tau}$ by $\hat{\eta}$ in the time-varying Brunt-Väisälä frequency, since $g(\hat{\xi})$ is nearly linear in $\hat{\xi}$. Hence, the definition

$$B(\hat{\eta})\equiv\left(\frac{2N_0}{\Omega}\right)^2+\left(\frac{2N_1}{\Omega}\right)^2\cos2\hat{\eta}\qquad(17a)$$

is made. Also the definition

$$b(\hat{\xi})=[1+(\tan\theta-\hat{\xi})^2]^{-1}\qquad(17b)$$

will be useful. After substituting (16) and (17a, b) into (15) and collecting terms of equal order in ϵ, the following sequence of equations is obtained:

$$\epsilon^0:\ g'^2\frac{\partial^2F_0}{\partial\eta^2}+B(\hat{\eta})b(\hat{\xi})F_0=0\qquad(18)$$

$$\epsilon^1:\ g'^2\frac{\partial^2F_1}{\partial\hat{\eta}^2}+B(\hat{\eta})b(\hat{\xi})F_1=-g''\frac{\partial F_0}{\partial\hat{\eta}}-2g'\frac{\partial^2F_0}{\partial\hat{\eta}\partial\hat{\xi}}.\qquad(19)$$

The slow time enters Eq. (18) as a parameter. Hence, when (17a) is substituted into (18), one finds that

$$\frac{\partial^2F_0}{\partial\hat{\eta}^2}+\left[\left(\frac{2N_0b^{\frac{1}{2}}}{\Omega g'}\right)^2+\left(\frac{2N_1b^{\frac{1}{2}}}{\Omega g'}\right)^2\cos2\hat{\eta}\right]F_0=0\qquad(20)$$

in Mathieu's equation. Nayfeh (1973) found approximate solutions to (20) under the condition that $N_1\ll N_0$. The zeroth order solution in $2N_1b^{\frac{1}{2}}/(\Omega g')$ is

$$F_0\approx A\left(\hat{\xi},\frac{2N_1b^{\frac{1}{2}}}{\Omega g'}\hat{\eta}\right)\exp\left[i\frac{2N_0b^{\frac{1}{2}}}{\Omega g'}\hat{\eta}\right]+C\left(\hat{\xi},\frac{2N_1b^{\frac{1}{2}}}{\Omega g'}\hat{\eta}\right)\exp\left[-i\frac{2N_0b^{\frac{1}{2}}}{\Omega g'}\hat{\eta}\right].\qquad(21)$$

The coefficient A (or C) is comprised of growing and decaying exponentials when $2N_0b^{\frac{1}{2}}/(\Omega g')\approx1$. If $2N_0b^{\frac{1}{2}}/(\Omega g')\approx1$, then A is independent of $\hat{\eta}$ to this order. As argued by Nayfeh,[2] the quantity $b^{\frac{1}{2}}/g'$ must be constant for a uniformly valid expansion (in ϵ). The constant is taken to be $b^{\frac{1}{2}}(0)\equiv b_0^{\frac{1}{2}}$ so that

$$g(\hat{\xi})=\frac{1}{b_0^{\frac{1}{2}}}\int_0^{\hat{\xi}}b^{\frac{1}{2}}(\hat{\xi}')d\hat{\xi}'.\qquad(22)$$

Thus, for slowly varying b, g is almost linear in $\hat{\xi}$ as asserted above. However, it is the small deviations from linearity of $g(\hat{\xi})$ which allow for the determination of

[1] The analysis to follow is similar to that by Nayfeh (1973, p. 282).

[2] p. 281.

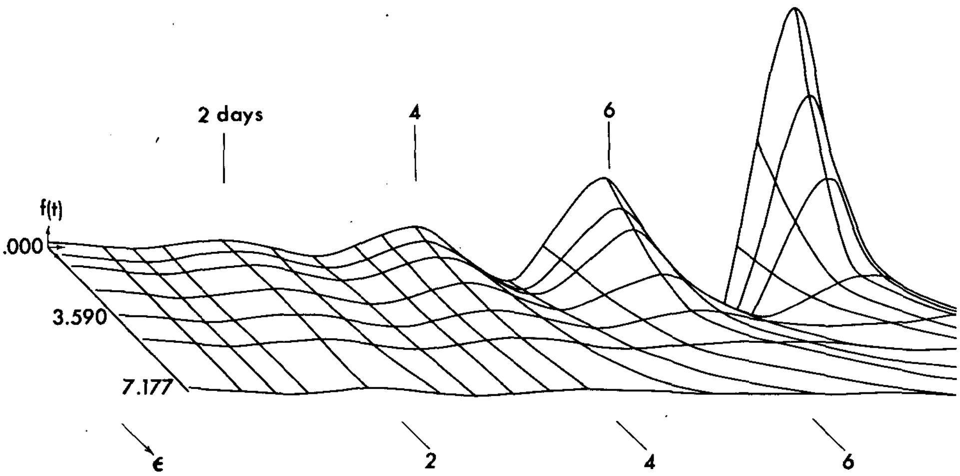

FIG. 1. Variation of the amplitude with time (days) and the value of ϵ.

$A(\xi)$. To find $A(\xi)$ one substitutes (21) into (19) and recalls that $b^{\frac12}/g'=b_0^{\frac12}$. The result is that

$$\left(\frac{\partial^2 F_0}{\partial\hat\eta^2}+B(\hat\eta)b_0F_1\right)g'^2$$

$$=\frac{2N_0b_0^{\frac12}}{\Omega}[-i(g''A+2g'A')\exp(i2N_0b_0^{\frac12}\hat\eta/\Omega)$$

$$+i(g''C+2g'C')\exp(-i2N_0b_0^{\frac12}\hat\eta/\Omega)]. \quad (23)$$

The derivative of A with respect to $\hat\eta$ has been neglected since it is of smaller order in $2N_1b_0^{\frac12}/\Omega$. The requirement of uniform validity of the solutions demands that[3]

$$g''A+2g'A'=0 \quad\text{and}\quad g''C+2g'C'=0. \quad (24a, b)$$

Therefore,

$$A=\frac{\tilde a_0}{g'^{\frac12}}\sim\frac{\tilde a_0}{b^{\frac14}},\quad C=\frac{\tilde c_0}{g'^{\frac12}}\sim\frac{\tilde c_0}{b^{\frac14}}. \quad (25a, b)$$

The actual vertical velocity $f(\tau)$ is related to $F(\tau)$ by $F(\tau)=[1+(\tan\theta-\Gamma\cos\phi\tau)^2]f(\tau)$ and therefore

$$f_0\sim\tilde a_0b^{\frac34}\exp(i2N_0b_0^{\frac12}\hat\eta/\Omega). \quad (26)$$

The expression given by Nayfeh[4] (1973) for $\tilde a_0$ is

$$\tilde a_0=\begin{cases}a_+\exp(+\frac14 2N_1b_0^{\frac12}\hat\eta/\Omega)+a_-\exp(-\frac14 2N_1b_0^{\frac12}\hat\eta/\Omega), & \dfrac{2N_0b_0^{\frac12}}{\Omega}\approx 1\\ 1, & \text{otherwise.}\end{cases}$$

If one neglects the decaying exponential and returns to the original coordinate system, one finds that Eq. (26) becomes

$$f_0(t)\sim\begin{cases}\exp\left[\frac12 N_1\int_0^t b^{\frac12}(t')dt'\right]\\ \quad\times b^{\frac34}(t)\exp\left[iN_0\int_0^t b^{\frac12}(t')dt'\right], \quad \dfrac{2N_0b_0^{\frac12}}{\Omega}\approx 1 & (25a)\\ b^{\frac34}(t)\exp\left[iN_0\int_0^t b^{\frac12}(t')dt'\right], \quad \text{otherwise} & (25b)\end{cases}$$

with

$$b(t)=\frac{1}{1+(\tan\theta-\Gamma\cos\theta t)^2},\quad b_0=\cos^2\theta.$$

Thus, for an initially downward propagating wave ($\tan\theta<0$), b is a decreasing function of time. Eq. (25) describes an exponentially unstable wave motion whose amplitude and period decrease as time goes on. Examination of (25a) reveals that the periodic variation of the diurnal stratification is a destabilizing influence while the effect of the shear is stabilizing. Eq. (25b) is simply Phillips' solution.

A qualitative picture of the behavior of internal gravity waves propagating in a mean shear flow with period variations in the Brunt-Väisälä frequency may now be constructed. As shown by (25a) the trapeze instability operates only for waves of 2-day period. For internal gravity waves, specification of the frequency is tantamount to specifying the inclination of the wave-number vector to the horizontal. Phillips' analysis has shown that the effect of shear is to change this inclination. If this change takes place gradually, then the instability can develop. As noted above, the rate of change of inclination depends on the parameter $\Gamma\cos\phi$.

Numerical solutions of (14) are shown in the three-dimensional graph (Fig. 1) which illustrates the above

[3] This procedure is equivalent to the WKB method.
[4] p. 256.

behavior. The values of the various parameters are given in Table 1. An initially downward propagating wave of 2-day period is considered. When $\epsilon=0$ the unstable wave develops exactly as predicted by Orlanski. As ϵ increases slightly, the motion grows at a reduced rate. A further increase in ϵ shows the solution being that of a wave of slowly decaying amplitude and frequency. This is because the action of the shear tends to lengthen the period and cause the absorption of the wave. The table shows that, as the value of the wind shear decreases, the trapeze instability can develop in an increased range of angles ϕ.

TABLE 1. Variation of $(\pi/2)-\phi$ with ϵ and Γ and the parameters used in the numerical solutions of Eq. (14).

$N_0=10^{-2}\ \mathrm{s}^{-1}$ $\quad \Omega=7.292\times10^{-5}\ \mathrm{s}^{-1}$
$N_1^2=0.9N_0^2$ $\quad \Gamma\equiv U_z$

		$\frac{\pi}{2}-\phi$	
$\epsilon=\frac{\Gamma\cos\phi}{\Omega/2}$	$\Gamma=5$ m $\mathrm{s}^{-1}\ \mathrm{km}^{-1}$	$\Gamma=1$ m $\mathrm{s}^{-1}\ \mathrm{km}^{-1}$	$\Gamma=0.5$ m $\mathrm{s}^{-1}\ \mathrm{km}^{-1}$
0.000	0.00	0.00	0.00
0.598	0.25	1.25	2.50
1.197	0.50	2.50	5.00
2.393	1.00	5.00	10.05
3.590	1.50	7.51	15.18
4.796	2.00	10.00	20.49
7.177	3.00	15.17	31.56

3. Summary and conclusions

The influence of weak mean vertical wind shear upon the trapeze instability of Orlanski (1973) has been investigated. Attention has been confined to the equatorial case where the most unstable wave has a 2-day period. A brief review of the trapeze instability mechanism of Orlanski (1973) and the wave-tilting mechanism of Phillips (1966) is presented. These results are combined to yield the result that, for typical values of wind shear Γ, only free waves which propagate at nearly right angles to the mean wind will be susceptible to the trapeze instability.

One of the ways that wave motion in the atmosphere may be detected is by the appearance of clouds at regularly spaced intervals. For a wave propagating in one horizontal dimension, one would expect cloud rows aligned perpendicular to the direction of propagation. It is evident from the present analysis that the unstable (trapeze) waves must propagate in one horizontal direction. In the equatorial region this direction is poleward, since the mean flow is directed about latitude circles. Thus, if the unstable waves become manifest by the appearance of cloud formations, then one should look for cloud rows which are aligned parallel to the equator. This conclusion may be modified by the fact that local variations of the mean wind direction such as that observed over Africa would allow the unstable waves to propagate longitudinally.

This analysis was performed for freely propagating waves. The asymptotic time dependence of Phillips' solution was also found by Booker and Bretherton (1967) for waves in a semi-infinite medium. This fact leads the author to believe that the presence of a boundary will not severely change the main conclusions. Numerical integrations by Orlanski (1976), under fairly realistic conditions, are in general agreement with the present results.

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Comments on "Instability Theory of Large-Scale Disturbances in the Tropics"

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In a recent paper by Kuo (1975), a condition was used in a CISK parameterization to explain the time and length scales of the observed mixed Rossby-gravity waves and the Kelvin waves in the tropics. This condition, as stated by Kuo on p. 2231, is that "the depletion of moisture through precipitation must be replenished