A Refined Charge Distribution in a Stochastic Electrical Model of an Infinite Cloud

ZEV LEVIN

Department of Geophysics and Planetary Sciences,[1] *Tel-Aviv University, Ramat Aviv, Israel*

(Manuscript received 10 February 1976, in revised form 9 June 1976)

ABSTRACT

The stochastic electrical numerical model of cloud growth and precipitation development of Scott and Levin (1975) has been refined to include a distribution of charge within each size class. Each size class is separated into three subclasses containing negative, neutral and positive charge, respectively. The results indicate that the electric field reaches values of around 4 kV cm^{-1} within about 1000 s and that both positive and negative charges are carried on the particles. In agreement with the previous model, most precipitation size particles carry negative charges while most smaller cloud particles carry positive charges. However, the electrification shows an enhancement in precipitation in the early stages of cloud development. The effect reverses when the field approaches its maximum value. At that point the electrical forces affect the particle interactions through their fallspeed, and the precipitation rate falls below the corresponding rate in the unelectrified case.

1. Introduction

Of late a number of articles have appeared which deal with the possibility of cloud electrification by the interaction of cloud elements. Some suggest that the polarization (induction) mechanism of charge separation is powerful enough for cloud electrification (Sartor, 1967; Mason, 1972; Paluch and Sartor, 1973; Ziv and Levin, 1974; Scott and Levin, 1975). Others, however, question the ability of such a mechanism to generate electric fields for lightning to occur (Moore, 1975a, b; Vonnegut, 1975; Kamra, 1975; Colgate, 1975).

Unfortunately most of the investigators used rudimentary mathematical models that omitted several physical effects. They ignored the cloud particle spectrum and considered only two size classes; they did not consider the effect of autoconversion of cloud water into rainwater; they did not allow for the mutual interdependence of rain formation and electrification; and some have linearized their equations and inappropriately optimized the final integral expressions (see Levin and Scott, 1975).

The recent published papers by Ziv and Levin (1974) and that of Scott and Levin (1975) have allowed for these effects. The last authors, in particular, have developed a stochastic model which simultaneously follows (i) the growth of the cloud elements by collision-coalescence processes, (ii) the development of the electrical charges on the cloud elements, and (iii) the growth of the electric field. This model also considers charge recombination, charge neutralization and the effect of the electric forces on the fallspeed of the particles. In the model the cloud was assumed to have infinite extent in the horizontal plane. The field growth at a plane midway between the top and the bottom of the cloud was calculated by summing the precipitation and discharge current fluxes across it. Although water drops were mainly used to simulate the cloud particles, results using ice spheres were also calculated. Most of the results have shown that the electric field can indeed grow to values of over 4 kV cm^{-1} even when there are large variations in parameters such as the contact angle between the separating particles, the liquid water content and the initial size distribution.

One of the most important results is that reduced efficiency of charge separation (e.g., reduced contact efficiency or increased value of the average collision angle) sometimes results in a higher maximum electric field, which occurs later in the cloud development. This result is due to the highly nonlinear nature of the problem and illustrates the simplified nature of the previous models.

This same model indicates that initially both the field growth and the particle growth are slow. However, when some of the particles reach precipitation size the growth of the field increases rapidly. This field development continues until the electrical forces on the precipitation and the cloud particles modify their fall velocities in such a way as to drastically reduce the number of interactions. This occurs because the precipitation particles receive negative charges and their fall velocity is slowed down by the electric forces, while the positively charged cloud particles are accelerated downward.

---

[1] Previously the Department of Environmental Sciences.

As a result the relative velocity between the interacting particles is reduced, producing a reduction in the interactions between particles, a lower precipitation rate and a termination of electrical field growth.

Of course all these results have to be considered in the proper perspective with full awareness of the assumptions and limitation of the model. One of the most restricting assumptions of the model is the lack of any spatial dimensions. Another limitation, as pointed out by Moore (private communication) and Colgate (1975), is the averaging of charge within each size category at the end of each time step of the integration. This averaging of charge tends to smooth out extreme charge buildup on the cloud elements and might modify the results. Our feelings (Levin and Scott, 1975) were that it would not result in a great modification of the field growth even though some changes are expected in the charge distribution. The purpose of this paper is to introduce some new results, which demonstrate the effect of refining the charge distribution on the electrical and precipitation development in the cloud, and to compare these with the results of Scott and Levin (1975; henceforth abbreviated as S-L).

2. Refined stochastic charging

The model utilized is the earlier S-L model with each size class divided into three charge subclasses containing positive, neutral and negative charges respectively. The notation here is the same as in S-L. Water drops only are simulated.

In the S-L model there are $N=57$ logarithmically distributed size categories each characterized as having charge Q_i cm^{-3} or q_i per particle. The subclasses (positive, neutral and negative) make a total of 171 charge bins. The appropriate charge transfer that accompanies collection is given by

$$\frac{\Delta Q_m}{\Delta t}=\sum_{n=1}^{3N}\sum_{r=1}^{3N}\frac{[q_n(t)+q_r(t)]}{x_n+x_r}x_m f_n(t)f_r(t)R_{nrm}-q_m(t)f_m(t)\sum_{\substack{n=1\\ n\neq m}}^{3N}f_n(t)V_{nm},\quad(1)$$

where the first term on the right represents the charge transferred into bin m by collection of particles from bins n and r, while the second term represents loss of charge from bin m by collection of particles from bin m with any other particles. Since in this model particles of the same size may carry different charges, their fallspeeds are different and collisions between them are allowed. As before, for lack of better experimental data we assume the collision and coalescence efficiencies to be unaltered by electrical effects. In this equation V_{nm} and R_{nrm} are the collection kernel and redistribution kernel, respectively, both defined in detail in S-L; x_n and $f_n(t)$ are the mass of a particle and the number of particles per cubic centimeter in bin n.

Particles in a given size class are separated into the three charge categories (positive, neutral and negative) according to the charge produced during a given interaction. That is, based on means or expected values of charge in a given bin and the mean or expected charge that occurs in a given interaction, the charge on the particles is altered. If the final charge is of the same sign, the particles will be placed in the appropriate size category but in the bin corresponding to the sign of its charge. The same procedure is followed when the charge is neutralized or reversed. This scheme is expected to produce a first approximation to a stochastic model with a full charge distribution in a given size class.

In the numerical procedure, final expected charges produced during interactions of particles of bins i and j are calculated. The polarization charging mechanism was used to charge the particles; its form was identical to that used by S-L. The field growth and the rate of charge buildup were also taken from S-L except that here summation was performed on 171 charge bins.

The S-L model allowed only a single mean charge on a given size class, generally with negative charges on larger particles and positive charges on the smaller particles. The radius at which the charge changed sign varied with time. In the present model both large and small particles carry charges of both signs. This results in events in which, for instance, some large particles are accelerated downward to overtake other slightly larger particles. In that case we assume wake capture and set the collision efficiency $E_1=1$.

3. Numerical procedure

The equations of charge buildup, electric field growth and particle growth are all integrated in parallel. The integration is performed with a standard second-order expansion scheme with 5 s time steps. The liquid water content is held constant and the initial cloud droplet distribution is assumed to be a gamma distribution (see Cotton, 1972). Most calculations are carried out with 100 droplets cm^{-3} and a radius dispersion of 0.28. As time increases the number of particles per cubic centimeter is calculated for 57 logarithmic size classes ranging from 3.00 to 3000.00 μm. The charges are calculated for 171 classes (three charge bins for each size class).

The minimum charge in a bin and in any interaction is limited to one electron; below this value the charge is set to zero. The number of particles in each class is set to zero if, within a particular Δt, it is below 10^{-16} cm^{-3}. With this simplification of the numerics the liquid water content is conserved to within 0.3% of the initial value and the total charge is conserved to within 10^{-8} esu. The accuracy in the work of S-L was much greater but had to be relaxed here in order to conserve computer time. A comparison with more stringent conditions revealed differences in the electric field growth of less than 5% but the time saved by the approximations is substantial.

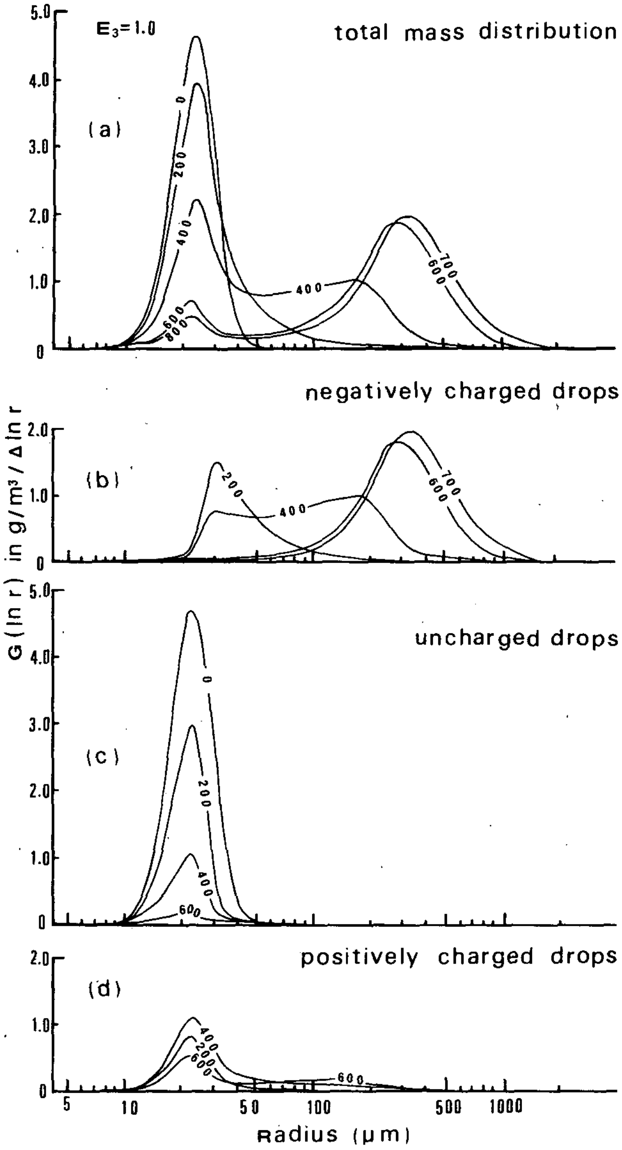

FIG. 1. The development of mass distribution in an electrified cloud. Contact efficiency $E_3=0.3$, liquid water content 3 g m^{-3}, 100 particles cm^{-3}, and radius dispersion 0.28 are considered.

4. Results

The resulting drop spectra for two values of the contact parameter E_3 are shown in Figs. 1 and 2 using the mass distribution function $G(\ln r)$. Fig. 3 shows the corresponding distribution for the unelectrified case ($E_3=0.0$). These figures display the total spectra as a function of time (as was done by S-L), but they also display the spectra of the positively and negatively charged particles. It can be seen that in all cases most of the negatively charged particles are of precipitation size while positively charged particles are mostly small cloud particles, even though in some cases (e.g., $E_3=0.3$) a non-trivial number of precipitation elements are also positively charged. The number of the uncharged drops decreases rapidly as the cloud develops. Comparison of the two figures reveals the rate of spreading of particles into the different charge classes. When charging is very effective ($E_3=1.0$) the field reaches maximum value quickly and the separation of charge is such that almost all precipitation particles are negatively charged and cloud particles positively charged. On the other hand, slower electrification process ($E_3=0.3$) permits longer time for cloud development before maximum field is reached and as a result more precipitation particles receive both positive and negative charges.

It appears, provided the electrification process is very effective, that the evolution of charge is similar to that

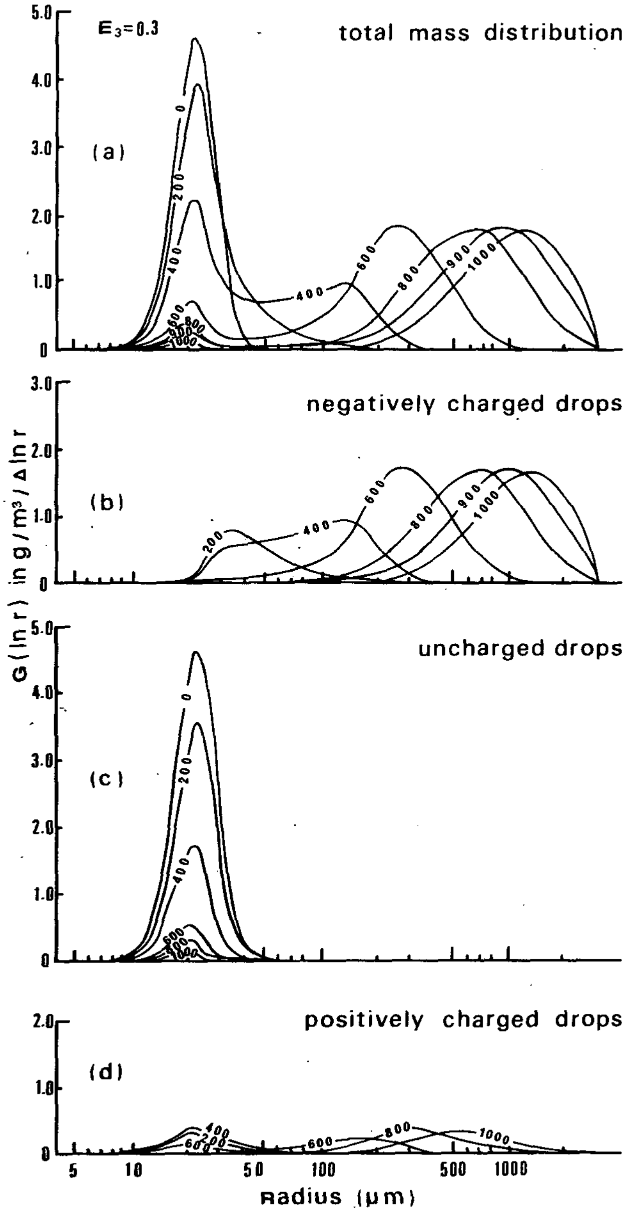

FIG. 2. As in Fig. 1 except $E_3=1.0$.

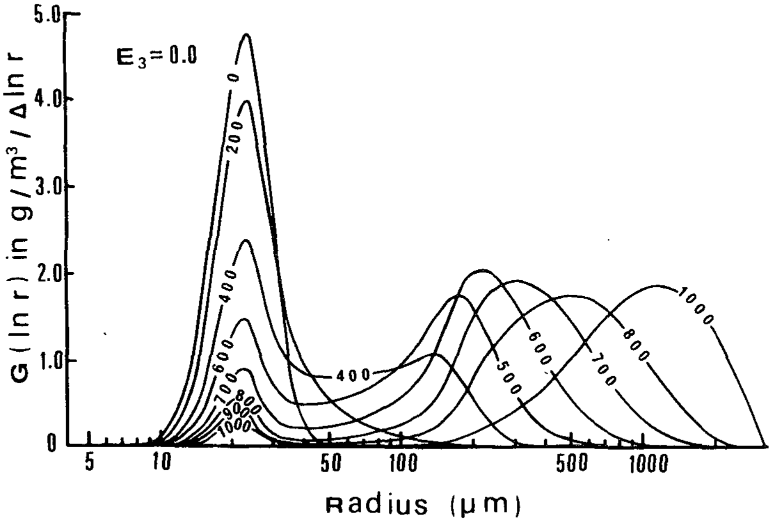

FIG. 3. The development of mass distribution in an unelectrified cloud, $E_3=0.0$.

calculated by S-L. That is, cloud particles are charged positively and precipitation particles are charged negatively. However, if the charging mechanism is not so effective ($E_3=0.3$) an appreciable number of negative cloud drops and positive raindrops are formed. In fact, as time proceeds there is a tendency for both positive and negative charges to reside on raindrops, the smaller raindrops being positive.

Fig. 4 shows the computed values of the charge per particle. Again these results are similar to those of S-L except they show that both charges are possible on either cloud or precipitation elements. The graph shows that the larger charges are always carried on the large particles.

Fig. 5 displays the electric field development for different values of E_3. The field growth for the case when the average interaction angle is 60° is also shown. It is seen that in all cases the field reaches a few kV cm^{-1}

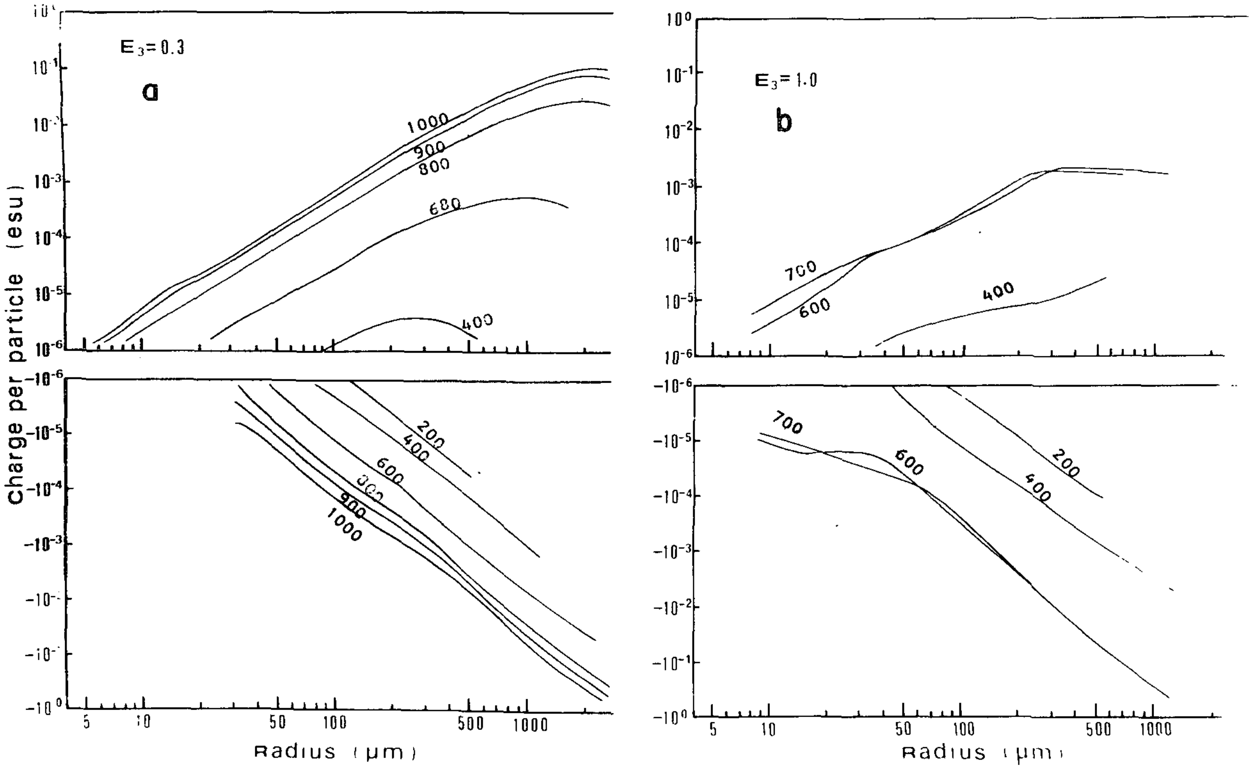

FIG. 4. The development of charge per particle for the cases shown in Figs. 1 and 2.

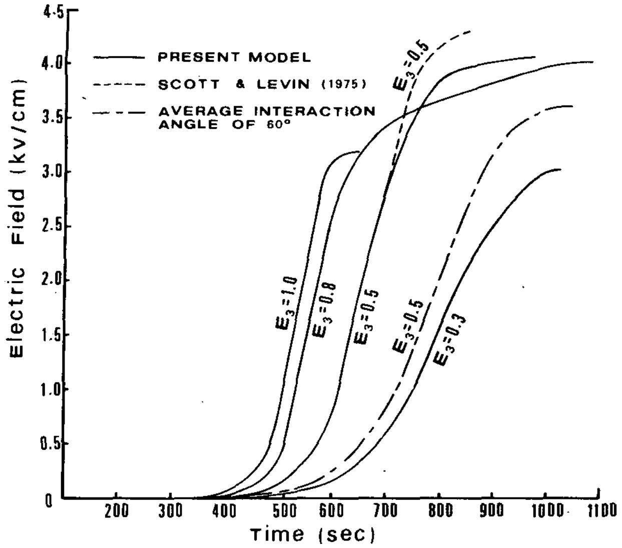

FIG. 5. The electric field growth with time as a function of the contact efficiency E_3.

within reasonable times as was previously observed by S-L. Also, the field growth is delayed when the effectiveness of the electrification is reduced (smaller E_3); however, the value of the maximum field is often higher (compare $E_3=0.5$ with $E_3=1.0$). This unexpected result is in general agreement with the results of Ziv and Levin (1974) and S-L. It stems from the fact that slow electrification permits longer time for precipitation development so that large particles are formed with relatively lower charge so that stronger electrical forces are required to reduce their interaction rate. When the field obtained here is compared with the value of the field in the model of S-L ($E_3=0.5$), it is observed that a slightly lower maximum is reached. This may also be a result of the effect of having positively charged precipitation particles that effectively reduce the value of the precipitation current to the point at which it balances the discharge currents at a lower value of the maximum electric field. In contrast to S-L the inclusion of charge distribution reduces the effectiveness of field buildup when E_3 is less than 0.5. In other words, a reduced efficiency of charge separation results in more positively charged precipitation particles that effectively reduce the space charge at the lower reaches of the modeled cloud. This results in a reduced final maximum field. The use of a higher angle of interaction reduces the maximum field to a value of 3.7 kV cm^{-1}. These reductions in the maximum field strength, when compared to the values obtained by S-L, are not so critical since the values of these fields are still high enough for lightning to occur (Gunn, 1948).

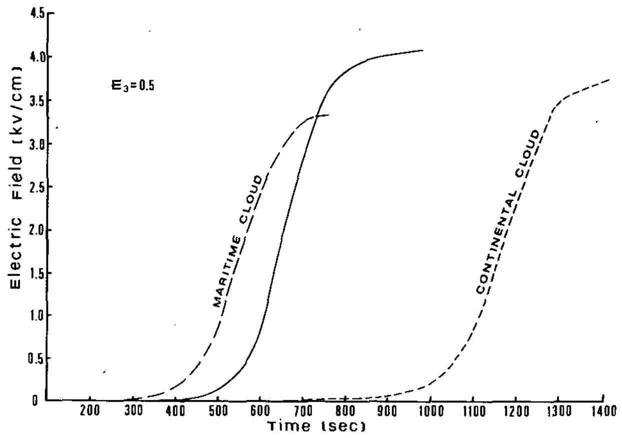

FIG. 6. The effect of the initial size distribution on the growth of the electric field. Maritime cloud is taken as 30 particles cm^{-3} and radius dispersion of 0.33; continental cloud as 300 particles cm^{-3} and radius dispersion 0.25. The solid line represents the reference cloud of 100 particles cm^{-3}, radius dispersion 0.28.

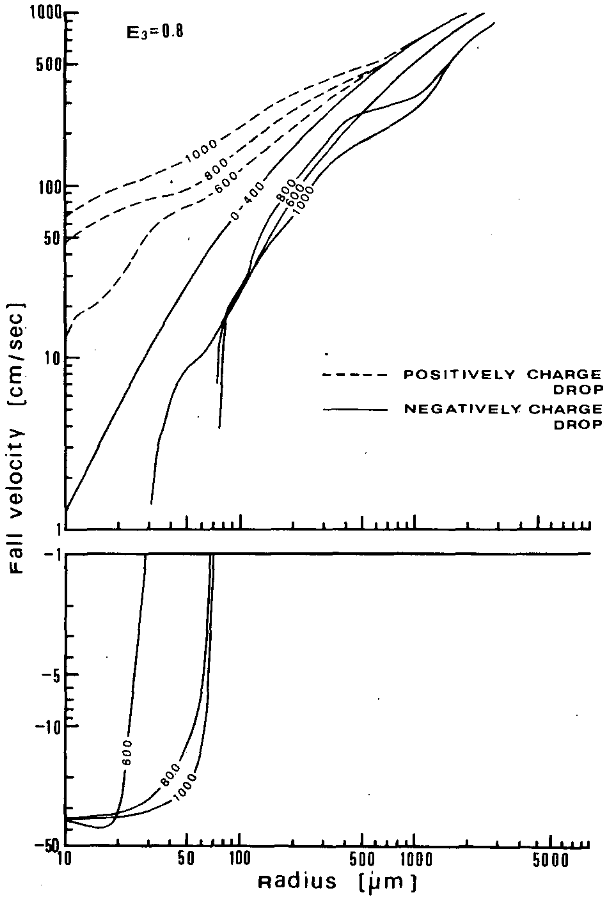

FIG. 7. Fall velocities of charged cloud and precipitation particles as a function of time for the case $E_3=0.8$.

Fig. 6 presents the effect of the initial size distribution on the field growth. The two selected cases are of maritime cloud (30 drops cm^{-3} and radius dispersion 0.33) and continental cloud (300 drops cm^{-3} and radius dispersion 0.25). The results are compared with our reference cloud (100 drops cm^{-3} and radius dispersion 0.28). Again as in S-L they all reach a field of a few kV cm^{-1} but the maritime distribution which tends to develop precipitation earlier due to the more efficient collection processes, reaches a lower maximum field earlier. On the other hand, the continental distribution results in a higher maximum field after substantially longer time.

Fig. 7 presents the fallspeeds of the particles as a function of time. It is shown that the small negatively charged particles are accelerated upward (negative velocity) only at the later stages of cloud and electrical development. Until then they are slowed down and can be captured by the other small particles which are oppositely charged. This tends to neutralize the extreme charges in each charge bin as was expected (see Levin and Scott, 1975).

Fig. 8 represents the development of the precipitation rate for different conditions. It is shown that the precipitation rate (precipitation particles are all larger than 100 μm) increases with time and reaches a maximum when the field is high. For certain choice of parameters ($E_3=1.0$) one can obtain a high electric field when the precipitation rates are very small (~ 1.0 cm h^{-1}). When compared with the unelectrified case ($E_3=0.0$) it is seen that precipitation is enhanced by the electrification at the early stages of cloud development and is reduced at the later stages when the field is near its maximum value. This can be accounted for by the fact that the increased relative velocities of some of the particles (e.g., positively charged precipitation particles and negatively charged cloud particles) increases the value of the kernel and therefore the collection rate. This result was not clearly observed by S-L since in their model precipitation particles carried only negative charges. On the other hand, when the electric field approaches its maximum value the electrical forces slow down the collision-coalescence process by reducing the interaction rate. This is primarily a result of reducing the fallspeed of the majority of precipitation particles (negatively charged) and of increasing the fallspeed of most of the cloud particles (positively charged). This result is in agreement with the results of S-L despite the fact that in this model some large drops are positively charged (Figs. 1d and 2d) and a few small cloud droplets are negatively charged (Figs. 1b and 2b) (remember that these graphs represent mass distribution). This implies that the improvement of including a charge distribution within a size class does not alter the basic results of S-L except to indicate some precipitation enhancement in the early stages.

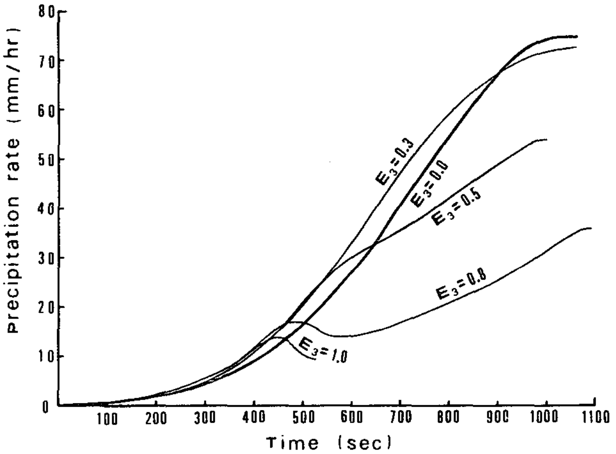

FIG. 8. The development of precipitation rate as a function of E_3. Heavy line represents the unelectrified case. In all cases water content 3 g m^{-3}, 100 particles cm^{-3} and radius dispersion 0.28 are considered.

5. Conclusions

The results presented here definitely show an effect of the electric charging on rain development such that there is an increase in rain in the early stages of cloud development and a decrease in the later stages. They also demonstrate that high electric fields can be developed by the polarization (induction) mechanism at least with the present choice of parameters.

The expansion of the previous model of S-L to include charge distribution within each size class does not change their basic conclusions. However, these results indicate that both positive and negative charges can be produced on particles of any given size. The results demonstrate that, if the effectiveness of the electrification is high, it is possible to reach very high electric fields before high precipitation rates occur.

An important conclusion of the model is that a slow electrification process (when E_3 is reduced from 1.0 to 0.5) results in higher maximum electric field and in a lower maximum field when E_3 is reduced even more. This puts a high priority on the investigation of the coalescence efficiency E_2 and the contact efficiency E_3 for testing the ability of the polarization charging to generate large electric field in thunderclouds.

A word of caution is in order: the model only looks at the cloud as an infinite volume and therefore averages all the variables over the cloud spatial dimensions. It is possible that in a more complete model which includes at least two dimensions the localized electric field could be quite different than suggested by these numerical results. In fact, Illingworth and Latham (1975) have recently shown that in clouds of finite diameter the maximum field produced by the polarization mechanism is smaller than in the infinite models. However, their model is highly simplified and it is difficult to predict the results when more realistic particles distributions are used. Also, the actual value of E_3 (the contact efficiency) is not really known, despite attempts to extract it from the limited available experimental results. It is possible that if E_3 is very much lower than the one chosen here, the field growth will be reduced.

Acknowledgments. The author wishes to thank Dr. W. D. Scott for his advice and for reading the manuscript. Thanks are also due to Mr. Israel Tzur for his assistance with the programming.

REFERENCES

Colgate, S. A., 1975: Comment on "The electrification of thunderclouds and the rain gush." *J. Geophys. Res.*, **80**, 3913–3914.

Cotton, W. R., 1972: Numerical simulation of precipitation development in supercooled cumuli—Parts I and II. *Mon. Wea. Rev.*, **100**, 757–784.

Gunn, R., 1948: Electric field intensity inside of natural clouds. *J. Appl. Phys.*, **19**, 481–484.

Illingworth, A. J., and J. Latham, 1975: Calculations of electric field growth within a cloud of finite dimensions. *J. Atmos. Sci.*, **32**, 2206–2209.

Kamra, A. K., 1975: The role of electrical forces in charge separation by falling precipitation in thunderclouds. *J. Atmos. Sci.*, **32**, 143–157.

Levin, Z., and W. D. Scott, 1975: Polarization charging may produce a large electric field before the radar echo maximum. *J. Geophys. Res.*, **80**, 3918–3923.

Mason, B. J., 1972: The physics of thunderstorms. *Proc. Roy. Soc. London*, **A327**, 433–466.

Moore, C. B., 1975a: Recombination limits on charge separation by hydrometeors in clouds. *J. Atmos. Sci.*, **32**, 608–612.

——, 1975b: Rebound limits on charge separation by falling precipitation. *J. Geophys. Res.*, **80**, 2658–2662.

Paluch, I., and J. D. Sartor, 1973: Thunderstorm electrification by the inductive charging mechanism: I. Particle charges and electric fields. *J. Atmos. Sci.*, **30**, 1166–1173.

Sartor, J. D., 1967: The role of particle interactions in the distribution of electricity in thunderstorms. *J. Atmos. Sci.*, **24**, 601–615.

Scott, W. D., and Z. Levin, 1975: A stochastic electrical model of an infinite cloud: Charge generation and precipitation development. *J. Atmos. Sci.*, **32**, 1814–1828.

Vonnegut, B., 1975: Comment on "The electrification of thunderclouds and the rain gush by Z. Levin and A. Ziv." *J. Geophys. Res.*, **80**, 438.

Ziv, A., and Z. Levin, 1974: Thundercloud electrification: Cloud growth and electrical development. *J. Atmos. Sci.*, **31**, 1652–1661.