NOTES AND CORRESPONDENCE

On the Convergence of Spectral Series—A Reexamination of the Theory of Wave Propagation in Distorted Background Flows

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ABSTRACT

Through a critical analysis of the convergence properties of spectral series, it is shown that Clark's method of solution leads to a divergent series; hence all his recent results on quasi-geostrophic wave propagation in distorted background flows are erroneous. A general condition for convergence is derived. The convergent solution (if it exists) to a general second-order recurrence formula is given, which is then applied to Clark's problem, yielding an exact closed form solution. The solution consists of an interacting trio of waves whose wavenumbers add up to zero. With results thus obtained, it is found that the propagation of wavenumber 2 disturbances is not affected by wavenumber 1 finite-amplitude distortions in the background flow, in disagreement with the result of Clark.

1. Introduction

Since its successful application by Lorenz (1972) the method of truncated spectral series approximation has recently been used widely in problems of barotropic and baroclinic instabilities, as well as in related problems of wave propagation in nonuniform basic states (e.g., Hoskins, 1973; Gill, 1974; Duffy, 1975; Kim, 1975; Clark, 1975). It must have been realized that for most problems in these fields the series often do not converge as rapidly as in the case of Lorenz, and that there may even exist instances when divergence actually occurs. To test for convergence the usual method has been to compare the values of the approximate solutions obtained through successively retaining more and more terms in the truncated series. The process can be tedious, especially when convergence is slow.

One of the purposes of this note is to point out that the convergence properties of a spectral series can be deduced from its recurrence formula *directly*. The condition for convergence will be derived, which turns out to be an eigenvalue equation relating the coefficients of the recurrence formula. For illustration the series of Gill (1974) and Lorenz (1972) will be discussed briefly as examples. Section 5 will be devoted entirely to the reexamination of the theory of wave propagation in distorted basic flows, a problem recently considered by Clark (1975) who obtained, by the method of truncation, a solution consisting of fictitious "critical levels". It shall be shown that Clark's solution is divergent. Using a method discussed in Section 3 the problem is solved exactly. The physical implications of the solution on the propagation of planetary waves in the presence of finite-amplitude distortions will also be discussed.

2. Classification

If the solution to a differential equation with periodic coefficients is expressed in a spectral series

$$\Psi(x)=\sum_n \phi_n e^{inx}, \tag{1}$$

a finite-difference equation results which serves as a recurrence relation for the quantities ϕ_n. Only second-order difference equations will be considered here, though extensions to higher orders are possible.

Consider the problem of absolute convergence of series (1) defined by the recurrence formula

$$\phi_{n+1}+\alpha_n\phi_n+\beta_n\phi_{n-1}=0. \tag{2}$$

By virtue of d'Alembert's ratio test, the convergence of (1) is related to the magnitude of $r_n=|\phi_{n+1}/\phi_n|$ for large n. The following classification shall be made on r_n:

a. Series (1) is said to be "slowly convergent" if $r=\lim_{n\to\infty}r_n$ exists and $0<r<1$, and "slowly divergent" if $r>1$ but finite.

b. Series (1) is said to be "rapidly convergent" if $r_n\to 0$ as $n\to\infty$, and "rapidly divergent" if $r_n\to\infty$.

The series solution to the difference equations of Gill

(1974) and Clark (1975) belongs to class a, while that of Lorenz (1972) belongs to class b.

3. Discussion of class a

Class a series are usually characterized by α_n and β_n of (2) having finite limits as $n\to\infty$. Let $\lim_{n\to\infty}\alpha_n=\alpha$ and $\lim_{n\to\infty}\beta_n=\beta$. For $n>N$, where N is a sufficiently large integer, (2) can be approximated by

$$\phi_{n+1}+\alpha\phi_n+\beta\phi_{n-1}=0. \tag{3}$$

The general solution to (3) is

$$\phi_n=a_1t_1{}^n+a_2t_2{}^n, \tag{4}$$

where a_1 and a_2 are constants, and t_1 and t_2 are the two roots to the quadratic equation

$$t^2+\alpha t+\beta=0. \tag{5}$$

If $|t_1|>|t_2|$, then since $\lim_{n\to\infty}(t_2/t_1)^n=0$, Eq. (4) implies

$$\lim_{n\to\infty}(\phi_{n+1}/\phi_n)=t_1, \tag{6}$$

i.e., the ratio tends to the root of (5) that has the larger modulus. If $t_1\neq t_2$, but $|t_1|=|t_2|$, it can be shown that $\lim_{n\to\infty}(\phi_{n+1}/\phi_n)$ does not exist, for there is no limit to $(t_2/t_1)^n$. This, essentially, is Poincaré's theorem (Poincaré, 1885). Generalizations to nth order equations can be found in Perron (1909).

Because the ratio (ϕ_{n+1}/ϕ_n) would, in general, tend to the root with the larger modulus, it appears that convergence is possible only if both roots to (5) have moduli less than 1. It turns out that this condition for convergence is too restrictive. There may exist special solutions to (3) that have $a_1=0$. For these solutions, one has, instead of (6),

$$\lim_{n\to\infty}(\phi_{n+1}/\phi_n)=t_2, \tag{7}$$

i.e., the ratio tends to the smaller root.

It can be proved (Perron, 1929; Milne-Thompson, 1933) that the solution to (2) that has the smaller limit (7) is given for $n\geqslant 1$ by

$$\phi_n/\phi_{n-1}=-\beta_n/(\alpha_n-\beta_{n+1}/(\alpha_{n+1}-\beta_{n+2}/(\cdots\quad\cdots))), \tag{8}$$

and for $n'\equiv -n\geqslant 0$ by

$$\phi_{n'}/\phi_{n'+1}=-1/(\alpha_{-n'}-\beta_{-n'}/(\alpha_{-n'-1}-\beta_{-n'-1}/(\cdots\quad\cdots))). \tag{9}$$

Consistency of (8) and (9) for the quantity ϕ_1/ϕ_0 gives the required eigenvalue equation

$$-\beta_1/(\alpha_1-\beta_2/(\alpha_2-\beta_3/(\cdots\quad\cdots)))=-(\alpha_0-\beta_0/(\alpha_{-1}-\beta_{-1}/(\alpha_{-2}-\beta_{-2}/(\cdots\quad\cdots)))). \tag{10}$$

If the coefficients of the recurrence formula satisfy (10), the ratio ϕ_{n+1}/ϕ_n will tend to the smaller root t_2. The series is convergent if $|t_2|$ is less than 1.

In his study of the problem of barotropic instabilities, Gill (1974) truncates his series to three terms. The convergence is then tested by comparing the result with those obtained through less severe truncations. Gill finds that, in some cases (*viz.*, his "weak interaction limit"), the results converge quickly, while in other cases ("strong interaction limit"), many terms of the series have to be included to obtain convergence. Using results obtained earlier, one can understand why this happens. Gill's recurrence relation [his Eq. (7.5)] can be written in the form of (2) with $\alpha_n=-i\mu/a_n$ and $\beta_n=1$, where $\lim_{n\to\infty}a_n=1$. Eq. (5) becomes

$$t^2-i\mu t+1=0,$$

with roots

$$t_1=i[\tfrac{1}{2}\mu+(1+\tfrac{1}{4}\mu^2)^{\frac{1}{2}}],\quad t_2=i[\tfrac{1}{2}\mu-(1+\tfrac{1}{4}\mu^2)^{\frac{1}{2}}],$$

so $|t_1|>1$ and $|t_2|=1/|t_1|<1$.

For convergence (10) has to be satisfied, which can be seen to be the same as his Eqs. (7.8) and (7.9). The rate of convergence, when (10) is satisfied, is

$$r=|t_2|=|\tfrac{1}{2}\mu-(1+\tfrac{1}{4}\mu^2)^{\frac{1}{2}}|.$$

For $\mu\gg 1$ ("weak interaction limit"), one has $r\ll 1$ and therefore the convergence is fast. For $\mu\ll 1$ ("strong interaction limit"), $r\approx 1^-$, therefore the convergence is slow. The latter case is not suited for application of the truncated spectral method.

4. Discussion of class b

For series of class b the coefficients α_n and β_n of (2) usually do not both tend to finite limits when n becomes large. Asymptotic solutions of (2) for large n are often more difficult to find than in class a, but they are nevertheless obtainable in most cases by methods available in calculus of finite differences (see e.g., Milne-Thompson, 1933). For our purpose here, one needs to know only that there exist two solutions: one diverges and the other converges "rapidly". We still have the result that the solution would tend to the larger limit (which in this case can be infinite) unless (10) is satisfied. Then the series converges rapidly and is well suited for the application of truncated approximations.

Consider as an example the series of Lorenz (1972). Its recurrence formula can be written as [cf. his Eq. (13)]

$$\phi_{n+1}+\alpha_n\phi_n+\beta_n\phi_{n-1}=0, \tag{11}$$

with

$$\alpha_n=2(n\beta a_n+\lambda)/(la_n),\ \beta_n=1,\ \lim_{n\to\infty}a_n=1.$$

Two asymptotic solutions can be found for large n:

$$\phi_{n+1}^{(1)}/\phi_n^{(1)}\approx -2n\beta/l\quad\text{and}\quad \phi_{n+1}^{(2)}/\phi_n^{(2)}\approx -\frac{l}{2n\beta}. \tag{12}$$

So $\phi_n^{(1)}$ leads to a rapidly divergent series while $\phi_n^{(2)}$ gives a rapidly convergent one. Perron's theorem tells us that the solution having the asymptotic behavior of $\phi_n^{(2)}$ is given by (8) and (9), which in turn requires that (10) be satisfied. Incidentally, it can be shown that the eigenvalue condition (10) is the same as the condition for the vanishing of the determinant of the set of equations generated by (2) for various n's. In particular if (10) is truncated by ignoring all α_n's except α_0, α_{-1} and α_1, one obtains for Lorenz's problem

$$-\alpha_0+1/\alpha_{-1}=-1/\alpha_1,$$

which can be shown to be the same as the characteristic equation obtained by Lorenz for a three-term series [cf. his Eq. (20)]. Eq. (10) offers the advantage that the errors of truncation can be estimated.

5. Propagation of quasi-geostrophic waves in distorted background flows

Matsuno (1970) has numerically calculated the amplitudes of stationary waves in the winter stratosphere using as a lower boundary forcing the observed waves in January 1967 at 500 mb. His predicted amplitude for wavenumber 1 is in good agreement with observation, both increasing with height to 30 km level (at around 60°N), but the predicted value for wavenumber 2 shows a deficiency above 20 km when compared with observed wave amplitudes. This discrepancy of wavenumber 2 magnitudes has been mentioned from time to time by some authors to justify the need for new theories on the propagation of planetary waves. This deficiency in wavenumber 2 amplitudes is easily accountable by the natural variations of that wave, without invoking any new theories. It seems[1] that Matsuno has used as his observed values the data from January 1958, during which time a major sudden warming occurred with wavenumber 2 growing to abnormally large amplitudes, while his calculations are based on data from January 1967, a "normal" year with no occurrence of major warming episodes. Had he compared his calculations with the observed value for a normal year, he might have obtained better agreement.[2]

The theory of Clark (1975) is one of the attempts at accounting for the apparent deficiency of wavenumber 2 disturbances in the stratosphere. Clark investigates the possibility for enhancements of wavenumber 2 propagations due to the presence in the basic flow of a finite-amplitude wavenumber 1 distortion, a conjecture originally forwarded by Matsuno (1970). The governing equation for linear quasi-geostrophic waves in a non-uniform background flow $[U(x,y,z),\, V(x,y,z),\, 0]$ is

$$\left[\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}+V\frac{\partial}{\partial y}\right]\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{f^2}{S}\left(\frac{\partial^2}{\partial z^2}-\frac{1}{4}\right)\right]\Phi'$$
$$+\left[\beta-U_{xx}-U_{yy}-\frac{f^2}{S}\left(\frac{\partial^2}{\partial z^2}-\frac{\partial}{\partial z}\right)U\right]\Phi_x'$$
$$-\left[V_{xx}+V_{yy}+\frac{f^2}{S}\left(\frac{\partial^2}{\partial z^2}-\frac{\partial}{\partial z}\right)V\right]\Phi_y'=0, \quad (13)$$

where t is the time, x east-west, y north-south and z log-pressure coordinates, f the Coriolis parameter and $\beta=df/dy$ (both evaluated at midlatitude); S is the static stability parameter and Φ' is the perturbation streamfunction normalized by the density factor $e^{-z/2}$. Clark uses as the background flow the Rossby finite-amplitude solution $U=-\partial\Phi/\partial y$, $V=\partial\Phi/\partial x$ with

$$\Phi(x,y,z)=-U_0y+\mathrm{A}ye^{z/2}\cos(kx+mz), \quad (14)$$

where U_0 is the constant zonal velocity, A the amplitude of the finite-amplitude distortion, and $(f^2/S)(m^2+\frac{1}{4})+k^2=\beta/U_0$. If the solution is assumed to be stationary with the y structure

$$\Phi'=y\Psi'(x,z), \quad (15)$$

the following will result:

$$[U_0-Ae^{z/2}\sin(x_0+mz)]$$
$$\times\left[\frac{\partial^2}{\partial x_0^2}+\theta\left(\frac{\partial^2}{\partial z^2}-\frac{1}{4}\right)+r\right]\Psi_{x_0}'$$
$$-[Ae^{z/2}\sin(x_0+mz)]$$
$$\times\left[\frac{\partial^2}{\partial x_0^2}+\theta\left(\frac{\partial^2}{\partial z^2}-\frac{1}{4}\right)+r\right]\Psi'=0, \quad (16)$$

where we have put $x_0=kx$, $\theta=f^2/(Sk^2)$ and $r=(\beta/k^2)/U_0$. To solve (16) Ψ' is spectrally decomposed, i.e.,

$$\Psi'(x_0,z)=\sum_{n=-\infty}^{\infty}\Psi_n(z)e^{inx_0}. \quad (17)$$

Note that the above series, when convergent, implies that the solution to Eq. (16) has the same period as the coefficients of that equation, i.e., $\Psi'(x_0+2\pi, z)=\Psi'(x_0,z)$. The existence of such periodic solutions can be justified by the theory of Floquet (see Struble, 1962).

When (17) is substituted into (16) a recurrence formula results:

$$(n+2)\epsilon^*\phi_{n+1}-nU_0\phi_n+(n-2)\epsilon\phi_{n-1}=0, \quad (18)$$

where $\epsilon\equiv\frac{1}{2}Ae^{z/2+imz}$, ϵ^* is the complex conjugate of ϵ and

$$\phi_n(z)\equiv\left[-n^2+\theta\left(\frac{d^2}{dz^2}-\frac{1}{4}\right)+r\right]\Psi_n\equiv L[\Psi_n]. \quad (19)$$

[1] Matsuno does not mention in his Fig. 7 which year his data are from, but in his Fig. 8 he mentions Muench's (1965) January 1958 data.

[2] Schoeberl and Geller (to be published) have shown that the discrepancy disappears when a different wind model is used in Matsuno's problem.

For $|n| \to \infty$, the limit equation (5) is

$$\epsilon^* t^2 - U_0 t + \epsilon = 0,$$

with roots

$$t_1 = \tfrac{1}{2}[U_0 + (U_0^2 - 4\epsilon\epsilon^*)^{\frac{1}{2}}]/\epsilon^*,$$
$$t_2 = \tfrac{1}{2}[U_0 - (U_0^2 - 4\epsilon\epsilon^*)^{\frac{1}{2}}]/\epsilon^*.$$

Applying Poincaré's theorem, one finds that for $U_0^2 < 4\epsilon\epsilon^*$, $\lim_{n\to\infty}\phi_{n+1}/\phi_n$ does not exist, suggesting that the convergence of the solution is in question. If Eq. (16) is examined, one finds that the coefficient of the highest derivative in x possesses zeroes when $U_0^2 < 4\epsilon\epsilon^*$. For this case, the solution cannot be obtained in a spectral form (see Section 6 for more comment on this point). For $U_0^2 > 4\epsilon\epsilon^*$ (i.e., the magnitude of the zonal wind is everywhere greater than that of the distortion), we have $|t_1| > 1$ and $|t_2| < 1$. Thus if (10) is not satisfied, the sequence $\{\phi_n\}$ will be divergent, as will the series (17).

Clark truncates his series to three terms ($\Psi_{n+1}, \Psi_n, \Psi_{n-1}$) and obtained a singular solution: $\phi_n(z) = D\delta(z - z_0)$. If his method of solution is followed, successive approximation can lead only to more δ-functions. It is clear that this solution can never be written in the form of (8) and (9) with $\alpha_n = -nU_0/(n+2)\epsilon^*$ and $\beta_n = (n-2)\epsilon/(n+2)\epsilon^*$. Therefore Clark's series is divergent and his solution not valid. As an indication of non-convergency, let us compare his eigenvalue solution with that obtained by retaining five waves—ϕ_n, $\phi_{n\pm1}$ and $\phi_{n\pm2}$. For the sake of comparison take $n = 2$ and $k = 1$ (a case considered by Clark). Clark's eigenvalue, from his Eq. (17), is

$$z_0 = (1 - 2m_i)^{-1} \ln[6U_0^2/A^2],$$

while that obtained from five waves is

$$z_0 = -\infty, \text{ or } z_0 = (1 - 2m_i)^{-1} \ln[8U_0^2/3A^2].$$

The correct eigenfunction should be obtained from Eqs. (8) and (9). However, before we do so, the following result shall be proved: If $\beta_n = 0$ for $n = N > 0$ and if the solution ϕ_n can be written in the form of (8), then $\phi_n = 0$ for $n \geqslant N$. From (8), ϕ_N can be written as $\phi_N/\phi_{N-1} = -\beta_N/(\alpha_N - \cdots)$, so that ϕ_N vanishes when $\beta_N = 0$. The recurrence formula for $n = N+1$ will in turn show $\phi_{N+2} = 0$. The result is thus proved. Similarly it can be proved that if $1/\alpha_{-n'} = 0$ for $n' = N \geqslant 0$, then $\phi_{-n'} = 0$ for $n' \geqslant N$, if the solution can be written in the form of (9). Returning now to our problem, since $\beta_2 = 0 = 1/\alpha_{-2}$, we have $\phi_n = 0$ for $n \geqslant 2$ and $n \leqslant -2$. *Exchange of potential vorticities occurs only among waves with zonal wavenumbers 0, k and −k; the other waves behave as if the distortion is absent.* From Eqs. (8) and (9) we have

$$\phi_1/\phi_0 = -\epsilon/U_0 \quad \text{and} \quad \phi_{-1}/\phi_0 = -\epsilon^*/U_0. \tag{20}$$

It can be shown that these two solutions are compatible, i.e., they satisfy Eq. (10). The horizontal structure of the solution Ψ' is now completely determined. Its vertical structure is given by

$$\left.\begin{aligned} L[\Psi_n] &= 0 \quad \text{if} \quad |n| \geqslant 2 \\ L[\Psi_1] &= -\epsilon/U_0 L[\Psi_0] \\ L[\Psi_{-1}] &= -\epsilon^*/U_0 L[\Psi_0] \end{aligned}\right\}. \tag{21}$$

They can easily be solved given the boundary conditions. If the radiation boundary condition is applied at the top we have

$$\Psi_n = a_n e^{il_n z}, \quad |n| \geqslant 2,$$

$$\Psi_1 = a_1 e^{il_1 z} - a_0 \exp[z/2 + i(m + l_0)z] \times \frac{A[-\theta(l_0^2 + \frac{1}{4}) + r]}{2U_0\{\theta[(\frac{1}{2} + im + il_0)^2 - \frac{1}{4}] + (r-1)\}},$$

$$\Psi_0 = a_0 e^{il_0 z},$$

$$\Psi_{-1} = a_{-1} e^{il_{-1} z} - a_0 \exp[z/2 + i(-m + l_0)]z \times \frac{A}{2U_0} \frac{[-\theta(l_0^2 + \frac{1}{4}) + r]}{\{\theta[(\frac{1}{2} - im + il_0)^2 - \frac{1}{4}] + (r-1)\}},$$

where l_n is given by (for $|n| \geqslant 1$)

$$\theta(l_n^2 + \tfrac{1}{4}) = (r - n^2)$$

and l_0 is to be obtained from lower boundary condition. It is seen that the amplitudes of the waves whose zonal wavenumbers have the same magnitude as that of the finite amplitude distortion are enhanced by the factor $e^{z/2}$.

To conclude this section let us state the results: The presence in the background flow of a finite-amplitude distortion of wavenumber k superimposed on a mean state of wavenumber zero has no effect on the propagation of disturbances whose zonal wavenumbers differ from 0, k and $-k$. The waves whose zonal wavenumbers are 0, k and $-k$ form an interacting trio exchanging potential vorticity among the three. The preceding conclusion should be qualified by noting the rather unrealistic meridional structure of the waves assumed. As far as Clark's solution is concerned, it has been shown that it is erroneous for both the large and small U_0 cases no matter how many terms are retained in his series. For the small U_0 case, the equation has a logarithmic singularity in the domain. Thus the "solution", if written in a spectral series, cannot be differentiated term by term as is done by Clark to obtain his solution. For the large U_0 case, spectral solution is possible, but the correct solution is not the one given by Clark. This is demonstrated by showing that the correct (convergent) solution should satisfy a certain eigenvalue equation which cannot be satisfied by Clark's series.

6. Concluding remarks

There has often been a misconception that series of the form of (1) are Fourier series with their convergence guaranteed by Bessel's inequality. It turns out that when divergence of (1) occurs, it can often be shown that the governing differential equation for $\Psi(x)$ has a singular point somewhere in x. In the case of Clark (1975), the coefficient for the highest derivative in x can be seen to vanish at some x if $U_0^2<4\epsilon\epsilon^*$. It is interesting to note that if $\Psi(x)$ is to be represented by a Fourier series in the interval $[0,2\pi]$, and if $\Psi(x)$ has a singularity somewhere, say at the boundary point $x=0$, its Fourier representation diverges not only at the point $x=0$, but over the whole domain.[3] Though the Fourier series can be modified so as to be well behaved away from $x=0$, the modified series cannot be written in the form of (1) (see Lanczos, 1966).

This note is not meant to be an exhaustive discussion of various cases on the convergence of spectral series. For example the case $|t_2|=1$ is not discussed (where Raabe's test has to be used instead of the ratio test used here). However, for most practical purposes, the series is "divergent" to someone trying to approximate it with a few terms. By a discussion of some typical examples, it is hoped that some useful mathematical results, relevant to the spectral series solution of equations, are brought to the attention of those working with spectral series.

[3] This remark applies to the first derivative of the solution in Clark's problem for it is the derivative that is singular.

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A Search for Lamb Waves Generated by the Solar Eclipse of 11 May 1975

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ABSTRACT

A search for eclipse-induced Lamb waves was made with ground-level pressure sensors at two sites during the eclipse of 11 May 1975. Cross-correlation analysis resulted in a preliminary upper limit being placed on the magnitude of the waves.

1. Introduction

Two classes of pressure waves have been predicted to arise as a result of atmospheric cooling induced by the moon's shadow during a solar eclipse. The first to be predicted were atmospheric gravity waves generated by cooling in atmospheric ozone (Chimonas, 1970); the second were Lamb waves from cooling in tropospheric water vapor (Chimonas, 1973). Neither have yet been identified (Jones and Bogart, 1975; Anderson and Keefer, 1975). This is not altogether surprising in the case of atmospheric gravity waves because their predicted magnitude at ground level is at least a factor