A Numerical Determination of the Efficiency with Which Spherical Aerosol Particles Collide with Spherical Water Drops Due to Inertial Impaction and Phoretic and Electrical Forces

S. N. GROVER AND H. R. PRUPPACHER

Department of Atmospheric Sciences, University of California, Los Angeles 90024

A. E. HAMIELEC

Department of Chemical Engineering, McMaster University, Hamilton, Ontario, Canada

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ABSTRACT

A theoretical model to compute the efficiency with which aerosol particles of radius $0.5 \leqslant r \leqslant 10\ \mu$m collide with water drops of radius $a = 42, 72, 106, 173, 309$ and $438\ \mu$m falling at terminal velocity in air is presented. Inertial impaction, thermophoresis, diffusiophoresis and electrical effects are considered. The computations were carried out for ambient conditions of 10°C, 900 mb, and 100%, 95% and 75% relative humidity. The drops and particles were assumed to carry electric charges of $0.2\ a^2$ and $0.2\ r^2$ [esu], respectively, and charges of $2.0\ a^2$ and $2.0\ r^2$ [esu], respectively, where a and r are expressed in centimeters. The external electric field strengths were assumed to range between $0 \leqslant E_0 \leqslant 3 \times 10^5\ V\ m^{-1}$. The results of our computations show 1) that the efficiency E with which aerosol particles collide with the drops considered is significantly raised by phoretic and electric forces over and above the efficiency resulting from inertial impaction, this effect being the more pronounced the smaller the collector drop; 2) hydrodynamic effects as well as phoretic effects tend to promote particle capture in the rear of a drop if particles are sufficiently small, resulting in a minimum of E versus r which lies in the "Greenfield gap" region and thus reinforces the gap; 3) electrical effects tend to eliminate this gap reinforcement; and 4) computations which consider phoretic effects only without simultaneously taking account of the particles' motion due to the hydrodynamic flow around the collector drop significantly overestimate E for $r \lesssim 1.5\ \mu$m.

1. Introduction

By far the largest portion of aerosol particles is removed from the atmosphere by interaction of these particle with clouds and precipitation. One rather obvious mechanism of removal hinges on the fact that drops and ice crystals in atmospheric clouds require the presence of aerosol particles for their formation. As a consequence, a significant portion of atmospheric aerosol particles are removed by precipitation from the atmosphere together with the drops and ice crystals they formed. In addition to this phase change mechanism of aerosol removal, aerosol particles may become removed (scavenged) from the atmosphere 1) by coagulation of aerosol particles with water drops and ice crystals caused by Brownian diffusion; 2) by the collision of aerosol particles with water drops and ice particles caused by phoretic forces (thermophoresis and/or diffusiophoresis); 3) by inertial impaction of aerosol particles on water drops and ice particles brought about by the hydrodynamic interaction of these particles with drops and ice particles both falling under gravity in air; and 4) by the collision of aerosol particles with water drops and ice particles caused by electrical forces which exist when the aerosol particles and the drops and ice particles are electrically charged and/or when an external electric field is present. Of course, these mechanisms also remove aerosol particles from the atmosphere only if cloud formation is followed by precipitation.

Greenfield (1957) was perhaps the first to determine the effect of various scavenging mechanisms combined. He computed the net effect for the removal of aerosol particles by drops due to Brownian diffusion, inertial impaction and turbulence. Unfortunately, at that time the methods available to determine the trajectory of aerosol particles past drops were rather inaccurate. In addition, his computations neglected the effects of phoretic forces. Also, the method for taking turbulence into account was rather crude. Greenfield also assumed that the collection kernels of all the individual scavenging mechanisms are additive which implies that the scavenging processes considered are not coupled to each other. This assumption cannot be considered accurate for his case since a turbulent flow field invariably will affect the diffusion of aerosol particles by Brownian motion.

A second study on the combined effect of various scavenging mechanisms was carried out by Slinn and Hales (1970) who considered Brownian diffusion, phoretic effects and inertial capture for the particular case of scavenging water drops which are 3°C colder than the environmental air. Aside from using a rather crude hydrodynamic model for determining the effect of inertial impaction, Slinn and Hales also assumed that the collection kernels of the individual scavenging processes considered are additive. Again, this assumption cannot be considered accurate for this particular case since the hydrodynamic flow around a drop may significantly alter both phoretic and Brownian diffusion, and since phoretic motion invariably affects Brownian diffusion.

In a third, more recent study, Young (1974) determined the net scavenging effect of Brownian diffusion and phoretic forces for the special case of water drops evaporating in an environment of 98% relative humidity, and growing by vapor diffusion in an environment supersaturated by 0.3%. Aside from the fact that scavenging by inertial impaction was not considered, Young again assumed that the collection kernels for the scavenging processes considered are additive. This assumption neglects the fact that the presence of phoretic forces will alter the Brownian motion collection rate. If, on the other hand, Brownian motion is negligibly contributing to particle collection, diffusiophoretic and thermophoretic effects can be considered uncoupled as long as the mole fraction of the diffusing species is small (Annis and Mason, 1975; Goldschmitt and May, 1966).

In addition to the deficiencies mentioned, all models discussed above suffer from not lending themselves to an extension which would make it possible to accurately include the effects coupled simultaneously to Brownian diffusion, and to phoretic and hydrodynamic forces.

In order to attempt eliminating some of the deficiencies of previous models, we have developed two new theoretical models which are to some extent complementary to each other. The first of these models (hereafter called model 1) computes particle capture due to the combined action of inertial forces, phoretic forces and electrical forces by means of a particle trajectory method, but ignores the effect of Brownian diffusion. The second model (hereafter called model 2) considers particle capture due to the combined action of Brownian motion, phoretic forces and electrical forces but ignores inertial impaction. Obviously, the first model applies to relatively large particles and the second to relatively small particles. Model 1 and the results derived thereof shall be the subject of the present paper. Model 2 will be discussed in a paper to be published elsewhere.

2. Physics and mathematics of model 1

The present model (model 1) attempts to determine the efficiency with which spherical water drops collect spherical aerosol particles by the simultaneous action of inertial impaction, diffusiophoresis and thermophoresis, and electrical forces due to the presence of electrical charges on the drops and particles and/or due to the presence of a vertical external electric field.

This efficiency is found from a determination of the trajectory of aerosol particles relative to the collector drop falling at terminal velocity in air. In the absence of Brownian motion the trajectory of such an aerosol particle (assuming that the flow around the aerosol particle does not affect the drop motion nor the flow field around it—which is justified if the ratio of the particle mass to drop mass is $\lesssim 10^{-3}$) is computed from its equation of motion:

$$m\frac{d\mathbf{v}}{dt}=m\mathbf{g}^*-[6\pi r\eta(\mathbf{v}-\mathbf{u})/(1+\alpha \mathrm{N_{Kn}})]+\mathbf{F}_{Th}+\mathbf{F}_{Df}+\mathbf{F}_e. \quad (1)$$

Here m is the mass of the aerosol particle, $\mathbf{v}$ its instantaneous velocity and r its radius, t is time, $g^*=g(\rho_P-\rho)/\rho_P$, with ρ_P the particle density and ρ the air density, η is the dynamic viscosity of air, $\mathbf{u}$ is the local velocity of the air flow past the drop, $(1+\alpha \mathrm{N_{Kn}})$ is the Stokes-Cunningham correction to the drag on a parricle with size of the order of the mean free path length of the air molecules, $\mathrm{N_{Kn}}=\lambda/r$ is the Knudsen number with λ the mean free path length of air molecules, and $\alpha=1.25+0.44\exp(-1.07\,\mathrm{N_{Kn}}^{-1})$, from Junge (1963). The first term on the right side of Eq. (1) is the buoyancy-corrected gravitational force on the particle. The second term is the slip-corrected hydrodynamic drag force acting on the particle. The terms $\mathbf{F}_{Th}$, $\mathbf{F}_{Df}$ and $\mathbf{F}_e$ are the thermophoretic, diffusiophoretic and electric forces acting on the aerosol particle, respectively. The particle's trajectory was found by numerically integrating Eq. (1) in the reference frame of the drop. For this task a stable, fourth-order predictor-corrector technique was used [for details see Beard and Grover (1974)]. From a knowledge of the particle trajectory around the drop, the collision efficiency

$$E=\frac{\pi {y_c}^2}{\pi(a+r)^2} \quad (2)$$

was deduced, where y_c is the largest initial horizontal offset the particle can have and still collide with the drop (y_c being measured from the drop's axis aligned along $\mathbf{g}$, and sufficiently far upstream from the drop). If necessary, the collision kernel K can easily be computed from Eq. (2) by considering that the terminal fall velocity $V_{\infty,r}$ and the radius r of the aerosol particle can be neglected as compared to the terminal fall velocity $V_{\infty,a}$ and radius a of the collector drop. For this condition

$$K=E\pi a^2 V_{\infty,a}. \quad (3)$$

Literature evidence further shows that we may assume

that all particles which collide with a water drop are retained by it. The collision efficiency E is then identical with the collection efficiency E_c and the collision kernel K identical with the collection kernel K_c.

It is important to note at this point that E and K obtained in the manner outlined above are the result of the simultaneous action of all the forces acting on an aerosol particle and are not the result of a summation of the collision kernel for each individual scavenging effect.

The slip-corrected hydrodynamic drag force acting on a particle was computed using the superposition method [for a justification of the usefulness of this method see Schlamp *et al.* (1975)]. For determining the local hydrodynamic velocity $\mathbf{u}$ necessary to determine the drag, we used the flow fields around spherical water drops generated by LeClair *et al.* (1972) who computed the flow fields inside and outside a circulating water drop in air by solving the complete Navier-Stokes equation of motion for steady, incompressible flow past a spherical drop.

The thermophoretic force $\mathbf{F}_{Th}$ exerted on a particle in air was computed from an expression due to Brock cited by Waldmann and Schmidt (1966) which, adapted to the present situation, is given by

$$\mathbf{F}_{Th}=-\frac{12\pi r\eta(k+c_t k_P \mathrm{N}_{\mathrm{Kn}})}{5(1+3c_m \mathrm{N}_{\mathrm{Kn}})(k_P+2k+2c_t k_P \mathrm{N}_{\mathrm{Kn}})p}k\nabla T, \quad (4)$$

where p is the air pressure, k and k_P are the thermal conductivity of air and the aerosol particle, respectively, ∇T is the local temperature gradient around the drop at a large distance from the aerosol particle, and the phenomenological coefficients c_t and c_m have the values 2.5 and 1.0, respectively. Apparently, Eq. (4) is applicable with fair accuracy over the entire range $0\leqslant \mathrm{N}_{\mathrm{Kn}}\leqslant \infty$ if k_P is not too large (Waldmann and Schmitt, 1966).

The diffusiophoretic force $\mathbf{F}_{Df}$ exerted on a particle in air through which water vapor is diffusing was computed from an expression cited by Hidy and Brock (1970) which, adapted to the present situation, is

$$\mathbf{F}_{Df}=-6\pi r\eta(1+\sigma_{va}x_a)\frac{D_v}{x_a}\nabla x_v, \quad (5)$$

where x_a and x_v are the mole fractions for air and water vapor, respectively, and D_v is the diffusivity of water vapor in air. Apparently, Eq. (5) is applicable with fair accuracy over the range $0\leqslant \mathrm{N}_{\mathrm{Kn}}\leqslant 0.25$ (or with $\lambda=7.24\times10^{-8}$ m, over the range $0.3\ \mu\mathrm{m}\lesssim r\leqslant\infty$) if for σ_{va} the empirical value -0.26 is inserted so that $(1+\sigma_{va})=0.74$ for $x_v\ll x_a$, i.e., $x_a\approx 1$ (Schmitt, 1961). Considering water vapor and air as ideal gases, $x_v=e/p$ and $x_a=p_a/p$ are the partial pressures of water vapor and dry air. With this result and the ideal gas law, Eq. (5) then becomes

$$\mathbf{F}_{Df}=-6\pi r\eta(1+\sigma_{va}x_a)\frac{D_v M_a}{M_w\rho_a}\nabla\rho_v, \quad (6)$$

where ∇p_v is the local gradient of the water vapor density ρ_v around the water drop of radius a at a large distance from the aerosol particle, and M_a and M_w are the molecular weights of air and water, respectively. Although Eq. (4) was derived for the case that no vapor density gradient exists while Eq. (5) was derived for the case that no temperature gradient is present, we applied those equations to the present case which involves the simultaneous presence of temperature and vapor density gradients. In so doing, we have implicitly assumed that terms which couple Eqs. (4) and (5) have negligible magnitude. Fortunately, as mentioned earlier, Annis and Mason (1975) and Goldschmitt and May (1966) showed that for a system like ours for which the mole fraction x_v of the diffusing vapor is much less than the mole fraction x_a of the gas through which it diffuses (i.e., $x_v\ll x_a$) the diffusiophoretic and thermophoretic forces are additive, i.e., cross-correlation terms can be neglected. Obviously, for computing the diffusiophoretic and thermophoretic forces on an aerosol particle located anywhere in the neighborhood of an evaporating or condensing drop, we must determine the vapor density distribution and the temperature distribution around the drop. In the present model the vapor density and temperature distribution around an evaporating or condensing ventilated water drop falling in air at terminal velocity were calculated by numerically solving the convective vapor diffusion and heat conduction equations around drops of specified Reynolds number. These equations have the dimensionless form

$$\tfrac{1}{2}\mathrm{N}_{\mathrm{Pe}}\mathbf{u}^*\cdot\nabla^*\beta=\nabla^{*2}\beta, \quad (7)$$

where N_{Pe}, the Peclet number, is either the product of the Reynolds number and the Schmidt number for water vapor, or the product of the Reynolds number and the Prandtl number for temperature, β is either the dimensionless vapor density $(\rho_v-\rho_{v,\infty})/(\rho_{v,a}-\rho_{v,\infty})$ or the dimensionless temperature $(T-T_\infty)/(T_a-T_\infty)$, and $\mathbf{u}^*$, the dimensionless air velocity, is given by the flow fields generated by LeClair *et al.* (1972). Eq. (7) was solved by the method developed by Woo and Hamielec (1971) who numerically determined the vapor density distribution around, and the evaporation rate of, ventilated water drops. In order to determine ρ_v and T from β, we specified T_∞ and the relative humidity $\mathrm{RH}=\rho_{v,\infty}/\rho_{v,\mathrm{sat}}(T_\infty)$ which in turn specifies $\rho_{v,\infty}$. The surface temperature T_a and surface vapor density $\rho_{v,a}$ of the drop were computed using the method outlined by Beard and Pruppacher (1971). We then assumed that this surface temperature and vapor density remained constant throughout the course of a particle's trajectory. This approximation was justified by showing

that the drop decreases negligibly in size during the time it takes for the aerosol particle to complete its trajectory. Thus, we found that for the most sensitive case studied, namely for RH=50%, $N_{Re}=2aV_{\infty,a}\eta/\rho=1$, the drop only lost ~0.4% of its mass by evaporation during the course of a particle trajectory. For larger drop Reynolds numbers and/or larger ambient relative humidities the relative mass loss is even less.

The electrical force $\mathbf{F}_e$ exerted on an aerosol particle due to electric charges Q_a on the drop, electric charges Q_r on an aerosol particle, and/or due to the presence of an external electric field $\mathbf{E}_0$, was computed from the force expressions given by Davis (1964a,b) for two conducting spheres:

$$\mathbf{F}_e=\{\epsilon r^2E_0^2(F_1\cos^2\psi+F_2\sin^2\psi)+E_0\cos\psi(F_3Q_a+F_4Q_r)+[(F_5Q_a^2+F_6Q_aQ_r+F_7Q_r^2)/(\epsilon r^2)]+E_0Q_r\cos\psi\}\hat{z}+\{\epsilon r^2E_0^2F_8\sin 2\psi+E_0\sin\psi(F_9Q_a+F_{10}Q_r)+E_0Q_r\sin\psi\}\hat{x}, \quad (8)$$

where it was assumed (see Fig. 1 of Davis, 1964b) that the z axis points from the center of the aerosol particle to the center of the drop, the x axis is perpendicular to the z axis, $\hat{\mathbf{z}}$ and $\hat{\mathbf{x}}$ are the unit vectors in z and x directions, respectively, that $\mathbf{E}_0$ makes an angle ψ with the z axis, with $E_0=|\mathbf{E}_0|$, and that ϵ is the dielectric constant of the medium ($\epsilon_{air}\approx 1$). The force coefficients F_1–F_{10} are complicated functions of r/a and of the separation distance between the drop and particle, and are given by Davis (1964a,b). Davis (1969), Hall and Beard (1975) and Grover (1976a) have shown that the deviation between the electric force on a water drop of $\epsilon_{water}\approx 80$ and that on a conducting sphere ($\epsilon=\infty$) is negligible. On the other hand, the force on a dielectric sphere of $2\leqslant\epsilon\leqslant 10$, which is typical for the materials of which the atmospheric aerosol particles are composed, was found to deviate significantly from that on a conductor. However, a detailed analysis shows that, for all collector drops considered, the trajectory of an aerosol particle of $\epsilon_P\gtrsim 2$ is negligibly different from that of an electrically conducting aerosol particle as long as the drop and aerosol particles are electrically charged but no external electric field is present. On the other hand, if an external electric field is present the assumption that $\epsilon_P=\infty$ leads to a serious overestimation of the collision efficiency except if we assume that the ambient relative humidity is sufficiently high so that the aerosol particles are not dry but have an adsorbed water layer.

In the present computations we assumed 1) that the aerosol particles are composed of a material which is electrically conducting and has a bulk density of 2×10^3 kg m^{-3} and a thermal conductivity of 4.19×10^{-1} J m^{-1} s^{-1} °C^{-1}, 2) that the ambient air has a dynamic viscosity of 1.77×10^{-5} kg m^{-1} s^{-1}, a density of 1.10 kg m^{-3}, a temperature of 10°C, a pressure of 900 mb and a mean free path length of 7.24×10^{-8} m, and 3) that the diffusivity of water vapor, given by the relationship recommended by Hall and Pruppacher (1975), is 0.255×10^{-4} m^2 s^{-1}. The collector drops were assumed to have sizes $a=42$ μm ($N_{Re}=1$), $a=72$ μm ($N_{Re}=4$), $a=106$ μm ($N_{Re}=10$), $a=173$ μm ($N_{Re}=30$), $a=309$ μm ($N_{Re}=100$), and $a=438$ μm ($N_{Re}=200$); and the aerosol particle sizes were assumed to range between $0.5\leqslant r\leqslant 10$ μm. The drops and aerosol particles were assumed to carry an electric charge of magnitude $Q_a=0.2a^2$ and $Q_r=0.2r^2$, respectively, or of a magnitude $Q_a=2.0a^2$ and $Q_r=2.0r^2$, respectively, with a and r expressed in centimeters, and Q_a and Q_r in esu. The later is equal to the mean thunderstorm charge on drops found by Takahashi (1973) from his own observations and those of others. In some of the earlier computations the calculations were made for the slightly different conditions of 1013 mb and 20°C, and therefore for slightly different drop sizes which correspond to the Reynolds numbers specified above. The external electric field was assumed to have a strength E_0 which varied between zero and 3×10^5 V m^{-1}. Since in the present computations the electric field was assumed nonzero only when the drop and the particle were uncharged, the present results are equally valid for a vertical field pointing upward or downward. For the same reason, the present results are also equally valid for positively or negatively charged drops, as long as the particle's charge has the opposite sign.

3. Results and discussion of present and previous calculations involving model 1

The results of the present and previous calculations involving model 1 have been derived from three successive endeavors. In the first Eq. (1) was solved for the special case of inertial impaction only, i.e., for $\mathbf{F}_{Th}=0$, $\mathbf{F}_{Df}=0$ and $\mathbf{F}_e=0$. The results of this study [discussed in detail by Beard and Grover (1974) and by Beard (1974)] show that for a given particle size the collision efficiency E increases with increasing collector drop size, as expected from computations of other investigators for drops colliding with other drops. Unexpectedly, however, it was found that for a given drop size E exhibits a minimum at a certain particle size near 0.3 to 0.5 μm if $N_{Re}(\text{drop})>20$.[1] This implies that below this critical particle size E increases although the particle size decreases. Trajectory analysis showed that this behavior is a result of rear capture of aerosol particles by the standing eddy at the downstream side of a drop. Small drops which have no eddy, or for which the eddy is still very small, have no such minimum in E. The significance of this finding lies in the fact that inertial impaction reinforces the gap in E found by Greenfield (1957) and later by others. This gap occurs in the particle size range $0.1\leqslant r\leqslant 1$ μm due to a trade-

[1] Later, more refined calculations of Grover (1977) show that $N_{Re}(\text{drop})>30$ for circulating spherical water drops in air.

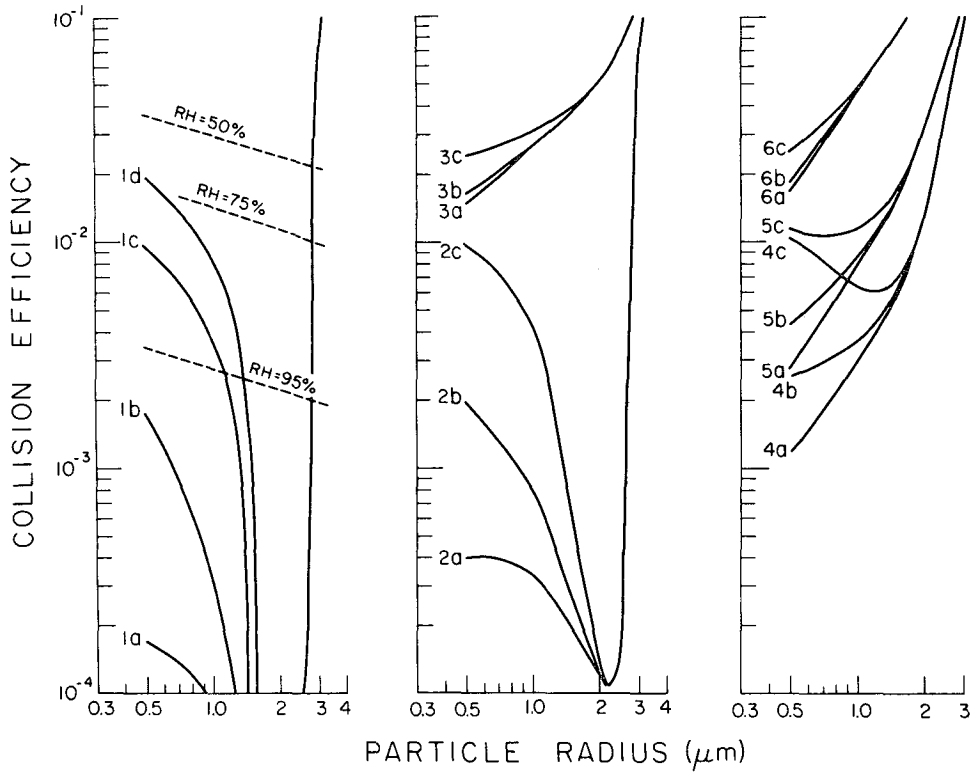

FIG. 1. Effect of electric field and electric charge on the efficiency with which a water drop of 42 μm radius at terminal velocity collides with aerosol particles of bulk density 2×10^3 kg m^{-3} in air of 10°C and 900 mb. Curves 1a, b, c, d: $q_a=0$, $q_r=0$, $E_0=0$ and RH=100%, 95%, 75%, 50%, respectively; curves 2a, b, c: $q_a=\mp0.2$ esu cm^{-2}, $q_r=\pm0.2$ esu cm^{-2}, $E_0=0$ and RH=100%, 95%, 75%, respectively: curves 3a, b, c: $q_a=\mp2.0$ esu cm^{-2}, $q_r=\pm2.0$ esu cm^{-2}, $E_0=0$ and RH=100%, 95%, 75%, respectively; curves 4a, b, c: $q_a=0$, $q_r=0$, $E_0=5\times10^4$ V m^{-1} and RH=100%, 95%, 75%, respectively; curves 5a, b, c: $q_a=0$, $q_r=0$, $E_0=1\times10^5$ V m^{-1} and RH=100%, 95%, 75%, respectively; curves 6a, b, c: $q_a=0$, $q_r=0$, $E_0=3.084\times10^5$ V m^{-1} and RH=100%, 95%, 75%, respectively. Here $q_a=Q_a/a^2$ and $q_r=Q_r/r^2$; dashed curves are due to phoretic effects only.

off between the effects of Brownian motion and inertial impaction.

In a second endeavor, Eq. (1) was solved for inertial impaction superimposed by electrical effects due to electrical charges on the drops and aerosol particles (Grover and Beard, 1975), and due to electrical charges on the drops and aerosol particles and/or an external electric field (Grover, 1976b). The results of this endeavor show, for the drop sizes considered, that an external field and/or electric charges of opposite sign raise the collision efficiency by up to two orders of magnitude. For a given surface charge density on drop and particle and for a given aerosol particle size this increase is the more pronounced the smaller the collector drop. For a given collector drop size and for a given surface charge density on drop and particle, the increase is the more pronounced the smaller the aerosol particle. For a given drop and particle size the increase is the more pronounced the larger the surface charge density on the drop and particle. An analogous behavior pertains to the effect of an external electric field. Both electric fields and charges of magnitude typical in atmospheric clouds were found to have a negligible effect on E for $r\gtrsim0.5$ μm if $a>700$ μm. An additional result of importance is the finding that rear capture of sufficiently small aerosol particles is significantly enhanced by electric forces which in tandem with the gravitational force may overcome the hydrodynamic force in the rear of the drop to pull the particle down to the drop surface. Trajectory analysis, however, shows that for sufficiently large electric fields and charges front-capture predominates, thus eliminating the minimum in E versus r for a given a. Electric effects therefore tend to eliminate the reinforcement of the "Greenfield gap" provided by rear capture.

In our third endeavor (present results), Eq. (1) was solved for inertial impaction in the presence of phoretic effects, and in the presence of electrical effects. Only evaporating drops were considered. The results of these computations are summarized in Figs. 1–5. Note from curves 1a–1d of Figs. 1–5 (inertial impaction and phoretic effects only) that phoretic effects significantly increase the efficiency with which aerosol particles are collected by drops, this increase being the greater the lower the relative humidity of the environmental air. Thus, for the collector drop sizes considered, the effi-

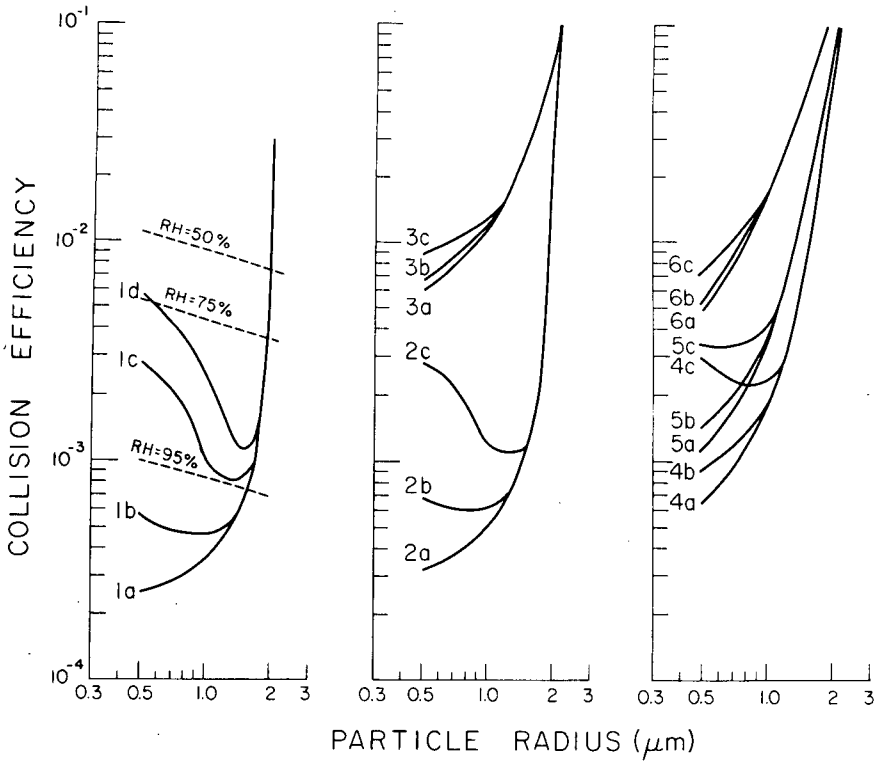

FIG. 2. As in Fig. 1 except for a water drop of 72 μm radius, and for 6a, b, c: $q_a=0$, $q_r=0$, $E_0=2.885\times10^5$ V m^{-1} and RH=100%, 95%, 75%, respectively.

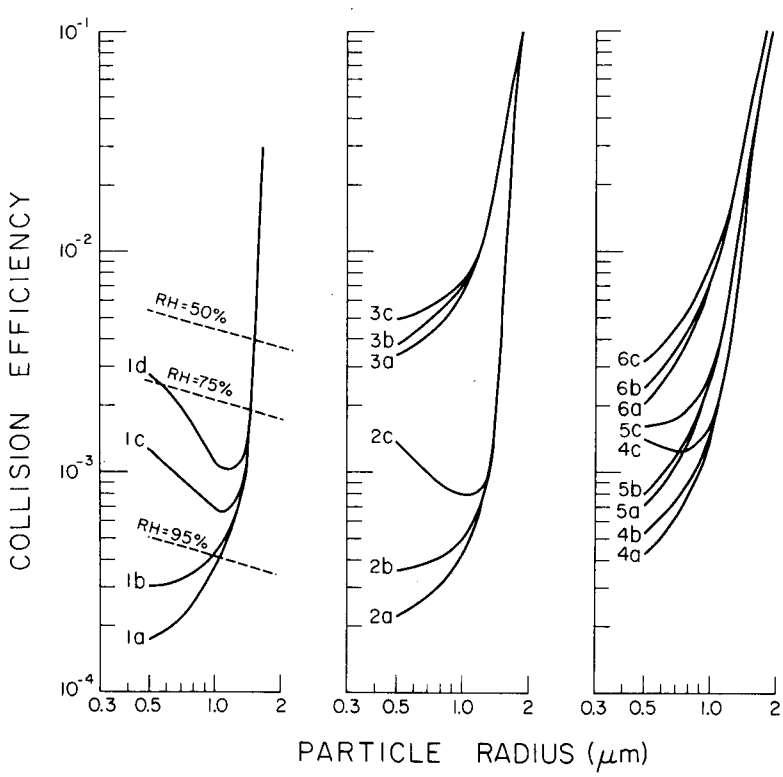

FIG. 3. As in Fig. 1 except for a water drop of 106 μm radius, and for 6a, b, c: $q_a=0$, $q_r=0$, $E_0=2.69\times10^5$ V m^{-1} and RH=100%, 95%, 75%, respectively.

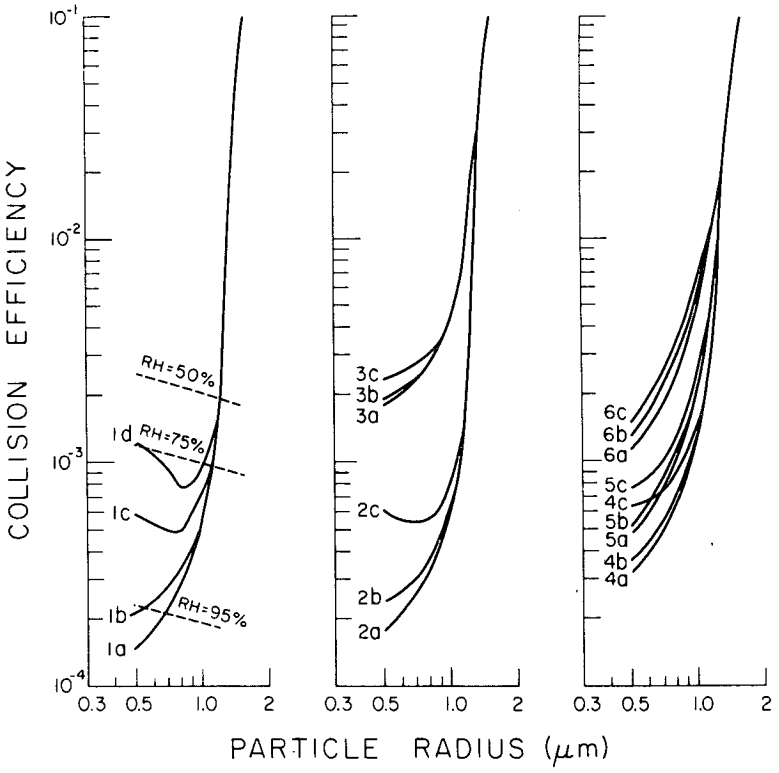

FIG. 4. As in Fig. 1 except for a water drop of 173 μm radius, and for 6a, b, c: $q_a=0$, $q_r=0$, $E_0=3\times10^5$ V m^{-1} and RH=100%, 95%, 75%, respectively.

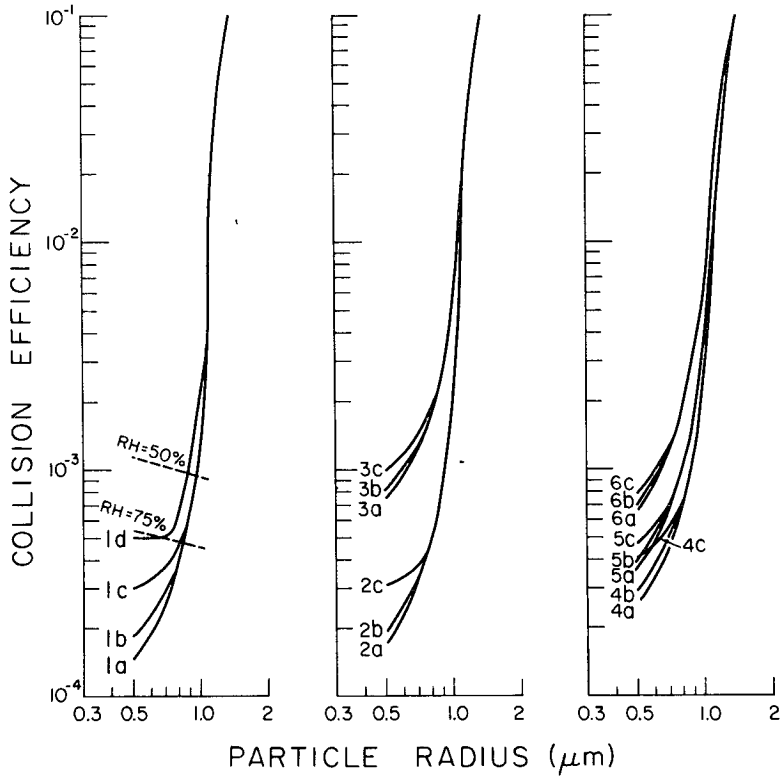

FIG. 5. As in Fig. 1 except for a water drop of 309 μm radius, and for 6a, b, c: $q_a=0$, $q_r=0$, $E_0=3\times10^5$ V m^{-1} and RH=100%, 95%, 75%, respectively.

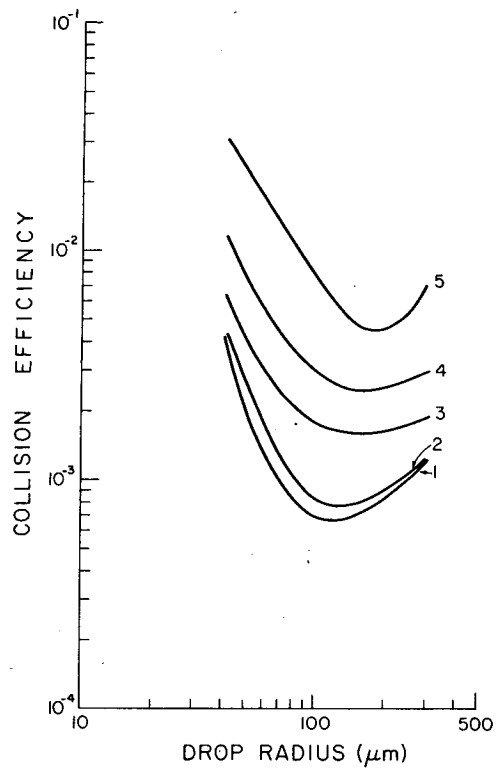

FIG. 6. Effect of electric field and electric charge on the efficiency with which an aerosol particle of 1 μm radius and bulk density 2×10^3 kg m^{-3} collides with various size water drops at terminal velocity in air of 10°C, 900 mb and RH=75%. Curve 1: $q_a=0$, $q_r=0$, $E_0=0$; curve 2: $q_a=\mp0.2$ esu cm^{-2}, $q_r=\pm0.2$ esu cm^{-2}, $E_0=0$; curve 3: $q_a=0$, $q_r=0$, $E_0=5\times10^4$ V m^{-1}; curve 4: $q_a=0$, $q_r=0$, $E_0=1\times10^5$ V m^{-1}; curve 5: $q_a=\mp2.0$ esu cm^{-2}, $q_r=\pm2.0$ esu cm^{-2}, $E_0=0$.

ciency is raised by up to two orders of magnitude over and above the value resulting from inertial impaction if RH=50%. Additionally, we note that phoretic and inertial effects combined tend to produce a minimum in E versus r due to rear capture of sufficiently small particles. In contrast to the minimum in E versus r due to hydrodynamic effects only, where the minimum is present for large collector drops which have a well developed eddy at their downstream side, the minimum in E versus r due to the combined effects of phoretic effects and hydrodynamic effects is the more pronounced the smaller the collector drop, i.e., the weaker the flow field around the drop and therefore the larger the relative effect of the phoretic forces. Also, this minimum appears to be near 1–2 μm if the drops are relatively small and the phoretic effect relatively large. As the phoretic effects decrease, this minimum shifts toward smaller particle sizes. It appears, therefore, that rear collisions due to phoretic effects tend to reinforce but also narrow the "Greenfield gap" by raising the collision efficiency for the smaller particles, an effect which is the more pronounced the smaller the collector drop.

The dashed lines in Figs. 1–5 are the values of E obtained by using the method of Young (1974), in which the effect of the particle's inertia is ignored, and in which the effect of the hydrodynamic flow around the collector drop is approximated by ventilation factors. A comparison between the dashed lines and curves 1a–1d of Figs. 1–5 shows that computations of the type carried out by Young (1974) cannot be considered sufficiently accurate for particles of $r\gtrsim0.5$ μm, so that the more detailed and accurate method outlined in the present work must be applied for such particles.

Fig. 6 displays the collision efficiency as a function of collector drop size for a 1 μm radius particle in an environment of 75% relative humidity for various electric charges and fields. The presence of a minimum in each curve is explained by the interplay of the phoretic, electric and hydrodynamic effects on the particle's trajectory. For the smaller collector drops, the collision efficiency decreases with increasing drop size because the hydrodynamic effect, which tends to counteract the attractive phoretic and electric forces by sweeping the particle around the drop, is the stronger the larger the drop. For the larger collector drops, the collision efficiency increases for increasing drop size because of the presence of the circulating eddy in the rear of the drop which works in coordination with the phoretic and electric forces to promote rear capture. The larger the drop the larger is the associated eddy, and thus the greater is the rear capture.

Returning to Figs. 1–5, it is seen that electrical effects tend to fill the gap in E versus r created by inertial and phoretic effects, and tend to raise the collision efficiency more the smaller the collector drop for a given particle size, and the larger the charge on the drop and aerosol particle and/or the larger the external electric field for a given drop and particle size. Whether or not the "Greenfield gap" by itself becomes reduced by electrical effects cannot be determined by computations with model 1. However, this question will be discussed in conjunction with our results from model 2 (to be published).

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