Maintenance of the Momentum Flux by Transient Eddies in the Upper Troposphere[1]

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ABSTRACT

The observed wintertime distribution of the poleward flux of westerly momentum by transient eddies at the jet stream (250 mb) level is characterized by 1) a strong convergence of momentum flux into "storm tracks" over the western and central oceans near 40°N and 2) strong poleward fluxes over the western parts of the continents. The former fluxes are strongly countergradient. This observed distribution is discussed in light of the flux maintenance equation, which is analogous to the equations for the local time rate of change of eddy kinetic energy. The terms of major interest in this equation are a so-called "mixing term," which is always acting to produce a down-gradient flux, and a pair of terms which involve temporal correlations between the eddy velocity components and their respective ageostrophic departures. It is shown that the observed countergradient fluxes over the oceans must be maintained by these ageostrophic correlations terms.

The geographical distributions of the terms described above are estimated on the basis of ten winters' hemispheric synoptic charts. The ageostrophic correlation terms tend to produce eddy fluxes of westerly momentum into the storm tracks, as observed. It is proposed that the convergence of westerly momentum into the storm tracks is a consequence of the fact that there is a strong tendency for air with high westerly momentum to be accelerated in the direction of the center of the storm track by the imbalance between pressure gradient and Coriolis forces.

1. Introduction

The poleward flux of zonal momentum by large-scale eddies plays a crucial role in the maintenance of the zonally averaged distribution of zonal winds in the earth's atmosphere. More than half a century ago Jeffreys (1926) recognized the need for such a flux in order to explain the observed distribution of surface winds in which easterlies predominate the tropics and westerlies poleward of 30° latitude. Jeffreys' ideas were not fully accepted until the early 1950's when the sign and order of magnitude of the eddy flux were established on the basis of direct observational evidence. Lorenz (1967) gives a detailed historical account of these pioneering studies which he views as marking the beginning of the modern era in the study of the general circulation.

A characteristic feature of the observed distribution of the meridional eddy flux of zonal momentum, first noted by Kuo (1951) and later elaborated on by Starr (1968) and others, is the tendency for it to be directed against the gradient of zonally averaged angular velocity. Such a countergradient flux is indicative of a transfer of kinetic energy from the large-scale eddies into the zonally averaged flow. In the theory of two-dimensional turbulence, this energy conversion represents one aspect of what is called a reverse energy cascade (Kraichnan, 1967; Tennekes, 1977).

An unfortunate consequence of the countergradient nature of the eddy flux of zonal momentum in the earth's atmosphere is the fact that the convenient and familiar "diffusion" or "mixing" hypothesis, so often used with moderate success in turbulence theory, cannot be employed to describe or parameterize the flux. Such formulation either yields down-gradient fluxes or, if one insists upon countergradient fluxes, negative values of the exchange coefficients are involved. The first option does not fit the data and the second leads to a serious thermodynamic dilemma. The failure of such simple schemes for treating the eddy fluxes of zonal momentum has been one of the major stumbling blocks in the development of zonally averaged general circulation models.

Over the past few years there have been a number of studies directed toward an understanding of the distribution of eddy transports of zonal momentum in the earth's atmosphere at a more fundamental dynamical level. Green (1970) showed that the convergence of

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eddy flux of westerly momentum is related to the eddy fluxes of heat and potential vorticity. Held (1975) investigated momentum fluxes in quasi-geostrophic flow on a beta-plane, subject to certain specified boundary conditions. He showed that under a wide range of conditions, eddies tend to transport westerly momentum into latitude belts where they are most strongly forced, either by baroclinic instability or by some external "stirring." Held's theoretical results are consistent with recent modeling studies by Simmons and Hoskins (1976, 1977) of baroclinic instability on a sphere. In these experiments amplifying baroclinic waves produce a convergence of westerly momentum into the latitude belt of strongest wave activity which turns out to be nearly coincident with the westerly "jets" in the assumed angular velocity profiles. Hence the simulated eddy fluxes of zonal momentum are primarily countergradient, just as in the real atmosphere.

In this paper, we will attempt to identify the specific dynamical processes which lead to the production and maintenance of countergradient fluxes of zonal momentum. For this purpose we will derive a budget equation for the local time rate of change of eddy flux of zonal momentum, starting with the primitive equations and proceeding in a manner analogous to deriving a budget equation for the time rate of change of eddy kinetic energy. We will then show that it is possible to evaluate the leading terms in this so-called *flux maintenance equation* on the basis of a 10-year set of hemispheric grid-point data produced in conjunction with the operational analyses from the National Meteorological Center. Finally, we will attempt to interpret the major "source" term in the flux maintenance equation in terms of a simple synoptic model.

Flux maintenance equations are becoming commonplace in the study of atmospheric turbulence. Observational evidence on the maintenance of fluxes in the atmospheric surface layer (Wyngaard *et al.*, 1971) has provided the insight necessary to understand the dependence of the mean temperature and velocity fields on the Richardson number; theoretical models based on this type of evidence can now explain the dynamics of inversion rise, the countergradient heat flux in the upper half of convective boundary layers, and a number of other phenomena (Zeman and Lumley, 1976).

To our knowledge, the only application of a flux maintenance equation to the poleward transport of westerly momentum in the atmospheric general circulation was made by Kao *et al.* (1970). Their study made use of the formalism developed by Kao (1968), in which the various terms in the flux-maintenance equation are evaluated separately for each component of the momentum flux in the wavenumber-frequency domain. Their results varied widely for different parts of the wavenumber-frequency domain and therefore do not readily lend themselves to physical interpretation in terms of the elementary physical processes represented in the flux maintenance equation.

This paper may be viewed as the third in a series of observational studies devoted to a three-dimensional description and interpretation of the Northern Hemisphere winter circulation. The first of this series by Blackmon (1976) dealt with the spatial and temporal decomposition of the variance 500 mb height field. Blackmon suggested that elongated maxima in the distribution of bandpass-filtered[3] variance can be identified with the major storm tracks. A second paper by Blackmon *et al.* (1977) was devoted to a description of the relationships between the storm tracks (as defined in terms of the band-pass filtered variance fields), the time-mean jet streams, and the distributions of the eddy fluxes of sensible heat and zonal momentum at selected levels. Topics to be dealt with in future papers in this series include a more comprehensive study of the exchange of kinetic energy between the eddies and the mean flow, a detailed study of the three-dimensional distribution of the eddy heat flux and its maintenance, and consideration of the local, time-averaged budgets of vorticity and potential vorticity.

In a recent paper in this journal, one of us (Tennekes, 1977) proposed a methodology for developing a time-averaged climate model based on a system of flux maintenance equations. He suggested that a climate model which retains more explicit dynamics, with parameterization efforts concentrated on the terms in the flux-maintenance equations, may be expected to perform better than models in which the fluxes are crudely parameterized. If experience with boundary-layer turbulence is any indication, however, we must expect that an improved understanding of the general circulation requires that the *entire* system of flux maintenance equations be studied, together with all the attendant feedback mechanisms (Tennekes, 1977).

2. The momentum flux maintenance equation

An equation for the time rate of change of the transient eddy flux of zonal momentum at any fixed geographical location can be readily formed in the following manner:

1) The equations for the local time rate of change of the zonal wind component u and the meridional wind component v are decomposed into time mean and fluctuating parts, where the time averaging operator $\overline{(\ )}$ is taken to be long enough to obtain a representative sampling of mean and covariance quantities. The prime operator $(\ )'$ refers to deviations from the corresponding time-averaged quantities.

2) The equation for du'/dt is multiplied by v' and the equation for dv'/dt is multiplied by u' and the two

[3] That is, comprising fluctuations with periods between about 3 and 7 days.

are added together and time-averaged. In pressure coordinates, neglecting the curvature terms due to the sphericity of the earth, the result is

$$\frac{\partial}{\partial t}\overline{u'v'}=-\bar{u}\frac{\partial}{\partial x}\overline{u'v'}-\bar{v}\frac{\partial}{\partial y}\overline{u'v'}-\bar{\omega}\frac{\partial}{\partial p}\overline{u'v'}-\frac{\partial}{\partial x}\overline{u'u'v'}$$
$$-\frac{\partial}{\partial y}\overline{v'u'v'}-\frac{\partial}{\partial p}\overline{\omega'u'v'}-\overline{u'v'}\frac{\partial\bar{u}}{\partial x}-\overline{v'v'}\frac{\partial\bar{u}}{\partial y}$$
$$-\overline{v'\omega'}\frac{\partial\bar{u}}{\partial p}-\overline{u'u'}\frac{\partial\bar{v}}{\partial x}-\overline{u'v'}\frac{\partial\bar{v}}{\partial y}-\overline{u'\omega'}\frac{\partial\bar{v}}{\partial p}$$
$$+f(\overline{v'v_a'}-\overline{u'u_a'})+\overline{v'F_x'}+\overline{u'F_y'}. \quad (1)$$

Here the subscript a refers to ageostrophic wind components, F_x and F_y refer to components of forces associated with subgrid-scale exchanges of momentum, and all the other symbols are used in their conventional meteorological context.

The first group of terms in (1) represents the advection of $\overline{u'v'}$ by the mean flow (including the advection by the mean vertical motion). Since these terms are proportional to the spatial gradients of $\overline{u'v'}$, they must vanish wherever $\overline{u'v'}$ exhibits a local maximum or minimum. This group of terms does not participate in the maintenance or destruction of features in the $\overline{u'v'}$ field; it only carries them from one location to another with a characteristic time scale on the order of a day or two, which, as will be shown presently, is about an order of magnitude longer than that of the leading terms in (1).

The second triad of terms represents the divergence of the flux of $\overline{u'v'}$ by transient eddy motions. These terms may act as a source of nonzero $\overline{u'v'}$ if the eddy flux of $\overline{u'v'}$ is countergradient or, conversely, they may act as a sink if the flux is directed down-gradient. We have evaluated these terms on the basis of observed data and found them to be about an order of magnitude smaller than the leading terms in the equation. The associated geographical patterns are extremely "noisy" and of dubious statistical significance.

The next six terms are similar in form to those that represent the gain or loss of kinetic energy to the mean flow in the maintenance equation for eddy kinetic energy. In the Appendix it is shown that the principal term in this group of six is $-\overline{v'v'}\partial\bar{u}/\partial y$; the other five terms are considerably smaller except at locations where $\partial\bar{u}/\partial y$ happens to be zero. Tennekes (1977) interpreted $-\overline{v'v'}(\partial\bar{u}/\partial y)$ as a "mixing" term, which tends to give $\overline{u'v'}$ a sign opposite to that of $\partial\bar{u}/\partial y$. Because this term contains the variance of the meridional eddy velocity component, there is no doubt about this interpretation: the very *presence* of v' in a field with nonzero $\partial\bar{u}/\partial y$ tends to destroy any countergradient eddy momentum flux that might exist, or to increase any down-gradient momentum flux. If there were no other dynamical mechanisms in Eq. (1), $-\overline{v'v'}(\partial\bar{u}/\partial y)$ would see to it that the eddy flux of momentum flows down the mean momentum gradient most everywhere, within a matter of hours (as will be demonstrated in Section 5). In the general circulation, of course, the eddy momentum flux runs *against* the gradient in those parts of the atmosphere in which baroclinic activity is concentrated. Since the principal interaction term with the mean flow tends to destroy this essential feature of the general circulation, two questions arise immediately. The first is—which terms *maintain* the countergradient flux? No less important, we also have to ask—what are the physical and dynamical processes by which these terms do the job required of them, and can we understand the physics of flux maintenance in the context of our present understanding of the nature of the general circulation?

The fourth group of terms in Eq. (1) represents the maintenance or destruction of $\overline{u'v'}$ by the net difference between the effects of pressure-gradient and Coriolis forces. It is useful to see how these terms come about. We have

$$\frac{\partial u'}{\partial t}=-\frac{\partial\phi'}{\partial x}+fv'+\cdots=fv_a'+\cdots, \quad (2)$$

$$\frac{\partial v'}{\partial t}=-\frac{\partial\phi'}{\partial y}-fu'+\cdots=-fu_a'+\cdots, \quad (3)$$

where ϕ represents the geopotential. Multiplying (2) by v', (3) by u', adding the two equations and time-averaging, we obtain

$$\overline{v'\frac{\partial u'}{\partial t}}+\overline{u'\frac{\partial v'}{\partial t}}=\frac{\partial}{\partial t}\overline{u'v'}=f(\overline{v'v_a'}-\overline{u'u_a'})+\cdots. \quad (4)$$

Since $-\overline{v'v'}\partial\bar{u}/\partial y$ and the two friction terms $\overline{v'F_x'}$ and $\overline{u'F_y'}$ tend to reduce the countergradient momentum flux, and since all other terms that we have thus far considered are relatively small, we conclude that the primary mechanism for maintaining the flux must be related to the two terms on the right-hand side of (4). In other words, the physics of flux maintenance is related to the correlations between eddy velocities and their ageostrophic components. Similar conclusions were reached by Kao *et al.* (1970), and by Tennekes (1977) for his buoyancy-forced, two-dimensional model of the general circulation. However, in Tennekes' model, the unbalanced wind components are defined in a somewhat different way.

It is well established that the eddy kinetic energy of the general circulation is maintained by terms whose form is similar to those on the right-hand side of (4).

More specifically, from an inspection of (2) and (3) it is easily verified that

$$\overline{u'\frac{\partial u'}{\partial t}}+\overline{v'\frac{\partial v'}{\partial t}}=\frac{\partial}{\partial t}(\overline{u'^2}+\overline{v'^2})/2=f(\overline{u'v_a'}-\overline{v'u_a'})+\cdots. \quad (5)$$

Hence, a thorough study of the statistical properties of departures from the geostrophic flow is necessary, not only for the understanding of the physics of momentum flux maintenance, but also for the understanding of the kinetic energy balance.

3. The data set and analysis procedures

The basic data set used in the present study consists of twice-daily (0000 and 1200 GMT) analyses of the wind and geopotential height fields for the Northern Hemisphere, proposed by the U. S. National Meteorological Center (NMC) and archived at the National Center for Atmospheric Research. Our study covers the winters of 1965–66 through 1974–75, with the exception of the 1969–70 winter which had a two-month-long data gap in the wind fields. Our winter season is taken to be the 120-day period starting from November 15.

The NMC analyses are produced on a 1977 point octagonal grid which provides for approximately uniform grid point spacing covering the region poleward of 20°N. Throughout the first nine winters covered by our study Cressman's (1959) scheme was still being used as a basis for these analyses. The first guess for the wind analysis was the gradient wind field derived from objective analysis of the geopotential height data on pressure surfaces. The first guess at each grid point was then corrected, in accordance with Cressman's scheme, to make it compatible with wind reports at nearby stations. Thus in regions of sparse data coverage the analyzed winds are likely to resemble the gradient winds and the cross-isobar flow likely to be underestimated. During the 1974–75 winter, the analyses were produced by Flattery's global analysis scheme with 12 h operational forecasts being used as first guess fields. Inspection of the calculations for individual winters indicates no major difference between the results for the 1974–1975 winter and those for the other winters in this data set.

The calculations described below have been carried out for 10 different pressure levels, but for the sake of brevity we will show in this paper only results for the 250 mb level, which is representative of conditions in the vicinity of the tropopause where the momentum fluxes are strongest. The definition of the winter season and the bandpass-filtering procedure, to be referred to in the following section, are the same as those described in Blackmon (1976) and Blackmon *et al.* (1977).

The ageostrophic terms in (1) and (4) were computed at each of the points on the NMC octagonal grid by making use of the relations

$$\overline{u'u_a'}=\overline{u'^2}-\overline{u'u_g'},$$

$$\overline{v'v_a'}=\overline{v'^2}-\overline{v'v_g'},$$

where the u and v components are now temporarily redefined in relation to the local orientation of the array

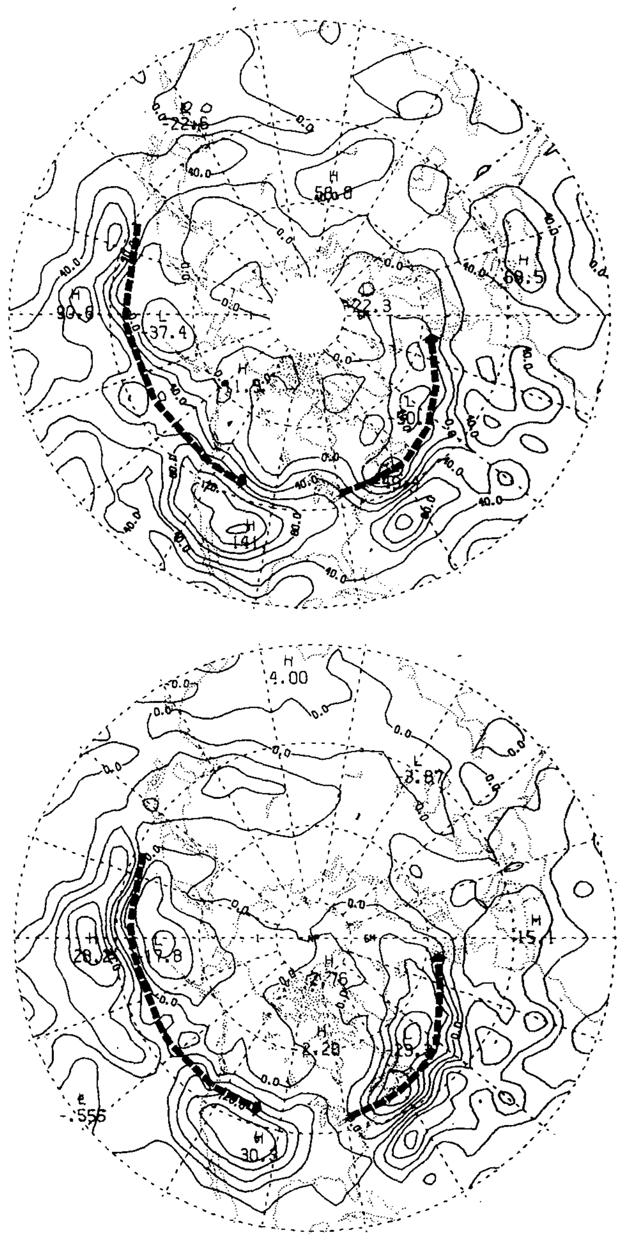

FIG. 1. The distribution of nine-winter average poleward momentum flux by transient eddies at the 250 mb level, computed from (a) unfiltered and (b) band-pass filtered wind data; contour intervals 20 and 5 m² s⁻², respectively. Arrows with dashed shafts denote positions of regions with maximum geopotential height variability in the band-pass period range.

of grid points and the subscript g refers to the geostrophic wind. The geostrophic components are estimated on the basis of a cubic fit to the geopotential height field at each grid point and the four surrounding points. Each field to be mapped was interpolated onto a $2\frac{1}{2}° \times 5°$ latitude-longitude grid.

4. The distribution of momentum flux

Fig. 1 shows the nine-winter average distribution of the poleward flux of zonal momentum (a) by all transient eddies and (b) by transient eddies with periods between about 3 and 7 days as determined by band-pass filtering of the time series of u and v at each grid point. The principal fluxes appear to be associated with two distinct types of features:

1) Dipole-like patterns over the western and central oceans with positive values (poleward fluxes) south of about 45° latitude and negative values (equatorward fluxes) to the north of that latitude. Blackmon *et al.* (1977) interpreted this pattern in terms of a convergence of the eddy flux of westerly momentum into the storm tracks [regions of large variability in ϕ and v in the 3–7 day (bandpass) period range].

2) Regions of strong poleward fluxes of westerly momentum over the western parts of the continents and particularly over the western United States. This feature did not show up as distinctly in the 500 mb data analyzed by Blackmon *et al.* (1977).

The features described above are most clearly seen in the bandpass filtered data (Fig. 1b), which are much more stable from a statistical point of view [see Blackmon *et al.*, Section 8, item (3)]. However, the similarity between filtered and unfiltered fields is worth noting. Although the magnitudes of the fluxes varied considerably from one individual winter to another the patterns were quite reproducible, particularly the bandpass-filtered one. Hence we are quite confident that the results displayed in Fig. 1 give at least a good qualitative description of the distribution of momentum flux over the Northern Hemisphere.

Fig. 2 shows the distribution of momentum flux by the geostrophic wind, together with the total contribution from the ageostrophic component of the wind, derived by taking the difference between the fields displayed in Figs. 1a and 2a. Note that since

$$\overline{u'v'}=\overline{u_g'v_g'}+\overline{u_g'v_a'}+\overline{u_a'v_g'}+\overline{u_a'v_a'} \qquad (6)$$

and $|u_g| \gg |u_a|$ most of the time, it is likely that the major contribution to Fig. 2b comes from the $\overline{u_g'v_a'}$ and $\overline{u_a'v_g'}$ terms. Over most of the hemisphere, $\overline{u'v'}$ and $\overline{u_g'v_g'}$ have the same sign and are of comparable magnitude. Over the oceanic storm tracks $\overline{u'v'}<\overline{u_g'v_g'}$, whereas over the western part of the continents the reverse is true.

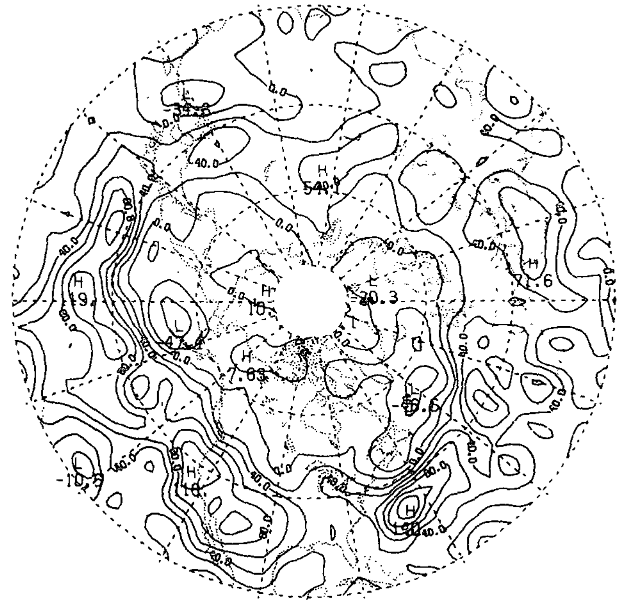

FIG. 2a. As in Fig. 1a except for transient eddy momentum flux by geostrophic winds; contour interval 20 m² s⁻².

The ageostrophic contribution to $\overline{u'v'}$ is particularly large over the western and central United States.

5. The major terms in the maintenance equation

The distribution of the "mixing term" $-\overline{v'^2}\partial\bar{u}/\partial y$ is shown in Fig. 3. As expected, this term exhibits a pronounced dipole-like pattern with respect to the

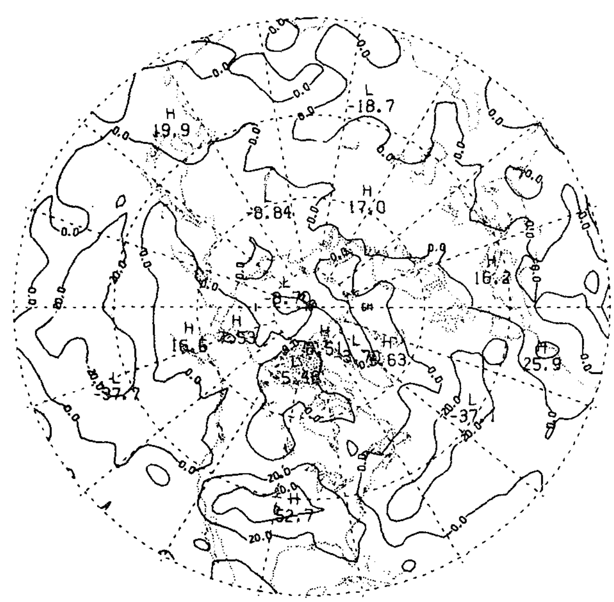

FIG. 2b. The contribution of ageostrophic effects to the total transient eddy momentum flux, $\overline{u'v'}-\overline{u_g'v_g'}$, averaged over nine winters at the 250 mb level; contour interval 20 m² s⁻².

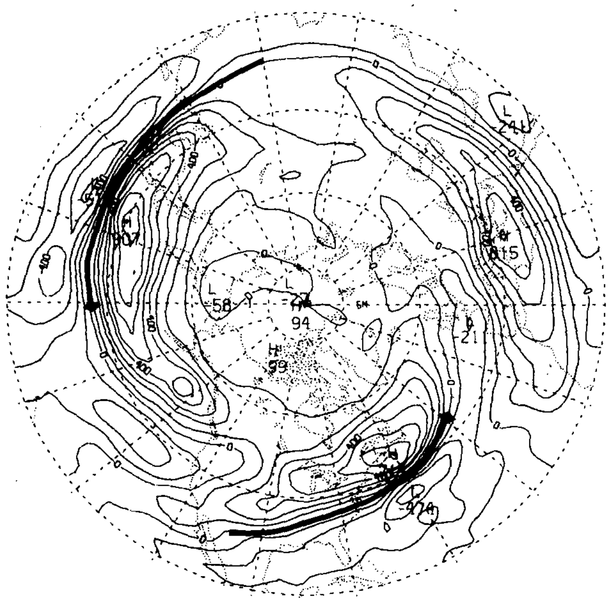

FIG. 3. The distribution of $-\overline{v'^2}(\partial\bar{u}/\partial y)$, averaged over nine winters at the 250 mb level; contour interval 10^{-5} m^2 s^{-3}. Heavy arrows denote positions of jet streams.

seasonal mean jet streams over the western oceans, with negative maxima to the south of the jets and positive maxima to the north of them. Note the negative correlation between this pattern and the actual distribution of momentum fluxes shown in Fig. 1, particularly in the vicinity of the storm tracks. In these

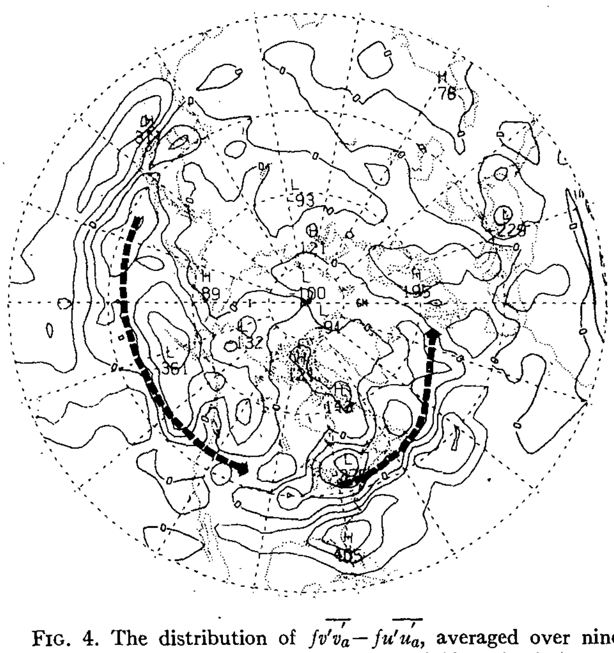

FIG. 4. The distribution of $\overline{fv'v_a'}-\overline{fu'u_a'}$, averaged over nine winters at the 250 mb level; contour interval 10^{-5} m^2 s^{-3}. Arrows with dashed shafts denote position of regions with maximum geopotential height variability in the bandpass-period range.

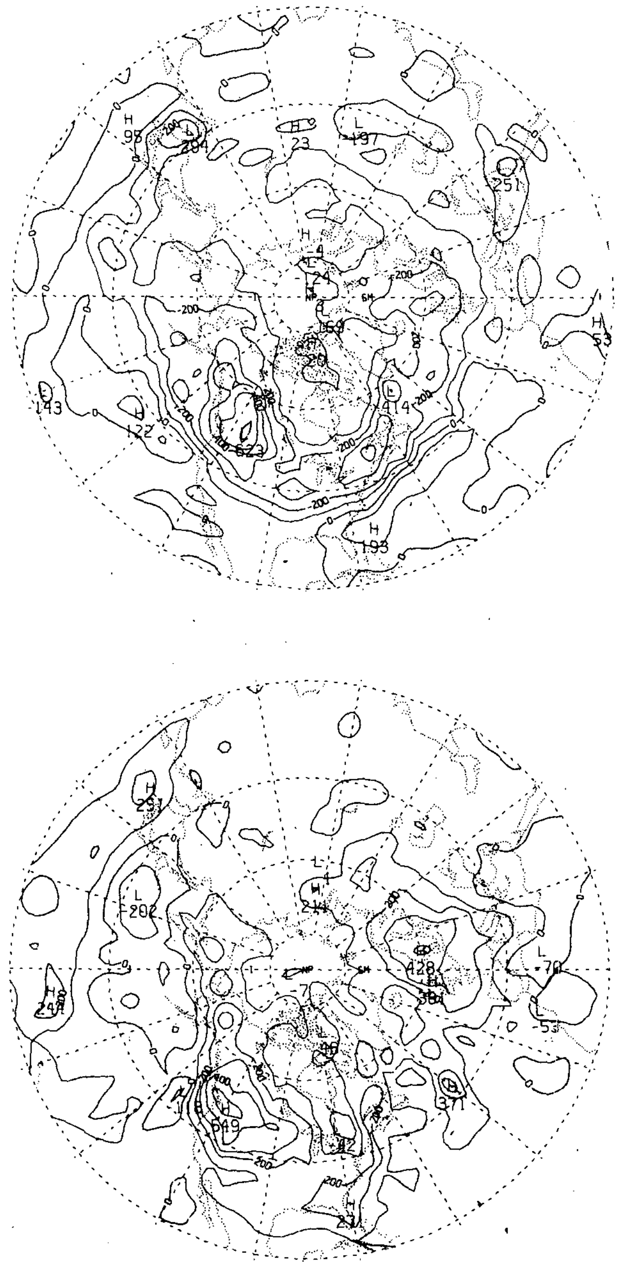

FIG. 5. The distribution of (a) $-\overline{fu'u_a'}$ and (b) $\overline{fv'v_a'}$, averaged over nine winters at the 250 mb level; contour interval 10^{-5} m^2 s^{-3}.

regions the mixing term acts to destroy the existing distribution of $\overline{u'v'}$ which is primarily countergradient. The time required for the mixing term, acting unopposed, to reverse the sign of the flux can be estimated simply by dividing the momentum flux at any point in Fig. 1a by the value of the mixing term at that point. The resulting values are on the order of a few hours in the vicinity of the storm tracks. In retrospect, we see that the mixing term is large, even in comparison to the advective terms in (1), which act on a time scale of a day or two.

The distribution of the ageostrophic correlation term is shown in Fig. 4. On the whole, the pattern is positively correlated with the distribution of momentum fluxes, with dipole-like patterns in the vicinity of the storm tracks. The magnitudes of this term tend to run somewhat smaller than those of the mixing term, which is contrary to what would be expected on the basis of flux maintenance considerations.

The two components of the ageostrophic departure term are shown separately in Fig. 5. The dominant contributor in the vicinity of the Atlantic storm track is the $-f\overline{u'u_a'}$ term, whereas the two terms make comparable contributions in the vicinity of the Pacific storm track. The $f\overline{v'v_a'}$ term contributes strongly to the maintenance of poleward fluxes over the western parts of the continents, but it is largely cancelled by the $-f\overline{u'u_a'}$ term over western North America.

6. Synoptic interpretation

There is reason for expecting a high degree of compensation between the mixing term $-\overline{v'^2}\partial\bar{u}/\partial y$ and the ageostrophic correlation term $f\overline{v'v_a'}$. As an air parcel moves meridionally in an eddy through a zone of strong lateral wind shear $\partial\bar{u}/\partial y$ there is a strong tendency for its zonal wind speed to adjust to the local value of $\bar{u}$. The mechanism for bringing about this adjustment is the term fv_a' which represents the effect of the meridional cross-isobar flow. If the adjustment were complete, we would have

$$\frac{du}{dt}=v'\frac{\partial\bar{u}}{\partial y}=fv_a', \tag{7}$$

in which case there would be complete compensation between the mixing and ageostrophic terms. Our observational results do show some compensation between these terms, particularly in the vicinity of the Pacific jet stream (see Figs. 3 and 5b). However, even in that region, the ageostrophic term, as calculated from the NMC data, is smaller than the mixing term by more than a factor of 2. We suspect that our estimates of the ageostrophic term shown in Fig. 7b may be too small, particularly in regions of sparse data, because of the inherent bias in the NMC analyses toward gradient winds and small cross-isobar flow.

Although one might expect a much stronger compensation between $-\overline{v'^2}\partial\bar{u}/\partial y$ and $f\overline{v'v_a'}$ than we have observed, the mixing term should still be the dominant one, since the adjustment process described above is likely to be incomplete to some degree. Thus, in order to qualitatively explain the observed countergradient fluxes of $\overline{u'v'}$, we must look to some other mechanism. The other ageostrophic term $-f\overline{u'u_a'}$ obviously contributes to the maintenance of the observed pattern. We now consider in more detail the physical interpretation of this term.

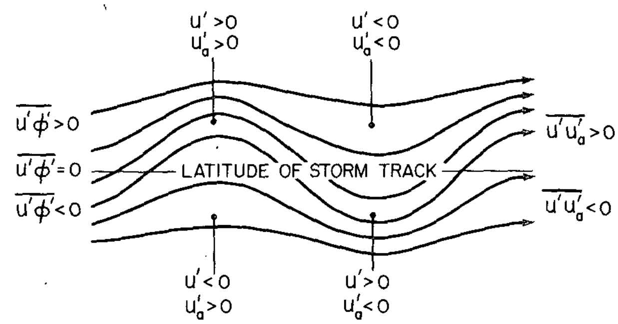

FIG. 6. Streamlines or geopotential height contours for the horizontal flow at the 250 mb level in an idealized wave moving along a storm track in a channel constrained by lateral boundaries, At this level the westerly zonal wind speed is faster than the speed of propagation of the wave.

Fig. 6 shows the streamfunction for the horizontal flow at the jet stream level in an idealized wave moving along a well-defined "storm track" in a channel bounded by rigid walls. Note that to the north of the storm track the zonal flow is strongest in the ridges and weakest in the troughs, whereas to the south of the storm track the reverse is true. Direct observational evidence of such a relationship is given in Fig. 7 which shows the distribution of $\overline{u'\phi'}$. This pattern shows a distinct dipole-like structure, with the zero line corresponding closely to the storm track, as defined in Blackmon *et al.*

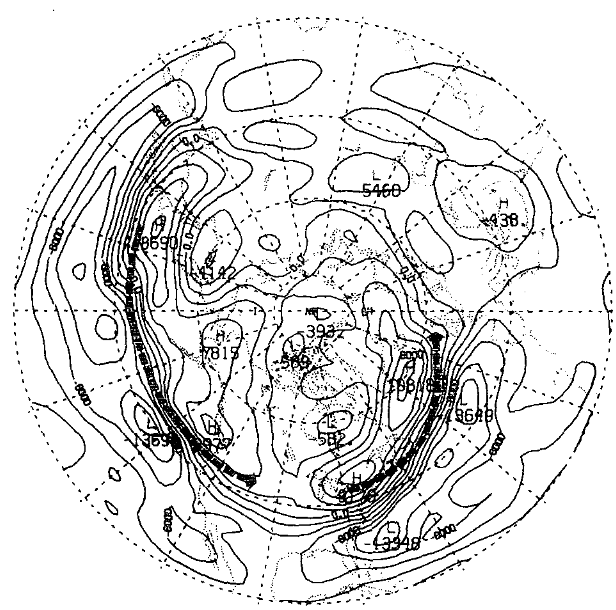

FIG. 7. The distribution of covariance $\overline{u'\phi'}$ between zonal wind and geopotential height, averaged over nine winters at the 250 mb level; contour interval 2×10^3 m^3 s^{-3}. Arrows with dashed shafts denote position of regions with maximum geopotential height variability in the bandpass-period range.

(1977). Poleward of the storm track $\overline{u'\phi'}>0$ which is indicative of strong westerlies in the ridges of transient waves and weak westerlies in the troughs. Equatorward of the storm track the opposite relationship is observed.

In determining the corresponding distribution of u_a' it is necessary to ascertain the sense of the curvature of the air trajectories in Fig. 6. At the jet stream level, the mean zonal wind speed is faster than the speed of propagation of the wave and thus the curvature of the air trajectories is in the same sense as that of the streamlines. Hence the flow is supergeostrophic in the ridges and subgeostrophic in the troughs, as indicated in Fig. 6. Thus we conclude that $-f\overline{u'u_a'}$ should be positive to the south of the storm track and negative to the north of it, in agreement with the observed distribution. We believe that this term is responsible for the observed convergence of westerly momentum into the storm tracks.

The synoptic interpretation of the flux maintenance over the western parts of the continents is difficult because of the tendency for compensation between the ageostrophic terms and the smallness of the mixing term. In these regions other terms in the equation may be of comparable importance to the ones we have singled out for discussion here.

7. Summary and concluding remarks

In view of the uncertainties in the magnitude of the ageostrophic terms, it does not appear feasible to undertake a complete budget type analysis for the momentum flux maintenance equation. Nevertheless, we believe that the processes of 1) formulating and identifying the major terms in the maintenance equation, 2) investigating the geographical distribution of the major maintenance terms in a semi-quantitative manner and 3) considering the synoptic interpretation of the major maintenance terms have yielded partial understanding of the dynamics of flux maintenance.

In the foregoing sections we have shown that in the vicinity of the oceanic storm tracks the observed distribution of momentum flux is a consequence of the ageostrophic correlation term $-f\overline{u'u_a'}$ associated with the gradient wind field, which is continually distorting the shape of the eddies so as to make them transport westerly momentum into the middle of the storm track. Although the observational evidence is not completely conclusive, it is clear that the other ageostrophic correlation term $f\overline{v'v_a'}$ must play an important role in counteracting the effects of the mixing term $-\overline{v'^2}\partial\bar{u}/\partial y$. This compensation is manifested in the tendency for the meridional cross-isobar flow to keep the zonal wind speed of air parcels relatively close to that of the local mean zonal flow, even in regions of very strong $\partial\bar{u}/\partial y$.

The flux-maintenance equation appears to be much less successful in explaining the large poleward fluxes over the western parts of the continents. The dynamical properties of the eddies in these regions appear to be quite different from those of the eddies in the storm tracks. For example, the results shown in Fig. 5 indicate that the winds in the eddies are strongly supergeostrophic (i.e., $\overline{u'u_a'}>0$ and $\overline{v'v_a'}>0$) over these regions at the jet stream level. Results shown in Fig. 11 of Blackmon *et al.* (1977) indicate that the eddy heat fluxes in the lower troposphere over these regions are very small, and more recent results (not yet published) indicate that the heat fluxes in the upper troposphere are strongly equatorward. The dynamics of these regions will be discussed in more detail in a forthcoming paper.

APPENDIX

The Terms in the Mean-Flow Interaction Group

Eq. (1) contains six terms that represent the effects of the mean shear on the maintenance of the momentum flux. They are

$$\left.\begin{aligned} S_1&=-\overline{u'v'}\frac{\partial\bar{u}}{\partial x}, & S_2&=-\overline{v'v'}\frac{\partial\bar{u}}{\partial y}, & S_3&=-\overline{v'\omega'}\frac{\partial\bar{u}}{\partial p}\\ S_4&=-\overline{u'u'}\frac{\partial\bar{v}}{\partial x}, & S_5&=-\overline{u'v'}\frac{\partial\bar{v}}{\partial y}, & S_6&=-\overline{u'\omega'}\frac{\partial\bar{v}}{\partial p}\end{aligned}\right\}.$$

In this Appendix, we show that S_2 is by far the largest member in this group, except near locations where $\partial\bar{u}/\partial y$ changes sign. First we note that

$$|\overline{u'v'}|\ll\overline{v'v'}\approx\overline{u'u'}. \tag{A1}$$

Typically, the covariance is about five times smaller than either variance; for example, compare Fig. 1 from Kao and Hurley (1962) with Fig. 6b from Blackmon *et al.*, (1977).

Since large-scale atmospheric flows are quasi-nondivergent

$$\frac{\partial\bar{u}}{\partial x}\approx-\frac{\partial\bar{v}}{\partial y}. \tag{A2}$$

Therefore, S_1 and S_5 nearly cancel each other. If the Rossby number of the large-scale flow is of order $\frac{1}{5}$, we obtain

$$|S_1+S_5|\sim\tfrac{1}{5}\left|\overline{u'v'}\frac{\partial\bar{u}}{\partial x}\right|. \tag{A3}$$

Since the mean flow is predominantly zonal, and since meridional gradients of zonal wind tend to be large compared to zonal gradients, we also have

$$|\bar{v}|<|\bar{u}|, \tag{A4}$$

$$\left|\frac{\partial\bar{u}}{\partial x}\right|<\left|\frac{\partial\bar{u}}{\partial y}\right|. \tag{A5}$$

But

$$\left|\frac{\partial \bar{v}}{\partial x}\right| \sim \left|\frac{\partial \bar{v}}{\partial y}\right|. \tag{A6}$$

In each of these, the inequality sign represents a factor of at least 3. Therefore, the ratio between S_2 and S_4 is typically at least 3. This implies that S_4 can be neglected in first approximation.

Returning to (A3), and making use of (A1) and (A5), we conclude that the sum of S_1 and S_5 is typically almost two orders of magnitude smaller than S_2. The terms S_3 and S_6 contain the vertical motion. In flows at low Rossby numbers, the vertical motion scales as $\omega/P \sim \text{Ro}V/L$, where P is an appropriate pressure scale and L represents the characteristic horizontal scale of the motion.

Now if the correlation coefficients between u' and ω', and v' and ω' are roughly comparable in absolute value to that between u' and v', it follows that

$$|\overline{v'\omega'}| \sim |\overline{u'\omega'}| \sim \text{Ro}\,|\overline{u'v'}|\,P/L.$$

Furthermore, we have

$$\left|\frac{\partial \bar{u}}{\partial p}\right| \sim \left|\frac{\partial \bar{u}}{\partial x}\right| \times \frac{L}{P} \quad \text{and} \quad \left|\frac{\partial \bar{v}}{\partial p}\right| \sim \left|\frac{\partial \bar{v}}{\partial y}\right| \times \frac{L}{P}.$$

Combining these relationships, we obtain

$$\left|\overline{v'\omega'}\frac{\partial \bar{u}}{\partial p}\right| \sim \text{Ro}\left|\overline{u'v'}\frac{\partial \bar{u}}{\partial x}\right|, \tag{A7}$$

$$\left|\overline{u'\omega'}\frac{\partial \bar{v}}{\partial p}\right| \sim \text{Ro}\left|\overline{u'v'}\frac{\partial \bar{v}}{\partial y}\right|. \tag{A8}$$

Even if there is no significant geostrophic cancellation between S_3 and S_6, they are of the same order of magnitude as the sum (S_1+S_5) which was shown to be roughly two orders of magnitude smaller than S_2. Therefore, we conclude that S_2 is the principal mean-flow interaction term in the maintenance equation for the eddy momentum flux.

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