

## On the Combined Infrared Cooling of Two Absorbing Gases in the Same Spectral Region

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### ABSTRACT

A simple criterion is presented for determining whether the combined local atmospheric infrared cooling rate at a given height in the presence of two gases, absorbing in the same spectral region, is less than or greater than the cooling rate of either of the gases alone. The criterion is applied to infrared cooling of a tropical atmosphere.

### 1. Introduction

In the course of a recent extensive study of infrared cooling rates in a gaseous atmosphere and their parameterization (Joseph and Bursztyn, 1976), it was found that under certain conditions there occurs the phenomenon illustrated in Fig. 1. Fig. 1 shows the computed vertical profiles of infrared cooling rate due to water vapor alone and due to water vapor plus carbon dioxide in a tropical atmosphere. It can be seen that with the addition of CO<sub>2</sub>, the infrared cooling rate in most of the troposphere is less than that for the water vapor alone. Carbon dioxide, acting by itself, will cool the troposphere (e.g., Plass, 1956) and so will water vapor alone. Fig. 1 shows that this is not necessarily so for the mixture water vapor and carbon dioxide at all heights. Similar results were presented for the effect of an increase of the CO<sub>2</sub> amount in a humid atmosphere (Gebhardt, 1967; Newell and Dopplick, 1970).

It is thus of interest to study the conditions under which the reduction in cooling in the lower troposphere due to the screening effect of the CO<sub>2</sub> aloft is not offset by the additional local cooling due to the CO<sub>2</sub>. More generally, these conditions would apply to the mixture of any two absorbers active in the same spectral region, including the case of an increase in the concentration of a single gas (Cox, 1973). It is the purpose of this paper to examine the conditions for which the infrared cooling rate due to water vapor plus carbon dioxide is less than that due to water vapor alone. We shall do this by deriving a general criterion for determining whether the combined infrared cooling rate of two absorbing gases in the same spectral region is less than or greater than that of one of the absorbing gases alone, and applying this criterion to some specific cases.

### 2. Method of calculation of infrared cooling

The method used for calculation of the IR cooling rates shown in Fig. 1 is the "exchange between layers"

(Joseph and Bursztyn, 1976). Exchange of radiant energy—heating or cooling—between any two layers in the atmosphere, including the surface and space, is calculated directly at 65 levels, separated by a distance

$$\Delta z = 0.11535,$$

where

$$\Delta z = - \frac{\ln(p_{\text{top}}/p_{\text{surface}})}{60}$$

except near the boundaries where  $\Delta z/2$  was used. Water vapor, carbon dioxide and ozone transmittances were taken from the literature (Rodgers and Walshaw, 1966). The temperature and humidity profiles used are those of a typical numerical model of the general circulation for a range of values appropriate for low-latitude conditions (Joseph and Bursztyn, 1976).

The overlap between CO<sub>2</sub> and H<sub>2</sub>O absorption in the 15  $\mu\text{m}$  band is treated following Rodgers and Walshaw (1966).

### 3. Analysis

The difference between IR cooling rates computed for water vapor alone and for water vapor plus carbon dioxide is due only to the difference in cooling rate for those regions of the spectrum where both H<sub>2</sub>O and CO<sub>2</sub> absorb; in our particular case, this is in the 15  $\mu\text{m}$  region. Fig. 2 shows the vertical profiles of IR cooling rate in the 15  $\mu\text{m}$  region for CO<sub>2</sub> alone, H<sub>2</sub>O alone and for H<sub>2</sub>O plus CO<sub>2</sub>, for the same tropical atmosphere as used in the calculations shown in Fig. 1. It is quite clear from Fig. 2 that although CO<sub>2</sub> alone causes cooling of the troposphere (except in a narrow layer near the tropopause at 200 mb), the combination of H<sub>2</sub>O plus CO<sub>2</sub> results in less cooling than for H<sub>2</sub>O alone for all levels below about 400 mb. Above 400 mb, the cooling due to H<sub>2</sub>O plus CO<sub>2</sub> is greater than that due to H<sub>2</sub>O alone (except in the immediate vicinity of the tropopause).

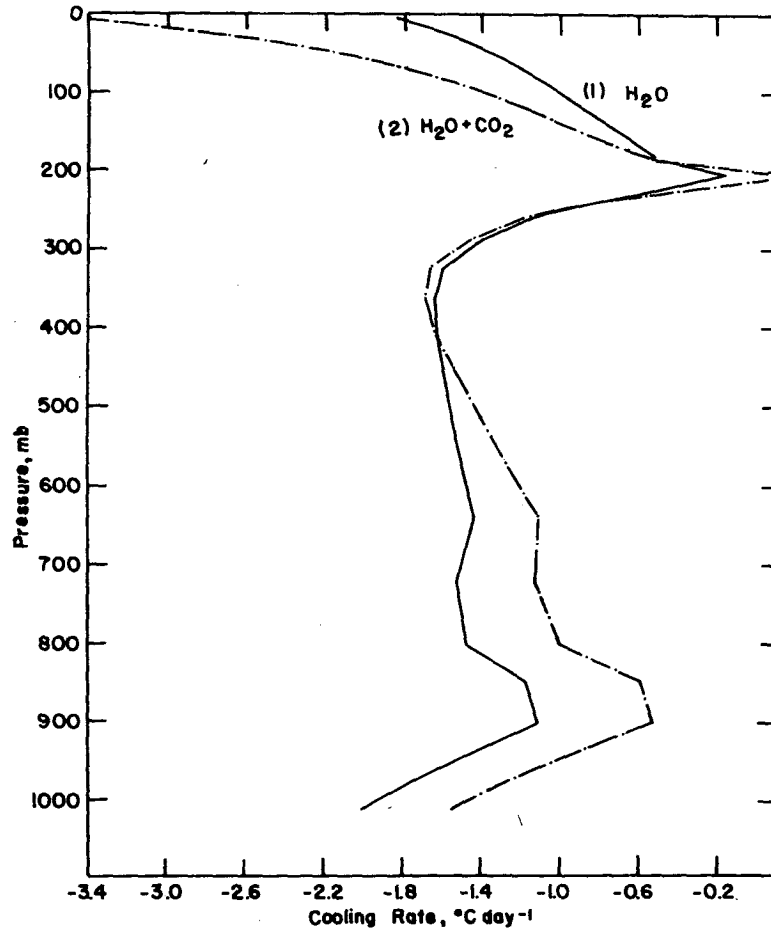


FIG. 1. Total infrared cooling rates in a tropical model atmosphere (Joseph and Bursztyn, 1976): (1) water vapor alone, (2) water vapor and carbon dioxide.

We shall now show by means of a simple analytical model under what physical conditions the combination of  $\text{H}_2\text{O}$  plus  $\text{CO}_2$  will cause less cooling than that due to  $\text{H}_2\text{O}$  alone. We shall assume that the IR cooling rate at any level can be approximated by the "cooling-to-space" term (Rodgers and Walshaw, 1966).

Such an approximation usually leads to a systematic overestimate by 0.2 to 0.5  $\text{K day}^{-1}$  of the cooling rate in the upper troposphere and lower stratosphere in humid, clear or cloudy atmospheres. Below clouds the cooling-to-space approximation should be replaced by cooling-to-cloud-base, which is similar. In the lowest 100 mb of the troposphere, the cooling-to-space approximation leads to an underestimate of the actual cooling rate by 0.1 up to about 1  $\text{K day}^{-1}$  depending on the humidity and its distribution in height (Rodgers and Walshaw, 1966; Fels and Schwarzkopf, 1975; Joseph and Bursztyn, 1976). The errors in using the cooling-to-space approximation, however, are in the same direction and of the same magnitude for both a single gas and for the mixture of two gases. This approximation will be therefore used here for simplicity, but only for the purpose of evaluating the *change* in cooling due to a

combination of two gases compared to the case of a single gas and thus does not lead to significant changes in our conclusions.

The cooling-to-space term for water vapor only may be written as

$$C_1(z) = \pi B(z) \frac{d}{dz} T_1(z, z_T), \quad (1)$$

where the subscript 1 represents  $\text{H}_2\text{O}$ ,  $C$  is the divergence of the radiation flux at the level  $z$ ,  $B$  is the Planck function,  $T_1$  is the flux transmittance of water vapor and  $z_T$  is the effective top of the radiating atmosphere. The vertical coordinate  $z = -\ln \phi$ , where  $\phi = p/p_s$ , where  $p$  is pressure and the subscript  $s$  refers to the surface. The actual cooling rate is proportional to  $C/\phi$ . For an atmosphere containing  $\text{H}_2\text{O}$  plus  $\text{CO}_2$  we may write

$$C_{1+2}(z) = \pi B(z) \frac{d}{dz} (T_{1+2}), \quad (2)$$

where  $T_{1+2}$  is the transmittance of  $\text{H}_2\text{O}$  plus  $\text{CO}_2$ ,

which is given by

$$T_{1+2} = T_1 \cdot T_2, \tag{3}$$

where  $T_2$  is the transmittance of  $\text{CO}_2$ .

From (1), (2) and (3) we can derive the following criterion for the infrared cooling of  $\text{H}_2\text{O}$  alone to be greater than that due to  $\text{H}_2\text{O}$  plus  $\text{CO}_2$ :

$$N > 0, \tag{4}$$

where

$$N = \frac{dT_1}{dz} - \left( \frac{T_1}{1-T_2} \right) \frac{dT_2}{dz}. \tag{5}$$

Eq. (5) is quite general and can be applied to any two gases that absorb in the same spectral region.

The calculations involved in evaluating (5) may all be done analytically with the use of some reasonable approximations. We use the Goody random transmission model as given by Rodgers and Walshaw (1966):

$$T = \exp \left[ - \frac{1.66 \text{ km}}{\delta} \left( 1 + \frac{1.66 \text{ km}}{\pi \bar{\alpha}} \right)^{-1} \right], \tag{6}$$

where  $k$  is the mean line intensity,  $\delta$  the mean line spacing,  $m$  the amount of absorbing gas, which for our

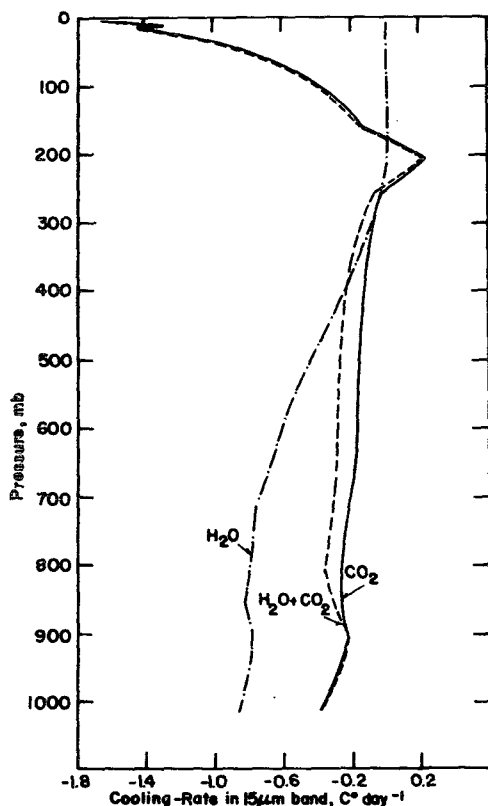


FIG. 2. Infrared cooling rates in a tropical model atmosphere in the 15 μm band region:  $\text{CO}_2$  alone,  $\text{H}_2\text{O} + \text{CO}_2$ ,  $\text{H}_2\text{O}$  alone.

TABLE 1.  $\phi_{\text{flip}}$  as a function of surface mixing ratio  $w_{s,1}$  and the ratio  $\lambda$  of atmospheric to water vapor scale height.

$\lambda$	$w_{s,1} \text{ [g g}^{-1}\text{]}$			
	$1 \times 10^{-3}$	$5 \times 10^{-3}$	$1 \times 10^{-2}$	$5 \times 10^{-2}$
3	0.366	0.327	0.300	0.264
4	0.400	0.350	0.336	0.290
5	0.450	0.388	0.364	0.334

problem is given by

$$m(\phi) = (p_s/g) \int_0^\phi w(\phi) d\phi, \tag{7}$$

where  $g$  is gravity,  $w(\phi)$  the mixing ratio of the absorbing gas, and  $\bar{\alpha}$  the mean half-width of the absorption lines along the path, given by

$$\bar{\alpha}(\phi) = \int_0^m \alpha dm / \int_0^m dm, \tag{8}$$

where  $\alpha = \alpha_s \phi$ ,  $\alpha_s$  being the half-width at standard temperature and pressure. The factor 1.66 is introduced to approximate flux transmission (e.g., Goody, 1964; Rodgers and Walshaw, 1966). The above development implicitly assumes that the atmospheric temperature is close to standard temperature so that the temperature correction to the line intensities and half-widths may be neglected. Although not particularly suitable for  $\text{CO}_2$  in the lower troposphere (Rodgers and Walshaw, 1966; Stone and Manabe, 1968), the random model is used here for both  $\text{H}_2\text{O}$  and  $\text{CO}_2$  to permit a simple analytical evaluation of (5). Its use here is also consistent with its use in the complete infrared cooling calculations shown in Figs. 1 and 2.

We assume that the vertical distribution of absorbing gas can be represented by

$$w = w_s \phi^\lambda, \tag{9}$$

where  $w_s$  is the mixing ratio at the surface.

Then

$$m = \frac{w_s}{g} \frac{p_s}{(\lambda+1)} \phi^{\lambda+1}, \tag{10}$$

$$\bar{\alpha} = \alpha_s \left( \frac{\lambda+1}{\lambda+2} \right). \tag{11}$$

The required spectral data for  $\text{CO}_2$  and  $\text{H}_2\text{O}$  in the 15 μm region are given by Rodgers and Walshaw (1966). For  $\text{CO}_2$ ,  $\lambda = 0$ , and with  $w_s = 5 \times 10^{-4}$ ,

$$\frac{1.66 \text{ km}}{\pi \bar{\alpha}} \gg 1$$

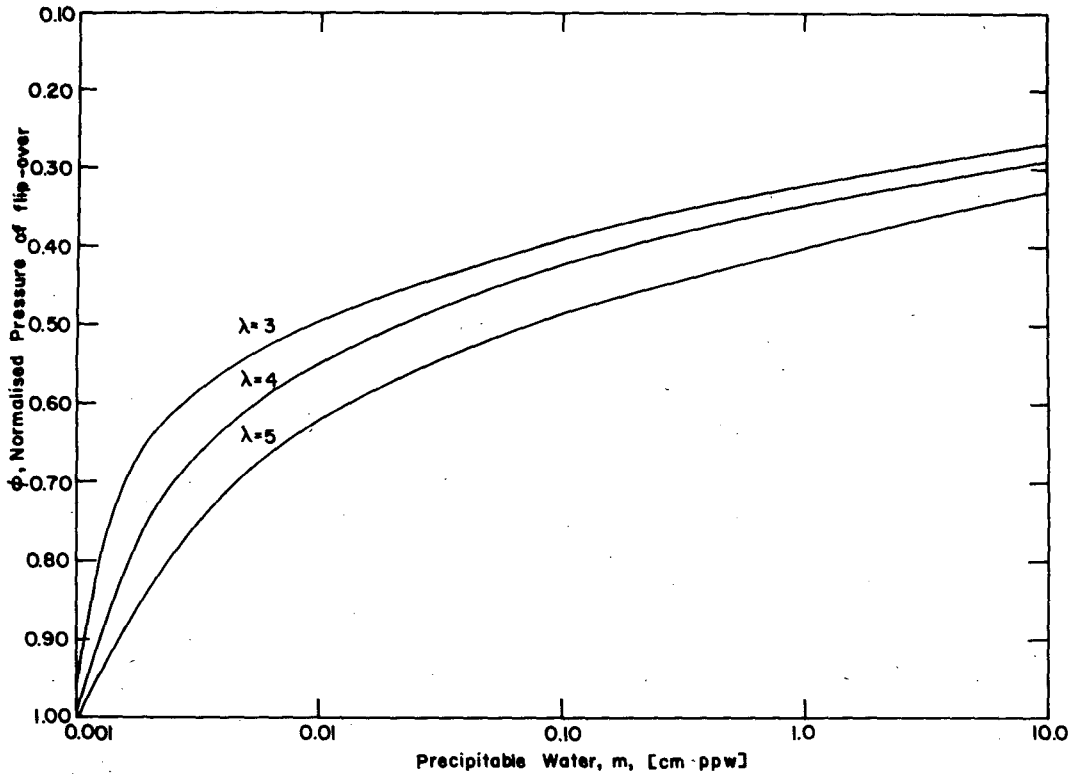


FIG. 3. The normalized flip-over pressure from reduced to increased infrared cooling as a function of total atmospheric water vapor content  $m$  and the ratio  $\lambda$  of atmospheric to water vapor scale height.

and the strong line approximation holds. We obtain

$$T_2(\phi) = \exp(-A\phi), \tag{12}$$

$$\frac{d}{dz} T_2(\phi) = AT_2(\phi)\phi, \tag{13}$$

where

$$A^2 = \frac{1.66\pi k_2 \alpha_{s,2} \rho_s w_{s,2}}{2\delta_2^2}. \tag{14}$$

For  $H_2O$  in the 15  $\mu m$  region, the strong line approximation is not generally valid and, hence, we use the complete form of (5) to obtain

$$T_1(\phi) = \exp[-B\phi^{\lambda+1}(1+D\phi^\lambda)^{-1}], \tag{15}$$

$$\frac{d}{dz} T_1(\phi) = T_1(\phi) [B(\lambda+1)\phi^{\lambda+1}(1+D\phi^\lambda)^{-1} - \frac{1}{2}BD\lambda\phi^{2\lambda+1}(1+D\phi^\lambda)^{-1}], \tag{16}$$

where

$$B = \frac{1.66k_1 w_{s,1} \rho_s}{\delta_{1g}(\lambda+1)}, \tag{17}$$

$$D = \frac{1.66k_1 w_{s,1} \rho_s (\lambda+2)}{\pi g \alpha_{s,1} (\lambda+1)^2}. \tag{18}$$

Based on these assumptions, the criterion for less cooling for the mixture of gases [Eqs. (4) and (5)]

may be written

$$\frac{T_2}{1-T_2} < \frac{B}{A} \phi^\lambda [(\lambda+1) - \frac{1}{2}\lambda D\phi^\lambda (1+D\phi^\lambda)^{-1}] (1+D\phi^\lambda)^{\frac{1}{2}}. \tag{19}$$

The results of solving Eq. (19) for that value of  $\phi$  for which equality occurs is given in Table 1. The value  $\phi_{flip}$  may be taken as the highest pressure where reduced cooling occurs.

The same quantity  $\phi_{flip}$  is shown in Fig. 3 as a function of the water vapor amount  $m$ , and  $\lambda$ .

Table 1 and Fig. 3 show that the normalized flip-over pressure  $\phi_{flip}$  is in the vicinity of 0.350 for those values of the mixing ratio and  $\lambda$  occurring most often in the atmosphere and for normal amounts of  $CO_2$ . The increase of  $\lambda$  from 3 to 5 reduces the height of flip-over from reduced to increased cooling or increases the value of  $\phi_{flip}$ . This is physically obvious because an increase in  $\lambda$  means a reduction of the scale-height of water vapor and a reduction in its total amount in the atmosphere.

These results show the directions of variation of  $\phi_{flip}$  as a function of  $w_s$  and  $\lambda$  in an atmosphere that is either isothermal or where the loss of radiation to space is the major reason for the radiative cooling. Our conclusions have application to more complex atmospheres, however (see beginning of this section).

We next apply our results to a specific case for which we have a complete profile of radiative cooling available in a non-isothermal, humid tropical atmosphere (Figs. 1 and 2).

For water vapor in the equatorial zone  $\lambda = 2.91$  (Smith, 1966). Using a value of  $w_s = 17 \text{ g kg}^{-1}$  for the  $\text{H}_2\text{O}$  surface mixing ratio of a tropical atmosphere, we have computed  $N/\phi$  as a function of  $\phi$  from the above equations. The results are plotted in Fig. 4. The expression  $N/\phi$  is proportional to the difference between the cooling rate of  $\text{H}_2\text{O}$  plus  $\text{CO}_2$  and the cooling rate of  $\text{H}_2\text{O}$  alone, a positive value indicating that the combined cooling rate is less than that of  $\text{H}_2\text{O}$  alone. The results of our simple analytical model indicate that the cooling rate due to  $\text{H}_2\text{O}$  plus  $\text{CO}_2$  should be less than that due to  $\text{H}_2\text{O}$  alone for all levels below about 300 mb. This is in good agreement with the results of Fig. 2 which show the cross-over point to be about 400 mb. The main reason for the small discrepancy is probably due only to our use of the cooling-to-space approximation, which neglects the cooling due to exchange between neighboring layers and heating of a layer by the surface emission compared to the results of the calculation shown in Fig. 2. Above 300 mb, the simple analytical model predicts greater cooling for  $\text{H}_2\text{O}$  plus  $\text{CO}_2$  than for  $\text{H}_2\text{O}$  alone. This is in agreement with the results in Fig. 2 except for the narrow layer immediately surrounding the tropopause where there are strong non-isothermal effects.

These analytical results may be interpreted physically with the help of Fig. 5, which shows the transmittance in the  $15 \mu\text{m}$  region from the level  $z$  to the top of the

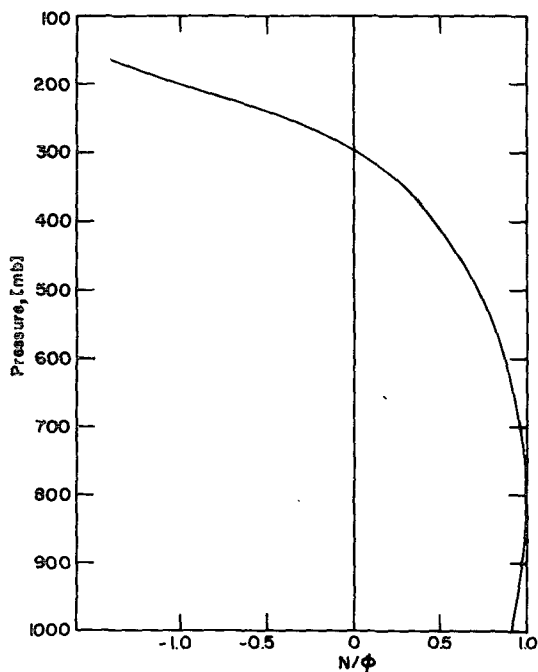


FIG. 4. The profile of  $N/\phi$  for the water vapor amount used for Figs. 1 and 2.

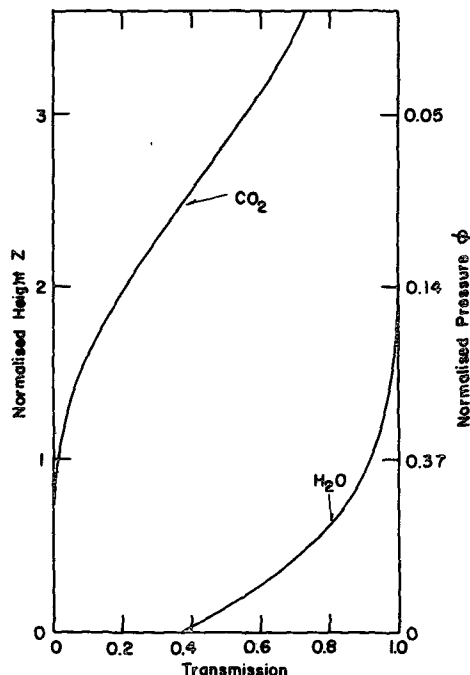


FIG. 5. The transmission profiles of  $\text{CO}_2$  and  $\text{H}_2\text{O}$  in a tropical atmosphere.

atmosphere for  $\text{CO}_2$  alone and for  $\text{H}_2\text{O}$  alone, as functions of  $z$ . For  $\text{H}_2\text{O}$  alone, it is clear that a large fraction of the emission from lower tropospheric levels will be transmitted to space. For  $\text{CO}_2$  alone, because of the small transmittances, only a small fraction of any emission from lower tropospheric levels will be transmitted to space. Since the transmittance of  $\text{H}_2\text{O}$  plus  $\text{CO}_2$  is the product of the respective transmittances, it is clear that the effect of adding the observed amount of  $\text{CO}_2$  to the  $\text{H}_2\text{O}$  is to partially dam up the  $\text{H}_2\text{O}$  emission of most of the troposphere and prevent it from escaping to space. Thus, the IR cooling rate of  $\text{H}_2\text{O}$  plus  $\text{CO}_2$  will be less than that for  $\text{H}_2\text{O}$  alone for most of the troposphere.

#### 4. Summary and conclusions

A simple criterion has been derived for determining whether the combined local infrared cooling rate at a given height of two absorbing gases in the same spectral region is less than or greater than that of one of the absorbing gases alone. Strictly speaking, the criterion is valid only for an isothermal or fairly dry atmosphere, but in actual practice it should be applicable to a large part of the real atmosphere—that part for which the cooling-to-space approximation either is applicable or systematically overestimates the cooling. With the use of some simplifying but realistic assumptions, the calculations necessary to evaluate this criterion for the case of the water vapor-carbon dioxide overlap in the  $15 \mu\text{m}$  region can be performed analytically. The criterion is used to explain the distribution of computed IR cooling

for H<sub>2</sub>O, and H<sub>2</sub>O plus CO<sub>2</sub> obtained for a tropical atmosphere.

Our results are quite general and can be applied to any two absorbing gases in the same spectral region. One may even include absorbers like aerosols, and extend the applications to three or more absorbing gases. It is of practical significance in such cases where one wants to know the effect on the IR cooling rate of eliminating, adding or changing the concentration of a particular absorbing gas or an absorber like aerosols. It may be also used to answer such questions as: Is the atmospheric infrared cooling overestimated or underestimated by neglecting the contribution of CO<sub>2</sub>?

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#### REFERENCES

Cox, S. K., 1973: Infra-red heating calculations with a water vapor pressure broadened continuum. *Quart. J. Roy. Meteor. Soc.*, **99**, 669–679.

Fels, S. B., and M. Daniel Schwarzkopf, 1975: The simplified exchange approximation: A new method for radiative transfer calculations. *J. Atmos. Sci.*, **32**, 1475–1488.

Gebhardt, R., 1967: On the significance of the short-wave CO<sub>2</sub>—absorption in investigations concerning the CO<sub>2</sub> theory of climate change. *Arch. Meteor. Geophys. Bioklimtol.*, **B15**, 52–61.

Goody, R. M., 1964: *Atmospheric Radiation*. Oxford University Press, 436 pp.

Joseph, J. H., and R. Bursztyn, 1976: A radiative cooling model in the thermal infrared for application to models of the general circulation. *J. Appl. Meteor.*, **15**, 319–325.

Newell, R. E., and T. G. Dopplick, 1970: The effect of changing CO<sub>2</sub> concentration on radiative heating rates. *J. Appl. Meteor.*, **9**, 958–959.

Plass, G. N., 1956: The influence of the 15 μ carbon dioxide band on the atmospheric infrared cooling rate. *Quart. J. Roy. Meteor. Soc.*, **82**, 310–324.

Rodgers, C. D., and C. D. Walshaw, 1966: The computation of infrared cooling rate in planetary atmospheres. *Quart. J. Roy. Meteor. Soc.*, **92**, 67–91.

Smith, W. L., 1966: Note on the relationship between total precipitable water and surface dew point. *J. Appl. Meteor.*, **5**, 726–727.

Stone, H. M., and S. Manabe, 1968: Comparison among various numerical models designed for computing infra-red cooling. *Mon. Wea. Rev.*, **96**, 735–741.