

## Diabatic Heating Climatology of the Zonal Atmosphere

MICHAEL HANTEL AND HANS-REINHARD BAADER

*Meteorologisches Institut der Universität Bonn, 5300 Bonn 1, Federal Republic of Germany*

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### ABSTRACT

The diabatic heating  $Q$  is the ultimate driving force of the general circulation and climate. We present seasonal and zonal mean estimates of  $Q$  (order of magnitude  $10^{-6}$  K s $^{-1}$ ) for the atmosphere from 15°S–90°N and from 50–1000 mb.  $Q$  comprises radiation, condensation, conduction, dissipation and diffusion; the first two terms are large, the last three are small. We compile  $Q$  indirectly by specifying (from the synoptic general circulation statistics of the MIT Library) sensible heat advective and storage terms, including the adiabatic heating, which together balance  $Q$  in the First Law of thermodynamics. An important component of the advective terms is subsynoptic-scale advection. We show that it is not restricted to boundary layer heating but also has convective-scale components of potential significance and seems to be active even in the stratosphere. However, we are not able to specify the total subsynoptic-scale advection since it is subject to considerable compensation. This causes a systematic error of the order of  $10^{-6}$  K s $^{-1}$  in our synoptic estimates of  $Q$ .

The meridional diabatic heating profiles show four latitude belts of different  $Q$  climate. The tropics and midlatitudes are characterized by net heating, the subtropics and the polar cap by net cooling. This pattern is visible throughout the year and reflects the net effect of the two governing, and partly balancing, components of  $Q$ : condensational heating dominates in the rainbelts, radiational cooling dominates in the dry belts. A new feature in the  $Q$  field is persistent strong heating at and above the jet stream level between 30–40°N throughout the year. We speculatively explain this effect with the subsynoptic advective terms.

### 1. Motivation

The First Law of thermodynamics reads

$$\frac{dT}{dt} + \frac{\kappa}{p} \omega T = \frac{Q}{c_p}, \quad (1.1)$$

where  $T$ ,  $t$ ,  $p$ ,  $\omega$ ,  $c_p$  denote temperature, time, pressure,  $dp/dt$ , specific heat at constant pressure, respectively;  $\kappa = 2/7$ . The symbol  $Q$  denotes the total diabatic heating.  $Q$  can be neglected in many cases of considerable meteorological significance (for example, short-term numerical weather forecasting). Nevertheless it is just the deviation from the idealized adiabatic case that eventually drives the motion system of the planetary atmosphere. Thus knowledge of  $Q$  and its distribution in space and time is material for an understanding of the general circulation. The objective of this study is to present estimates of  $Q$  for the zonal mean atmosphere from about 15°S to the North Pole and from 50 to 1000 mb; time domain shall be a climatological average for the seasons.

There are two ways to obtain the diabatic heating. The first and direct way is to separately estimate the internal components of  $Q$  on the right of Eq. (1.1). This has been done by Newell *et al.* (1969; hereafter referred to as N1969) and has been repeated by Newell *et al.* (1974; hereafter referred to as N1974). They

split  $Q$  into radiative heating, latent heat release and boundary layer heating and presented tables of the components and patterns of  $Q$  for the zonal mean atmosphere from the South Pole to the North Pole. Their results are redrawn in Fig. 1 in vertically averaged form and show consistent net heating in the inner tropics and net cooling in the subtropics and over the polar caps. In the extratropics, however, the two estimates differ considerably. In summer N1969 reported cooling in the latitudinal belt 40–80°N while N1974 reported heating. The discrepancies become even more visible with vertical resolution (cf. Fig. 8 of N1969 and Fig. 7.18 of N1974). They are largely due to the different radiation heating rates employed in the two papers. The uncertainties in the  $Q$  fields are further increased by strong compensation between radiative heating and latent heat release and by the unknown vertical distribution of the latter. For example, in N1974 the authors were forced to distribute the surface precipitation in the vertical on the basis of cloud statistics at middle and high latitudes and on model profiles of vertical motion in the vicinity of the equator. In summary, this approach is of limited value due to the limitations of the input data which, with the present global observing system, cannot be measured but must be parameterized.

The second and indirect way is to determine the

left-hand side of Eq. (1.1) which ought to balance  $Q$ . This has been done by Brown (1964) with a two-parameter quasi-geostrophic model on the basis of objective analyses of the geopotential height field, leading to a vertical average of  $Q$  for the 400–800 mb layer. Brown obtained net heating in the tropics and net cooling in the extratropics poleward of about 45°N. The limitations of his approach seem to be due to the model restrictions. Determination of the left of Eq. (1.1) on the basis of observations was also tried in N1969 with a negative result; the authors concluded that in view of the many uncertainties involved a comparison between computed atmospheric transport (from the first way above) and observed atmospheric transport had little meaning.

Nevertheless, the purpose of the present study is to try the second way again. We feel that the uncertainties of the direct method are more severe than the uncertainties of the indirect method. We shall use the high-quality general circulation statistics of the MIT Library (Oort and Rasmusson, 1971) along with a parameterization of the synoptic vertical eddy heat flux (Hantel and Baader, 1976). The principal limitation of this approach will be that we cannot specify from observation or parameterization the *sub-synoptic* contributions to the zonal and time average of  $dT/dt$  and  $\omega T$ . We shall discuss the implications of this shortcoming in the next section and shall speculate about its order-of-magnitude impact on  $Q$  in Section 4. The results shall be presented in Section 3.

**2. Method**

*a. Various conventional forms of the First Law*

The shortest versions of the First Law are obtained by applying the concepts of entropy  $\sigma$  or potential temperature  $\theta$  to Eq. (1.1), i.e.,

$$T \frac{d\sigma}{dt} = Q \quad \text{or} \quad \left(\frac{p}{p_c}\right)^{\kappa} \frac{d\theta}{dt} = \frac{Q}{c_p} \quad (2.1)$$

Since both  $\sigma$  or  $\theta$  must be computed from the measured  $T$  field on isobaric surfaces, there is no practical advantage in using either of Eq. (2.1) instead of (1.1).

Another quasi-exact form of the First Law can be obtained (e.g., Kung, 1966) by combining it with the equation for horizontal kinetic energy  $K \equiv |\mathbf{V}|^2/2$ . Along with the hydrostatic approximation we get

$$d/dt(c_p T + K) + \nabla_p \cdot \mathbf{V}\Phi + \partial\omega\Phi/\partial p - \mathbf{V} \cdot \mathbf{F} \approx Q, \quad (2.2)$$

where  $\Phi$  is the geopotential,  $\nabla_p$  the two-dimensional gradient operator and  $\mathbf{F}$  the frictional force. The main difference is that  $\omega\alpha$  ( $\alpha \equiv$  specific volume) in Eq. (1.1) has been replaced by  $\mathbf{V} \cdot \nabla_p \Phi - (\nabla_p \cdot \mathbf{V}\Phi + \partial\omega\Phi/\partial p)$  in (2.2). This is advantageous since the fluxes involving  $\Phi$  can be readily estimated with synoptic data while  $\omega\alpha$  or  $\omega T$  require evaluation of the  $\omega$  field (e.g., Kung, 1973).

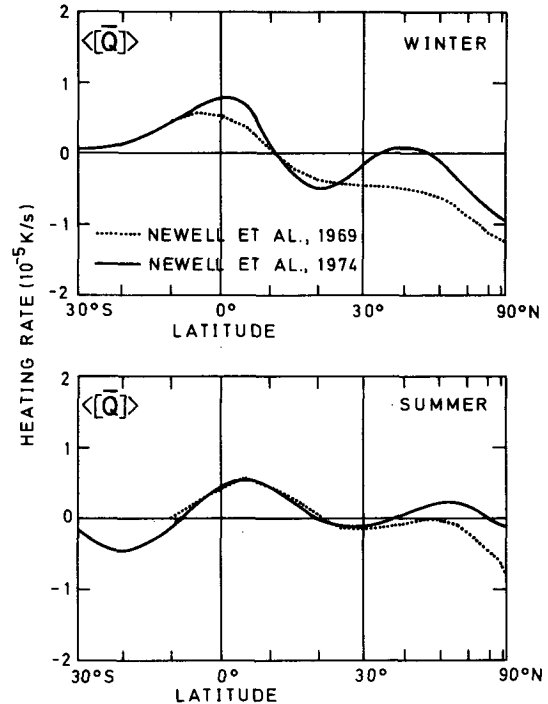


FIG. 1. Vertical average of total diabatic heating ( $10^{-5} \text{ K s}^{-1}$ ) compiled from Newell *et al.* (1969, 1974) on the basis of their radiation, condensation and boundary layer heating estimates.

A further approximation of (2.2) may be gained by assuming that the terms with  $K$ , including dissipation, are negligible compared with the components of the flux divergence; this is suggested by observations. Introducing the potential heat  $s \equiv c_p T + \Phi$  transforms Eq. (2.2) into

$$\frac{ds}{dt} \approx Q. \quad (2.3)$$

This form of the First Law has been discussed and applied by Betts (1974) and others.

Another way of arriving at (2.3) is to apply the hydrostatic plus geostrophic assumptions to (1.1), without reference to the kinetic energy budget,

$$\partial c_p T / \partial t + \nabla_p \cdot \mathbf{V}s + \partial\omega s / \partial p \approx Q. \quad (2.4)$$

Since the storage terms of all forms of energy are again small compared with the components of the flux divergence (Hantel *et al.*, 1976), Eqs. (2.3), (2.4) are practically equivalent. The storage term in (2.4) can be replaced by  $(\partial/\partial t)(c_p T + \Phi)$ , where  $c_v$  is the specific heat at constant volume. It is in this form that the First Law (2.4) has been applied recently (Hantel and Langholz, 1977) for an estimate of the precipitation flux in the free atmosphere. In the present study we shall apply the potential heat equation (2.4) in vertically averaged form for an independent estimate of the total value of  $Q$ .

### b. Components of the diabatic heating

The diabatic heating splits into the terms (e.g., van Mieghem, 1973)

$$Q = Q_{\text{RAD}} + Q_{\text{LH}} + Q_{\text{COND}} + Q_{\text{F}} + Q_{\text{D}}. \quad (2.5)$$

For most of the indexing we have adopted the convention of N1974. The terms describe, respectively, heating by radiation, condensation (latent heating), conduction, friction and diffusion. The first two are due to the fact that the atmosphere is an open system; this applies also to condensation processes (as long as we do not carry separate budgets for water in form of rain, snow or ice). The last three terms in (2.5) are due to internal molecular processes of the system.

In order to characterize the molecular terms without extensive theoretical reasoning (e.g., Landau and Lifshitz, 1959) we may generalize the arguments of Lorenz (1967, p. 14) in his qualitative discussion of the irreversible diabatic heating. One of the most important irreversible processes in the atmosphere is the mixing of different masses of air. Following Lorenz, we may distinguish between the mixing of masses of *different temperature* (i.e., conduction  $Q_{\text{COND}}$ ), the mixing of masses of *different velocity* (i.e., friction  $Q_{\text{F}}$ ) and the mixing of masses of *different composition* (i.e., diffusion  $Q_{\text{D}}$ ).

The nonfrictional  $Q_{\text{COND}}$  describes heating by molecular conduction. This process is active, even in the absence of motion, provided there are temperature gradients in the fluid. We shall not adopt the notion of "turbulent conduction" in spite of the often applied conductive parameterization of turbulent heat fluxes; consequently, our  $Q_{\text{COND}}$  is quite different from the respective quantity in N1974. We maintain that turbulent heat fluxes on scales below the synoptic scale are also generally of an ordered nature and thus should be considered advective; this applies not only to the mesoscale (*convection*) but down to scales of local wind gusts and below. Therefore, we shall collect all temperature changes which involve correlations between wind and temperature (advection in the strict sense, irrespective of scale) on the left of Eq. (1.1).

Similar remarks apply to the other irreversible diabatic terms.  $Q_{\text{F}}$  denotes the frictional heating or dissipation. Dissipation is due to velocity gradients and does not involve temperature fluctuations, irrespective of the fact that it acts to drive the field of sensible heat. Likewise,  $Q_{\text{D}}$  is active in an atmosphere with gradients of concentration (most importantly: specific humidity) even in the absence of velocity or temperature structure.

In many meteorological applications of the First Law the terms  $Q_{\text{COND}}$ ,  $Q_{\text{F}}$  and  $Q_{\text{D}}$  have been, and can safely be, neglected. We note in passing that the irreversible terms also comprise flux divergence expressions which are generally considered reversible,

and which we have skipped in the present qualitative discussion. The governing terms of (2.5) are  $Q_{\text{RAD}}$  and  $Q_{\text{LH}}$  (Hantel and Peyinghaus, 1976). In the present study we shall not distinguish between the various components of  $Q$ , except for qualitative explanations of the net diabatic heating.

When adopting the viewpoint above that all turbulent (or Reynolds) correlation terms principally belong to the left side of Eq. (1.1) we become faced with the problem of how to separate the synoptic-scale from the subsynoptic-scale advection. We shall address this question by considering the so-called boundary layer heating. This virtual heating term is intended to parameterize a certain part of the correlation terms. It results from the scale separation generated on the left of Eq. (1.1) by the averaging process.

### c. The boundary layer heating

We consider the averaged form of (1.1). With the familiar averaging symbols

$$\text{zonal: } [\ ] + *; \quad \text{time: } \overline{\quad}, \quad (2.6)$$

the total time derivative of temperature in pressure coordinates becomes (meridional coordinate  $\eta = a \sin\phi$ )

$$\overline{dT/dt} = \frac{\partial[\overline{T}]}{\partial t} + \frac{\partial[\overline{vT}]}{\partial \eta} \cos\phi + \frac{\partial[\overline{\omega T}]}{\partial p}. \quad (2.7)$$

The time- and zonally averaged fluxes split into mean and eddy terms. For the standing plus transient eddies in the horizontal and vertical directions we shall use the concise notation (Hantel and Baader, 1976)

$$[\overline{\omega^* \overline{T}^*}] + [\overline{\omega' T'}] = [\overline{\omega^E T^E}]. \quad (2.8)$$

The eddy fluxes split again into a synoptic plus a subsynoptic contribution; thus we have

$$[\overline{\omega T}] = [\overline{\omega}][\overline{T}] + [\overline{\omega^E T^E}]_{\text{SYN}} + [\overline{\omega^E T^E}]_{\text{SUBSYN}}. \quad (2.9)$$

This formulation is quite general; it can be considered the simplest form of a spectral expansion. Eq. (2.9) does not depend on the particular scale at which the section is made between synoptic and subsynoptic. Eq. (1.1) now reads

$$\frac{\partial[\overline{T}]}{\partial t} + \frac{\partial[\overline{vT}]}{\partial \eta} \cos\phi + \left( \frac{\partial}{\partial p} - \frac{\kappa}{p} \right) \{ [\overline{\omega}][\overline{T}] + [\overline{\omega^E T^E}]_{\text{SYN}} + [\overline{\omega^E T^E}]_{\text{SUBSYN}} \} = \frac{[\overline{Q}]}{c_p}. \quad (2.10)$$

The operator affecting the vertical eddies consists of two parts: the terms with  $\partial/\partial p$  are eddy components of the regular flux divergence and the terms with

$-\kappa/p$  constitute the conversion between eddy available potential and eddy kinetic energy. Since we will not apply the concept of available potential energy in the subsequent discussion, we shall consider all terms in (2.10) which involve  $[\overline{\omega^E T^E}]$  as (synoptic or subsynoptic) advective components of the First Law. If necessary, we shall refer to the terms with  $-\kappa/p$  as the adiabatic terms.

Newell *et al.* (1974) consider, but do not evaluate, the synoptic-scale vertical eddy heat flux [their terms (9) and (10), p. 45, equivalent to our terms in (2.10) with index SYN]. For the subsynoptic-scale vertical eddy heat flux (equivalent to our terms with index SUBSYN) they introduce two separate quantities: a boundary layer heating term  $Q_{BLH}$  plus a turbulent conduction term. The difference between the two is not clear. According to N1974, the "boundary layer heating . . . represents . . . small-scale turbulent motions in the boundary layer." And, the "conductive term serves to represent the heat transport by subsynoptic-scale motion . . ." However, the authors evaluate only  $Q_{BLH}$  in the 1000–850 mb layer. van Mieghem (1973), on the other hand, treats the boundary layer heating as a form of eddy conduction.

We shall not follow these conventions for two reasons. First, the activity of the vertical eddy heat flux is not restricted to the boundary layer. Second, the nature of the subsynoptic eddies cannot be restricted to Austausch-like mechanisms which are typical for conductive parameterizations. We maintain that the correlation between  $\omega$  and  $T$  indicates an advective-like transport mechanism which should be described as *subsynoptic temperature advection*.

*d. Parameterization of the vertical eddy heat flux*

Concerning the synoptic-scale vertical eddy heat flux component we shall adopt a parameterization introduced by Saltzman and Vernekar (1971):

$$[\overline{\omega^E T^E}]_{SYN} = \left( \frac{\partial p}{\partial y} \right)_{[\theta]_{SYN}} [\overline{v^E T^E}]_{SYN}. \quad (2.11)$$

Here the vertical eddy heat flux is set proportional to the meridional eddy heat flux; the constant of proportionality is the pressure gradient on mean synoptic isentropic surfaces. Eq. (2.11) and its implications for the diabatic heating have been discussed recently by Hantel and Baader (1976). Note that (2.11) is different from  $Q_{BLH}$ ; for example, Hantel and Baader showed that (2.11) has sizable values up to the 400 mb level and above while in N1974  $Q_{BLH}$  is limited to levels between 1000–850 mb. Hantel and Baader extended their parameterization to still higher orders. However, this is not necessary for our present purpose since  $[\overline{\omega^E T^E}]_{SYN}$  is altogether not very big (Fig. 2).

TABLE 1. Estimate of vertical heat flux components.

Vertical heat flux component	Assumptions	Order of magnitude (10 <sup>-4</sup> K mb s <sup>-1</sup> )
Cell: [ $\overline{\omega}$ ][ $\overline{T}$ ]	[ $\overline{\omega}$ ] $\approx 0.4 \times 10^{-4}$ mb s <sup>-1</sup> ; $T \approx 250$ K	100
Synoptic eddy: [ $\overline{\omega^E T^E}$ ] <sub>SYN</sub>	Parameterized [see Hantel and Baader (1976)]	10
Subsynoptic eddy: [ $\overline{\omega^E T^E}$ ] <sub>SUBSYN</sub>	Sensible heat flux at earth's surface: 1 kcal cm <sup>-2</sup> month <sup>-1</sup> (Budyko, 1963) Contribution in single hail cloud (see text)	10  3 × 10 <sup>4</sup>

Concerning the subsynoptic-scale vertical eddy heat flux component we feel it is not possible at present to offer a plausible parameterization. Ideally,  $[\overline{\omega^E T^E}]_{SUBSYN}$  should come from observations. As long as representative measurements are lacking it might be tempting to employ the  $Q_{BLH}$  as a preliminary parameterization. We have abandoned this idea for the reasons mentioned above.

We now discuss the order of magnitude of the various vertical heat flux terms (Table 1). For  $[\overline{\omega}][\overline{T}]$  we have adopted conservative estimates of Oort and Rasmusson (1971) yielding about 100 flux units (1 unit = 10<sup>-4</sup> K mb s<sup>-1</sup>). For  $[\overline{\omega^E T^E}]_{SYN}$  we have adopted the parameterization (2.11) yielding about 10 units. This estimate is supported by Kung and Smith's (1974) figure for the eddy conversion between available potential and kinetic energy in midlatitude cyclones; they estimate  $[\overline{\omega^E \alpha^E}] \approx 11$  W m<sup>-2</sup> which corresponds to 11 units of  $[\overline{\omega^E T^E}]_{SYN}$ . Concerning  $[\overline{\omega^E T^E}]_{SUBSYN}$  it is clear that it must match the figures measured in the constant flux layer close to the earth's surface, and in this sense might be considered a boundary heat flux. Adopting the surface sensible heat flux values of Budyko (1963) this would correspond to about 5–30 units, depending on latitude.

In higher levels, however, there are also sizable contributions to the subsynoptic heat flux which are not of a microturbulent nature. A striking example has just been given by Sartor and Cannon (1977) who investigated a precipitating convective cloud with a sailplane. The cloud air temperature was about 3.5 K higher than the ambient air temperature, the updraft velocity in the cloud was of the order 10 m s<sup>-1</sup>, both values being characteristic for at least 1 km in the horizontal and several kilometers in the vertical. This observation yields locally 3 × 10<sup>4</sup> units of vertical eddy heat flux. Comparing this estimate with our synoptic heat flux it requires only some 10<sup>-4</sup>–10<sup>-3</sup> of the area covered with active convective clouds of the Sartor and Cannon type to generate the same subsynoptic vertical heat flux as is measured on the synoptic scale.

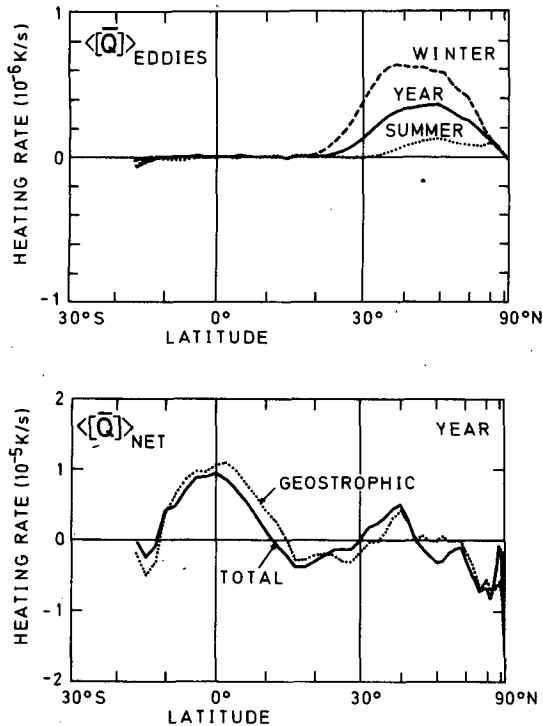


FIG. 2. Net diabatic heating of zonally and vertically averaged northern atmosphere. Upper diagram: contribution by synoptic eddies (parameterized) to net heating. Lower diagram: full curve ("total"), net annual diabatic heating calculated from synoptic advective terms of the First Law; dotted curve ("geostrophic"), calculated from potential heat equation. Note different units of diagrams.

This example suggests that the subsynoptic-scale, but nevertheless highly organized, eddy heat flux mechanisms might contribute significantly to the left of Eq. (2.10) and thus influence indirect estimates of the diabatic heating. On the other hand, these processes are presumably of a transient and intermittent nature. Their regional distribution and net impact upon the diabatic heating remain unknown with present knowledge.

Therefore, we are forced to shift the subsynoptic term from the left of (2.10) to the right and lump it into  $Q$ . It follows that (2.5) becomes spoiled by an additional heating term defined as

$$\left(\frac{\partial}{\partial p} - \frac{\kappa}{p}\right) \overline{[\omega^E T^E]}_{\text{SUBSYN}} \equiv -\overline{[Q_{\text{SUBSYN}}]} / c_p. \quad (2.12)$$

Note that  $Q_{\text{SUBSYN}}$  is formally equivalent to, but physically different from, the  $Q_{\text{BLH}}$  discussed above.

#### e. Specification of synoptic heat fluxes

The meridional heat flux component  $[\bar{v}T]$  splits in a way similar to (2.9). Following Manabe *et al.* (1974) we consider the subsynoptic eddy contribution to the meridional heat flux as negligible. This approximation

should eventually be scrutinized with subsynoptic-scale observed data.

Our discussion of the First Law may now be summarized by combining Eqs. (2.10) and (2.12) into

$$\begin{aligned} \frac{\partial[\bar{T}]}{\partial t} + \frac{\partial[\bar{v}][\bar{T}]}{\partial \eta} \cos \phi + \frac{\partial[\overline{v^E T^E}]_{\text{SYN}} \cos \phi}{\partial \eta} \\ + \left(\frac{\partial}{\partial p} - \frac{\kappa}{p}\right) \{[\bar{\omega}][\bar{T}] + \overline{[\omega^E T^E]}_{\text{SYN}}\} \\ = \overline{[Q_{\text{SUBSYN}} + \bar{Q}]} / c_p. \quad (2.13) \end{aligned}$$

We shall refer to the right of (2.13) as  $Q_{\text{NET}}$ . Available data for the terms on the left are zonal mean fields of  $[\bar{T}]$ ,  $[\bar{v}]$ ,  $[\bar{\omega}]$  and  $[\bar{v}^* \bar{T}^*]$  plus  $[\bar{v}' T']_{\text{SYN}}$  yielding  $[\overline{v^E T^E}]_{\text{SYN}}$  according to (2.8). Data base is the seasonal MIT Library (Oort and Rasmusson, 1971) with  $2^\circ$  resolution in meridional and 50 mb in vertical direction.

The actual computation of  $Q_{\text{NET}}$  was made after manipulating the mean advective terms on the left with the mass continuity equation

$$\frac{\partial[\bar{v}][\bar{T}]}{\partial \eta} \cos \phi + \frac{\partial}{\partial p} [\bar{\omega}][\bar{T}] = [\bar{v}] \frac{\partial[\bar{T}]}{\partial y} + [\bar{\omega}] \frac{\partial[\bar{T}]}{\partial p}, \quad (2.14)$$

where  $y \equiv a\phi$  is the meridional coordinate corresponding to  $\eta$ . This transformation proved to be of only spurious impact on the actual fields but offered an opportunity to independently carry out the hemispheric averaging and thus to test the truncation errors involved in our integration scheme (see Table 2 below).

The original  $[\bar{\omega}]$  and  $[\bar{v}]$  fields in the format available did not exactly fit the continuity equation in  $p$  coordinates. They were adjusted to give exact mass balance. This point is of significance since even small imbalances can cause visible errors in the diabatic heating. The maximum adjustment was about 10% of the mass flux streamfunction. The differential expressions in (2.13) and (2.14) were evaluated with finite differences and the integrations were carried out with a spline function technique.

### 3. Results

Fig. 2 (upper diagram) shows the latitudinal distribution of the parameterized  $[\overline{v^E T^E}]_{\text{SYN}}$  component. It is of the order of  $10^{-7}$  K s $^{-1}$  and, therefore, sufficiently small as compared to  $Q_{\text{NET}}$ . The maximum of the eddies is located in the extratropics (Hantel and Baader, 1976).

Fig. 2 (lower diagram) shows the latitudinal distribution of the diabatic heating for the annual case.  $Q_{\text{NET}}$  is of the order of  $10^{-5}$  K s $^{-1}$ . The full curve has been calculated from the left of the vertically averaged Eq. (2.13). The dotted curve has been calculated

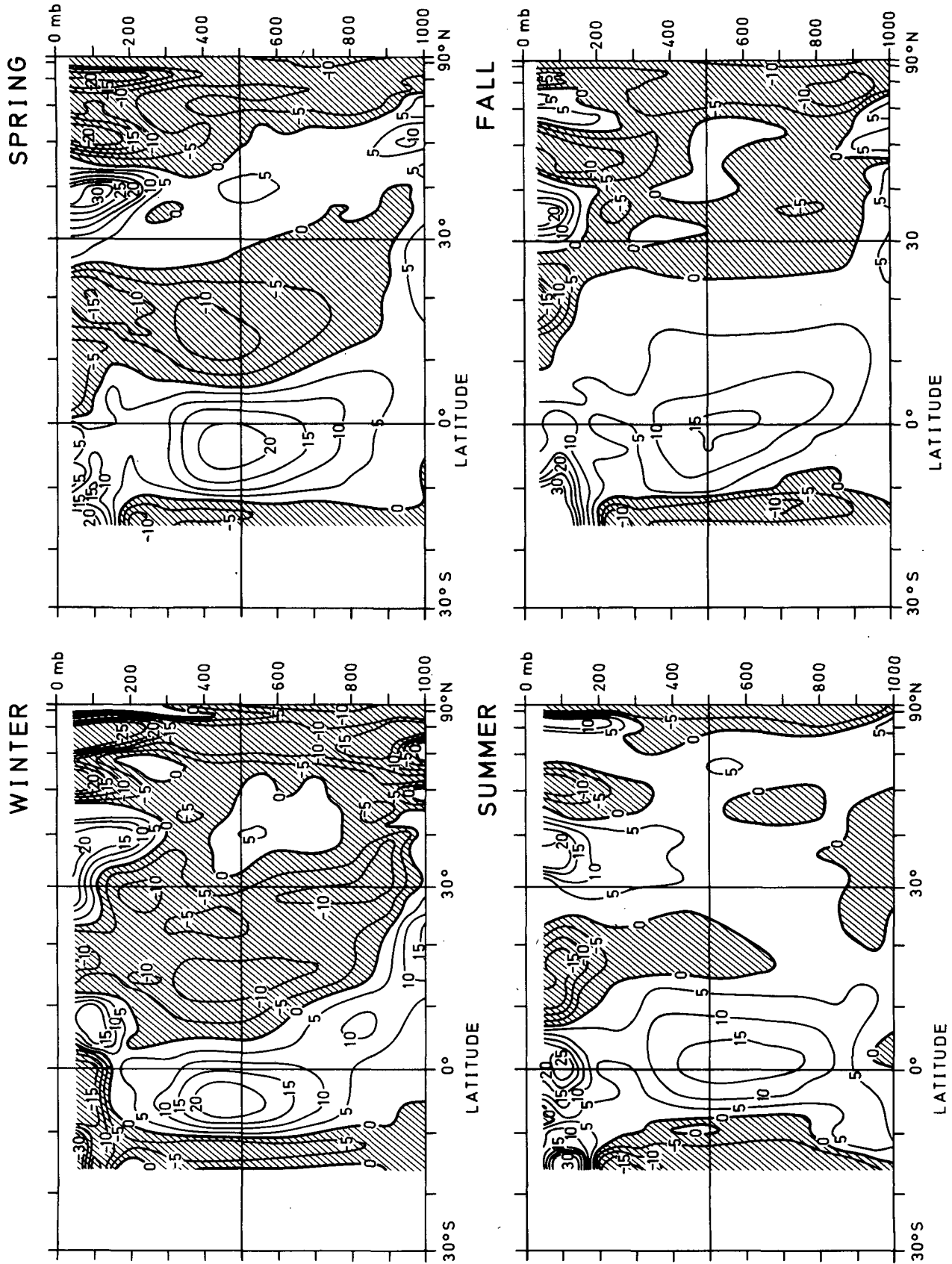


FIG. 3. Net diabatic heating ( $10^{-6} \text{ K s}^{-1}$ ) of zonally averaged northern atmosphere for the seasons determined from synoptic general circulation data.

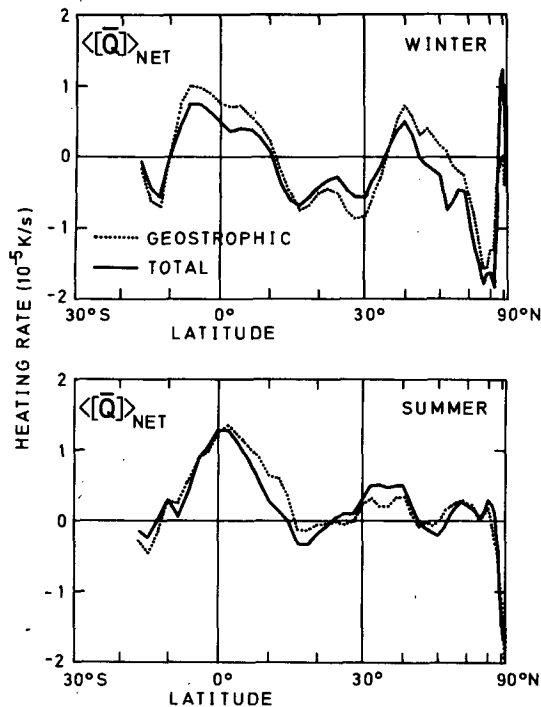


FIG. 4. Net diabatic heating of zonally and vertically averaged northern atmosphere for extreme seasons. Full curves, vertical averages from Fig. 3; dotted curves, independent compilations from potential heat equation. Difference between curves is estimate of error.

from the left of the vertically averaged Eq. (2.4) for potential heat. The main difference between the two versions is that the averaged Eq. (2.4) requires no vertical fluxes: mean plus eddy vertical fluxes at the earth's surface are zero on the synoptic scale. On the other hand, it requires, in addition to sensible heat, horizontal fluxes of geopotential. This makes estimates of  $Q$  based on either (2.4) or (2.13) sufficiently independent.

Fig. 2 indicates that the patterns of the total and of the geostrophic approximation to  $Q_{NET}$  are relatively similar. We consider the difference as a first estimate of the error of our compilation (see also Fig. 4). This error is partly due to the systematic truncations made in our approximate equations and partly due to data inconsistencies.

The latter we may estimate by employing the 10% adjustment which was necessary to balance the meridional and vertical velocity fields. Since  $\omega$  determines the mean adiabatic heating which is the most important single term on the left of (2.13) we conclude that the accidental error of  $Q_{NET}$  is less than 10% or less than  $10^{-6} \text{ K s}^{-1}$ .

Fig. 3 shows  $Q$  for the seasons. We note the following large-scale details:

1. The tropical zone between about  $10^{\circ}\text{S}$ – $10^{\circ}\text{N}$  is throughout the year a region of considerable heating, up to 20 units ( $10^{-6} \text{ K s}^{-1}$ ).

2. The subtropics are regions of net cooling, most pronounced in winter and spring, up to 10 units. This is visible in both hemispheres.

3. The midlatitudes are characterized by a more irregular pattern. By and large, there is tendency for net heating in the troposphere, up to 5 units, most pronounced in spring and summer.

4. The polar cap is characterized by net cooling north of about  $50^{\circ}\text{N}$  (except in summer where the cooling begins at  $70^{\circ}\text{N}$ ), with highest intensity in winter.

This four-latitude belt pattern is in accord with the annual distribution of  $Q$  in Fig. 2 and has already been apparent in the vertical mean results compiled from N1974 (see Fig. 1). We might explain it qualitatively with the combined action of radiation and latent heating.  $Q_{RAD}$  is negative throughout most of the atmosphere (Dopplack, 1972; Falconer and Peyinghaus, 1975) and exhibits a fairly smooth pattern with little latitudinal structure. What causes the latitudinal structure in Figs. 2 and 3 is the condensation heating  $Q_{LH}$  with maxima in the climatological rainbelts and minima in the dry zones (Hantel and Langholz, 1977). Consequently,  $Q_{LH}$  dominates in the tropics (maximum located on the summer hemisphere) and to a lesser degree in the mid-latitudes.  $Q_{RAD}$  dominates in the subtropics (most pronounced during the dry season) and over the polar cap.

Fig. 3 differs from N1969 and N1974 in a number of details. To give a first example, N1974 reports cooling in the 700–1000 mb layer over the tropics and subtropics in all seasons while our data indicate moderate heating in this layer. This discrepancy can be solved, in favor of N1974, by the following speculation. Suppose for a moment we would employ the  $Q_{BLH}$  of N1974 as a substitute for  $Q_{SUBSYN}$ . We would have to subtract it on both sides of Eq. (2.13). The resulting pattern would be an estimate of  $Q$ , not of  $Q_{NET}$ ; let us refer to it as Fig. 3'. Since  $Q_{BLH}$  is positive over almost the entire hemisphere throughout the year (of the order  $1$ – $2 \text{ K day}^{-1}$ ) we would get cooling in our speculative Fig. 3' in the boundary layer of about  $10$ – $20 \times 10^{-6} \text{ K s}^{-1}$ . This result could be compared with N1974 after also subtracting  $Q_{BLH}$ . Both estimates would now end up with net diabatic cooling in the boundary layer (still with considerable latitudinal structure).

The result of net negative heating in the boundary layer now seems quite meaningful when we keep in mind the considerable cooling which must occur in the lowest atmospheric layers through evaporation of falling rain. Nevertheless we maintain that the uncertainties of  $Q_{BLH}$  are too serious to allow a quantitative estimate. The speculation above remains to be tested with observed data.

A second conspicuous example are the strong heating regions in Fig. 3 in the  $30$ – $40^{\circ}\text{N}$  belt in the upper

troposphere and stratosphere throughout the year. They are in contrast to N1969 and N1974 notably in the upper troposphere in spring and summer, when Newell *et al.* show cooling. These regions seem to be related to the subtropical jet. It is difficult to explain such strong heating (with values above  $2 \times 10^{-5} \text{ K s}^{-1}$ ) with radiation or condensation processes. We shall speculate about a possible mechanism in the next section.

The vertically averaged net heating in the extreme seasons is shown in Fig. 4, in the same format as the annual pattern of Fig. 2. The independent dotted curves on the basis of the geostrophic assumption show reasonable agreement.

4. Discussion

When averaged over the mass of the entire northern atmosphere (i.e.,  $p$  average from  $p=0$  down to surface pressure, denoted by angle braces;  $\eta$  average from the equator to the North Pole, denoted by a tilde the First Law (2.13) reads

$$\begin{aligned} & \textcircled{1} \quad \textcircled{2} \quad \textcircled{3} \\ & \frac{\partial \langle [\tilde{T}] \rangle}{\partial t} - a^{-1} \langle [\tilde{v}] [\tilde{T}] \rangle_{\text{EQ}} - a^{-1} \langle [\tilde{v}^E T^E] \rangle_{\text{EQ, SYN}} \\ & \quad \textcircled{4} \quad \textcircled{5} \\ & \quad - \kappa \langle [\tilde{\omega}] [\tilde{T}] / p \rangle - \kappa \langle [\omega^E T^E]_{\text{SYN}} / p \rangle \\ & \quad \textcircled{6} \\ & = \kappa \langle [\omega^E T^E]_{\text{SUBSYN}} / p \rangle \\ & \quad + c_p^{-1} \langle [\tilde{Q}] \rangle = c_p^{-1} \langle [\tilde{Q}] \rangle_{\text{NET}}. \quad (4.1) \end{aligned}$$

The terms on the left along with their sum are presented in Table 2 for the seasons and the year. Dominating terms are as follows: first, the cross-equatorial flux  $\textcircled{2}$  which makes a positive contribution in winter and a negative in summer due to the gov-

erning influence of the cross-equatorial branches of the Hadley cell (Oort, 1971); and, second, the adiabatic term  $\textcircled{4}$  which makes a negative contribution in winter and a positive in summer, due to the governing influence of the downward or upward branches of the Hadley cell.

Terms of minor importance in the  $Q_{\text{NET}}$  balance are the storage term  $\textcircled{1}$  and the vertical synoptic eddy flux term  $\textcircled{5}$ , both of 0.1–0.3 units. The synoptic cross-equatorial eddy flux term  $\textcircled{3}$  is below 0.1 unit in all seasons (Oort, 1971) and is not explicitly listed. The boundary value at the earth's surface of the vertical advective heat flux is clearly zero since there is no advective flux across a solid boundary. It follows that the vertical integral of  $(\partial/\partial p)[\omega T]$  must vanish in every single term, synoptic or subsynoptic. The heat exchange across the earth's surface is conductive and thus implicitly contained in  $Q$ . Therefore it does not need to be specified for our present purpose. Note that this argument is not touched by the practical identity of the conductive cross-surface flux and the vertical sensible heat flux in the constant flux layer immediately above the earth's surface (Budyko, 1963).

The main result of this study, the patterns of Fig. 3, cannot be directly compared with independent data. The reason has been stated above: the two governing components of the diabatic heating,  $Q_{\text{RAD}}$  and  $Q_{\text{LH}}$ , are not available from measurements in their vertical profile but must be parameterized. Even their vertical average is not very reliable, mostly due to erroneous surface data. We have discussed our results in comparison with the parameterized estimates of N1969 and N1974. When taking the difference between Figs. 1 and 4 we arrive at about  $2 \times 10^{-6} \text{ K s}^{-1}$ . We consider this the approximate error of our compilations.

An indirect check of our detailed Fig. 3 is nevertheless possible by employing additional information. For example, Hantel and Langholz (1977) combined a  $Q_{\text{NET}}$  estimate of the present type with radiation calculations from a model (Hantel and Peyinghaus, 1976) plus Budyko heat flux to arrive at the condensation heating in form of the vertical *precipitation flux*.

TABLE 2. Northern atmosphere diabatic heating budget, in units of  $10^{-6} \text{ K s}^{-1}$ . Single components from Eq. (4.1).  $Q_{\text{NET}}$  is average from left sides of Eqs. (2.13) and (2.14). This results in imbalance with sum of  $\textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} + \textcircled{5}$ ; imbalance is of order 0.3 unit and was lumped into term  $\textcircled{6}$ . "Mean" is calculated from seasonal, "Year" from annual data.

Component	Winter	Season			Mean	Year
		Spring	Summer	Fall		
$\textcircled{1}$ $\partial \langle [\tilde{T}] \rangle / \partial t$	-0.1	0.4	0.1	-0.4	0.0	0.0
$\textcircled{2} + \textcircled{3}$ $-a^{-1} \langle [\tilde{v} T] \rangle_{\text{EQ, SYN}}$	3.3	1.7	-3.4	-1.8	-0.1	-0.1
$\textcircled{4}$ $-\kappa \langle [\tilde{\omega}] [\tilde{T}] / p \rangle$	-5.5	-2.9	5.6	2.9	0.1	0.0
$\textcircled{5}$ $-\kappa \langle [\omega^E T^E]_{\text{SYN}} / p \rangle$	0.3	0.2	0.0	0.1	0.2	0.1
$\sum_{i=1}^5 \textcircled{i}$ $c_p^{-1} \langle [\tilde{Q}]_{\text{NET}} \rangle$	-2.1	-0.6	2.3	0.9	0.2	0.1



Their fields compared reasonably well with independent surface precipitation observations. The most pronounced discrepancies were found in the extratropics in winter where Hantel and Langholz overestimated the surface precipitation by about 50%. They partly attributed this result to the tendency of the radiosonde network to underestimate peak horizontal wind velocities and further to a tendency of the precipitation measurements to be too low in winter. We shall now offer a further, and more plausible, explanation on the basis of the strong heating at and above jet stream levels in Fig. 3.

Considering the strong vertical  $\theta$  gradient at and above the tropopause we speculate that there might be a downward heat flux of eddy potential temperature  $[\overline{\omega^E \theta^E}]_{\text{SUBSYN}}$  from the upper stratosphere into the lower stratosphere and upper troposphere. If this heat flux is of quasi-turbulent nature on the meso-scale, it should be maximum in regions of excessive vertical wind shear, and should converge above the 200 mb level, i.e., above the core of the jet. Combining the left of (2.12) with the potential temperature to obtain

$$(p/p_0)^\kappa \frac{\partial [\overline{\omega^E \theta^E}]_{\text{SUBSYN}}}{\partial p} = -[\overline{Q_{\text{SUBSYN}}}] / c_p, \quad (4.2)$$

we see that a vertical heat flux of only  $20 \text{ W m}^{-2}$ , provided it converges within a layer 100 mb thick, would account for heating rates of  $2 \times 10^{-5} \text{ K s}^{-1}$  in this layer and could explain the positive regions in Fig. 3.

This effect would also lead to a pronounced overestimate of  $Q_{\text{LH}}$  in jet stream latitudes. The winter precipitation excess found by Hantel and Langholz was about  $5 \text{ g cm}^{-2} \text{ month}^{-1}$ . This corresponds to a downward vertical heat flux of even  $50 \text{ W m}^{-2}$  and fits well into the speculative assumption of considerable subsynoptic downward fluxes converging at and above jet stream levels and confined to jet stream latitudes.

The results of this study may be summarized as follows:

1) The total diabatic heating of the seasonal and zonal mean northern atmosphere has been estimated by specifying synoptic heat flux data in the First Law. The basic assumption has been that sensible heat synoptic advective plus storage terms balance  $Q$ . This indirect method is complementary to the approach of previous investigators who obtained  $Q$  by parameterization of its components, notably radiation and condensation.

2) The vertical-meridional pattern of  $Q$  is characterized by heating in the tropics and midlatitudes and cooling in the subtropics and over the polar cap due to the combined action of radiation and latent

heat release. This four-latitude belt pattern of diabatic heating climate is principally in accord with, but in details different from, previous estimates.

3) A systematic error in the present approach is that subsynoptic-scale vertical advection could not be specified or parameterized. We have speculated that  $Q_{\text{SUBSYN}}$  may be sizable at and above jet stream levels. For its overall order we have estimated 10% of  $Q$ . This constitutes the main error of the method.

4) The other errors are inaccuracy of synoptic data (mean and eddy) which is most influential in adiabatic term, due to governing role of Hadley cell; non-coincidence of surface pressure and the 1000 mb surface; neglect of subsynoptic horizontal eddy fluxes; systematic errors in parameterization of synoptic vertical eddy fluxes; and truncation error due to vertical and meridional integration procedures, apparent in adjustments necessary to fit  $[\bar{v}]$ - and  $[\bar{\omega}]$ -fields.

This study could be repeated with more confidence when sensible heat flux data become available for the southern atmosphere which should have the same accuracy standards as the ones presently available for the northern atmosphere. In the long-term averages of the components of (4.1) the large cross-equatorial fluxes would then vanish and (4.1) would reduce to the following balance (the tilde denotes the average from the South Pole to the North Pole):

$$-\kappa \langle \{ \overset{\textcircled{4}}{[\bar{\omega}]} [\bar{T}] + \overset{\textcircled{5}}{[\overline{\omega^E T^E}]_{\text{SYN}}} + \overset{\textcircled{6}}{[\overline{\omega^E T^E}]_{\text{SUBSYN}}} \} / \bar{p} \rangle = c_p^{-1} \langle [\bar{Q}] \rangle. \quad (4.3)$$

The global average of  $Q_{\text{RAD}}$  plus  $Q_{\text{LH}}$  plus the flux divergence part of  $(Q_{\text{COND}} + Q_{\text{F}} + Q_{\text{D}})$  must be zero. The global average of the irreversible remainder would be determined by the global dissipation. The dissipation rate has been placed by various authors (e.g., Lorenz, 1967; Oort and Peixoto, 1974) at about  $2 \text{ W m}^{-2}$ , corresponding to an overall heating rate of  $2 \times 10^{-7} \text{ K s}^{-1}$ . Thus with reliable estimates of  $\textcircled{4}$ ,  $\textcircled{5}$  and  $\textcircled{7}$  it should eventually be possible to arrive at a global estimate of the subsynoptic advection  $\textcircled{6}$ . This might also be of interest for the available potential energy budget (Oort, 1964).

The principal limitation of this study, which it shares with previous investigations, is the nonseparability of  $Q_{\text{SUBSYN}}$ . Like others, we have been forced to lump this term into  $Q$  to yield a net diabatic heating which is different from  $Q$ . The consequence is that the degree to which  $Q_{\text{NET}}$  in Fig. 3 is an approximation of  $Q$  remains unknown. We have indicated that the so-called boundary layer heating term  $Q_{\text{BLH}}$  is only a part of  $Q_{\text{SUBSYN}}$ ; the rest is probably not

negligible, but its exact pattern is still a matter of speculation. Thus we could not adopt existing parameterizations of  $Q_{BLH}$  as a substitute for  $Q_{SUBSYN}$ .

Let us consider this shortcoming from another viewpoint. The estimates in Table 1 suffer from the non-identity of heat flux and heat flux convergence. Although the heat flux across the earth's surface can be reasonably well estimated, the thickness of the layer in which the flux converges is a matter of speculation. Newell *et al.* decided to let the flux converge in the 1000–850 mb layer. This is equivalent to neglecting any subsynoptic heat flux above 850 mb.

For practical applications it might be irrelevant whether one estimates  $Q$  or  $Q_{NET}$ . From a theoretical position it seems preferable to have  $Q$  since  $Q$  comprises all physical processes which cannot be described by temperature advection (and storage). It is hoped that the gap between the synoptic and the micro-turbulent scale will eventually be closed so that we get reliable measurements of  $Q_{SUBSYN}$  and, therefore, quantitative estimates of the diabatic heating  $Q$ .

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