A Model of Katabatic Winds

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ABSTRACT

A new model of katabatic winds is presented. A hydraulic approach is employed in which the detailed vertical structure of the flow is replaced by a quiescent stably stratified environment and an equivalent flowing layer which is subject to sustained layer cooling, surface stress and interfacial entrainment. A scaling which contains most of the parametric behavior is found. It shows that interfacial entrainment is the dominating retardation mechanism of the flow and that surface stress may be relatively unimportant.

Steady solutions are presented to show that katabatic winds are essentially supercritical on all practical slopes (slope angles $>0.1°$), and are affected by ambient stratification only at large distances. The model is in satisfactory quantitative agreement with the limited field data available.

1. Introduction

Katabatic winds are local winds due to downslope flow of air which is continuously cooled by radiative processes near the ground. They provide the major ventilation mechanism in mountainous regions at night when synoptic pressure gradients are slack. This paper presents a new model of these flows. An understanding is required of the development of katabatic flow with distance and time, the mass of air involved, and the mixing within the flow and between the flow and the ambient air. Then the properties of such a flow may be predicted and its effect on the transport and dispersion of participating air may be assessed.

Thyer (1966) has reviewed the historical development of the subject. There have been three essentially separate approaches. The analytic approach taken by Prandtl [1942; see Sutton (1953)] regards the katabatic wind as being steady and invariant with position along the slope. By taking exchange coefficients to be constant and the slope to be small the equations of motion can be solved to obtain the velocity and temperature fields as functions of distance normal to the slope. The predicted functional form for the velocity profile agrees well with experiment (Defant, 1949) but quantitative comparison reduces to a determination of the exchange coefficients. Other writers have extended Prandtl's approach to include diurnal variations, or height-dependent exchange coefficients or nonstationary conditions (Gutman, 1953), or have included the Coriolis force as well (Gutman and Malbakhov, 1965; Lykosov and Gutman, 1972; see also Gleeson, 1953). In each case quantitative comparison with experiment is limited.

The second approach involves a numerical gridpoint solution of the full primitive equations or a subset of them. This has the advantage of allowing the inclusion of the important advection processes. Only two papers appear to have taken this course. The first, by Thyer (1966), is complex but achieves little in developing an understanding of the physics involved. Fair agreement between some observations of wind and the model is achieved. Leslie and Smith (1974) explore numerically the effectiveness of katabatic circulations in dispersing an atmospheric pollutant but make no appeal to observations.

The third approach to understanding katabatic flow was first taken by A. Defant [1933, quoted by F. Defant (1949)] and later by Fleagle (1950). They considered only the "average" flow within an identified cooled layer. All internal structure of the flow was eliminated and only variations with time were considered. Petkovsek and Hocevar (1971) extended Fleagle's ideas to include a stable background temperature stratification but in so doing obtained the anomalous result that predicted katabatic velocity becomes infinite as the ambient stratification approaches adiabatic. Streten *et al.* (1974) have recently compared their wind observations with the model of Petkovsek and Hocevar, finding "some degree of similarity," and came to the conclusion that more detailed observations are required. For a model of drainage flow down a slope from a cold source (as distinct from our definition of katabatic flow for which cooling is maintained along the slope), Ball (1956) took a similar approach to Fleagle's, drew on the similarity with the theory of open-channel hydraulics, and took the flow to be steady and to have

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a thickness which was to be determined. While not allowing the flow to mix with the ambient air (assumed to be adiabatic and at rest), Ball included advection and Coriolis effects. His work has been successful in describing some aspects of the intense surface winds which occur along the east Antarctic littoral (Ball, 1957, 1960).

In a study of turbulent entrainment in stratified flows, Ellison and Turner [1959; see also Turner (1973, Chap. 6)] extended Ball's hydraulic model of a drainage flow to include entrainment of ambient fluid into a turbulent flowing layer. Laboratory experiments were conducted to determine the entrainment physics, and it was shown how this might be included in a model of a drainage flow. However, it seems that the approach has not been developed except for recent work by Smith (1975) relating to oceanic gravity currents on the continental shelves.

Each approach has its limitations. A detailed analytical description of the vertical structure of katabatic flows is only possible through omission of advection processes and restrictive assumptions about exchange coefficients. Analytic descriptions of the temporal and downslope development of the flow do not appear attainable. Therefore, numerical grid-point simulations, while still requiring parameterization of mixing processes and having limited vertical resolution, are necessary for a full description of the flow. The parameterization of mixing has been a major impediment to achieving an understanding of katabatic flows by this approach, but development of higher order turbulence models may improve the situation (e.g., Brost and Wyngaard, 1978). The extended hydraulic approach, while lacking a detailed treatment of the vertical structure, does offer a convenient simplification, providing a framework within which to study the dynamics of katabatic flow and some parameterizations necessary for grid-point modeling.

To realize the potential of this third approach in modeling katabatic flows, in Section 2 we present a generalization which includes time dependence, down-slope development of the two-dimensional flow by turbulent entrainment of environmental air (which may be stably stratified and is taken to be at rest), radiation cooling of the flowing layer and surface stress. The detailed vertical structure of the flow is absorbed into three profile factors which are assumed known. There are no "free" parameters in the model. The entrainment formulation is specified by appeal to laboratory and related field experiments, and all other parameters may be specified independently.

The model admits steady solutions and their nature is explored in Section 3. Several possibilities exist, associated with different limits, including an analytic solution for weak to neutral ambient stratification. Numerical solution is required for other cases and the results are discussed.

The recent field study by Streten *et al.* (1974) appears to be the most comprehensive study of katabatic flow published and in Section 4 the results are compared with predictions of the new model. Finally, we draw attention to the possibility of strong modifying influences due to ambient winds.

2. A generalized hydraulic model of katabatic flow

a. Derivation of katabatic flow equations

Consider the flow of cool air down a long uniform slope inclined at an angle α to the horizontal as shown in Fig. 1. Axes centered at the crest of the slope are taken along the flow (s direction) and normal to it (n direction) so that there is no mean motion perpendicular to the plane (s,n) in the cool air. The flow may be regarded as being constrained by distant walls to the slope so that lateral variations need not be considered and the flow may be treated as two-dimensional. There is imposed by processes of no concern here a constant ambient stable stratification with a

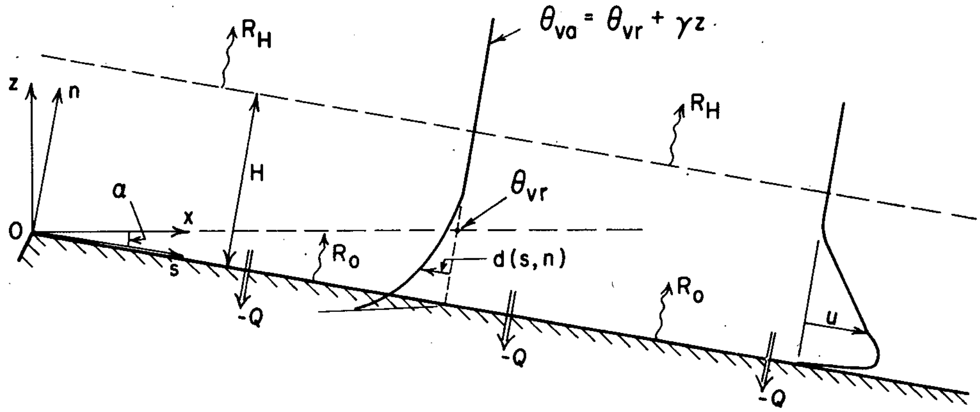

FIG. 1. Schematic of an idealized katabatic flow in a stably stratified environment.

buoyancy frequency N defined by

$$N^2 \equiv \frac{g}{\theta_{vr}}\frac{d\theta_{va}}{dz} \equiv \frac{g}{\theta_{vr}}\gamma. \tag{2.1}$$

Here z is the vertical coordinate, θ_{va} the potential virtual temperature field of the ambient air, θ_{vr} the reference value at O (see Fig. 1), and g the acceleration due to gravity. The ambient air is assumed to be at rest except for possible perturbations due to the downslope flow itself.

The slope and air are cooled by longwave radiation to space. There results a heat flux $\dot{Q}$ from the adjacent air into the ground; this flux together with radiation divergence within the adjacent air layer increases the density of the air relative to the environment at the same level up to some height small compared with the length of the resulting slope flow. Heights of no more than a few hundred meters are relevant so the air may be regarded as a Boussinesq fluid (e.g., Gray and Giorgini, 1976), justifying the use of θ_{vr} in (2.1). In (s,n) coordinates the potential virtual temperature distribution is

$$\begin{aligned}\theta_v &= \theta_{vr} - \gamma(s\sin\alpha - n\cos\alpha) - d(s,n,t) \\ &= \theta_{va} - d(s,n,t),\end{aligned} \tag{2.2}$$

where $d(s,n,t)$ is the deficit in θ_v for the katabatic flow, relative to the ambient field θ_{va} and is taken to be a function of time t.

Coriolis forces will be neglected since except in some Antarctic applications the flow is not extensive enough for rotation to be important. Further, even in mountainous terrain extensive ground slopes are generally small, say, less than 10°, so that in direction n normal to the slope the fluid may be regarded as being in hydrostatic balance. Then the mean tangential and normal momentum equations for the thin two-dimensional katabatic flow are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s} + w\frac{\partial u}{\partial n} = -\frac{1}{\rho_r}\frac{\partial}{\partial s}(p - p_a) + g\frac{d}{\theta_{vr}}\sin\alpha - \frac{\partial}{\partial n}\overline{u'w'}, \tag{2.3a}$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho_r}\frac{\partial}{\partial n}(p - p_a) - g\frac{d}{\theta_{vr}}\cos\alpha \approx 0, \tag{2.3b}$$

where (u,w) are fluid velocities in directions (s,n), ρ_r (corresponding to θ_{vr}) is the reference density, p is the local pressure, p_a the ambient pressure field associated with θ_{va}, and $\overline{u'w'}$ is the only turbulent Reynolds stress considered to be of importance.

The thermal energy equation is

$$\frac{\partial\theta_v}{\partial t} + u\frac{\partial\theta_v}{\partial s} + w\frac{\partial\theta_v}{\partial n} = -\frac{1}{\rho_r C_p}\frac{\partial R}{\partial n} - \frac{\partial}{\partial n}\overline{w'\theta_v'}, \tag{2.4}$$

where R is the outgoing radiation flux and $\overline{w'\theta_v'}$ is the flux of potential virtual temperature at height n.

Continuity may be expressed as

$$\frac{\partial u}{\partial s} + \frac{\partial w}{\partial n} = 0. \tag{2.5}$$

The governing equations of motion [(2.3), (2.4), (2.5)] are simplified if the flow is treated as an extended hydraulic model in which the detailed structure is replaced by a layer with equivalent velocity, depth and buoyancy scales. Each of the equations [(2.3a), (2.4) and (2.5)] is integrated in the direction n through the flow from the surface $n=0$ out to a constant height H sufficiently distant so as to be largely unaffected by the katabatic flow—see Ellison and Turner (1959), Kotsovinos and List (1977) and Niiler and Kraus (1977). Then on substitution of twice-integrated (2.3b) (first from n to H and then from 0 to H) into the integrals of (2.3a) and since $u \to 0$ and $p \to p_a$ as $n \to H$ there results

$$\frac{\partial}{\partial t}\int_0^H u\,dn + \frac{\partial}{\partial s}\int_0^H u^2 dn = -\frac{\partial}{\partial s}\int_0^H g'n\cos\alpha\,dn + \int_0^H g'\sin\alpha\,dn + (\overline{u'w'})_0. \tag{2.6}$$

Eqs. (2.4) and (2.5) become

$$\frac{\partial}{\partial t}\int_0^H g'dn + N^2\sin\alpha\int_0^H u\,dn - N^2\cos\alpha\int_0^H w\,dn + \frac{\partial}{\partial s}\int_0^H ug'dn = B, \tag{2.7}$$

$$\frac{\partial}{\partial s}\int_0^H u\,dn + w_H = 0, \tag{2.8}$$

where w_H is the velocity normal to the slope at height H, $(\overline{u'w'})_0$ is the surface shear stress and $g' = gd/\theta_{vr}$. From (2.4) the term B, which is the flux of buoyancy from the cooled layer at any section, is given by

$$B = \frac{g}{\rho_r C_p \theta_{vr}}[(R_H - R_0) - \dot{Q}], \tag{2.9}$$

where $(R_H - R_0)$ is the radiation divergence over height H, $\dot{Q} = \rho C_p \overline{w'\theta_v'}$ is the surface heat flux and at $z=H$ the heat flux is zero.

Early in the temporal development of the downslope flow, i.e., while the air adjacent to the surface is near neutral, the surface heat flux dominates the layer cooling. $\dot{Q}$ is related to the net radiation flux from the surface and the ground heat flux by the surface energy balance, and may then be comparable to R_0 in magnitude (say, 50 W m^{-2} with a clear sky). Subsequently, as the thermal stratification of the downslope flow increases due to the sustained cooling, $|\dot{Q}|$ decreases. $\dot{Q}$ may then approach a value of no more than a few watts per square meter (Hicks, 1976; Kondo *et al.*, 1978). The radiation divergence $R_H - R_0$, which is a complex interactive function of the water vapor and temperature structure of the layer of air, becomes more important and probably dominates the layer cooling. For the present purpose B [Eq. (2.9)] is taken to be an imposed constant and no attempt is made to distinguish between or parameterize these interacting processes.

We now define scales of velocity U, thickness h and buoyancy deficit Δ by

$$Uh=\int_0^H udn, \qquad U^2h=\int_0^H u^2dn, \tag{2.10, 2.11}$$

$$U\Delta h=\int_0^H ug'dn, \qquad S_1\Delta h^2=2\int_0^H g'ndn, \tag{2.12, 2.13}$$

$$S_2\Delta h=\int_0^H g'dn, \qquad \int_0^H wdn=w_H\cdot H-S_3w_Hh. \tag{2.14, 2.15}$$

These scales make fluxes of volume, momentum and buoyancy deficit (per unit breadth of flow) identical for the layer flow and the actual katabatic flow. Profile factors S_1, S_2 and S_3 take the value unity for well-mixed katabatic flow. Actual values probably depend on the scale of the flow; Ellison and Turner (1959) found for laboratory-scale drainage flows average values of 0.25, 0.75 for S_1, S_2, respectively, with S_2 increasing to 0.9 at the highest Reynolds numbers studied. No detailed temperature and velocity data for katabatic flows which would enable computation of these profile factors appear to have been published. However, we have used the results of a recent field experiment (to be published) to estimate values of $S_1 \approx 0.5$ and $S_2 \approx 0.9$, but no data for S_3 could be obtained. Since, except in special cases of little practical importance, it will be shown that results are insensitive to the values of the profile factors, they are taken to be constants—a similarity assumption—and have values

$$S_1=0.5, \quad S_2=0.9, \quad S_3=1. \tag{2.16}$$

Then the group

$$S_4\equiv\frac{S_2}{S_1}S_3=1.8. \tag{2.17}$$

b. Closure of the equations

While eliminating the need to invoke constitutive relations as in the differential approach, the integrated layer approach has introduced two boundary functions, *viz.*, $(\overline{u'w'})_0$ and w_H. These must be parameterized before the scales (2.10)–(2.15) are incorporated.

1) THE SURFACE STRESS

The usual quadratic drag formulation for the surface stress,

$$-(\overline{u'w'})_0=C_DU^2, \tag{2.18}$$

will be used here, although C_D is strictly not well defined because U is not a velocity measurable at a predetermined level above the surface. However, the field observations mentioned above show that U and u at, say, 10 m elevation are little different in practice.

In using relation (2.18), Ball (1960) took $C_D \approx 5 \times 10^{-3}$ to fit observed Antarctic katabatic flows, but from the data of Lettau (1966) $C_D \approx 1 \times 10^{-4}$ may be more realistic there. Now it is certainly the case that values of C_D based on conditions of neutral stability are improbably large when an intense surface inversion is present as happens during katabatic flow. As Carson and Richards (1978) have shown, C_D may be expressed as a function of a bulk Richardson number Ri_B in stable conditions and then C_D may be no larger than about 3×10^{-4} for $\mathrm{Ri}_B > 0.3$, regardless of surface roughness. As exemplified by Hicks (1976), clear-sky light-wind nocturnal values of Ri_B are of this order or larger and support the conclusion that the flow may be dynamically isolated from the surface.

2) THE ENTRAINMENT ASSUMPTION

All the fluid participating in katabatic flow is entrained from the environment. The resulting small induced ambient velocity has been observed to be a feature of katabatic flows in the Mirny region of Antarctica (Tauber, 1960). It may be parameterized as is done in the theory of plumes and turbulent interfaces by the *entrainment assumption* (Turner, 1973, Chaps. 6 and 9). This states that the velocity of inflow into the turbulent downslope flow is proportional to the velocity scale of the layer, i.e.,

$$w_H=-EU. \tag{2.19}$$

Ellison and Turner (1959) made this assumption in interpreting their laboratory study of turbulent entrainment into inclined wall plumes. It was found that E was a function of only the layer Richardson number Ri which in turn was a function of the slope of the surface. The layer Richardson number is defined here as $\mathrm{Ri}=\Delta h \cos\alpha/U^2$. The term $S_1\mathrm{Ri}$ is an inverse internal Froude number of the flow and is of order unity for katabatic flows.

Several fundamental studies have been made of the entrainment parameter in contexts similar to the present. In reviewing these, Long (1975) strongly argues for a relation of the form $E=A/(S_1\,\mathrm{Ri})$, where A is a constant and $\mathrm{Ri}>0$. It has a simple energetics interpretation. The change in potential energy of the layer by incorporation of fluid is proportional to the turbulent kinetic energy made available by shear production at the interface between layer and environment. More detailed parameterizations of the turbulent kinetic energy equation are used by modelers of the atmospheric boundary layer (e.g., Stull, 1976; Zeman and Tennekes, 1977) and the oceanic thermocline (see Niiler and Kraus, 1977). They reduce to approximately the above form except in certain limits, one of which is of concern here. As $\mathrm{Ri}\rightarrow 0$, E must approach an upper bound which the review by Turner 1973, Chap. 9) indicates to be $E_0\approx 0.1$. Thus for E we will take the form

$$E=A/(S_1\,\mathrm{Ri}+K). \tag{2.20a}$$

Following Hansen (1975), we take A to be 2×10^{-3}; it is probably correct to within a factor of 2. Hence

$$K\equiv A/E_0=2\times10^{-2}. \tag{2.20b}$$

It may be observed that (2.20) fits Ellison and Turner's (1959) experimental results well, with A perhaps slightly larger than 2×10^{-3}.

The existence of the small normal velocity w_H induced in the environment by entrainment into the katabatic flow has a further consequence not yet accounted for when the ambient air is stably stratified. For then any sufficiently weak velocity induced in the environment will, after an initial potential flow response lasting for a time of order N^{-1}, be directed parallel to the isotherms (more strictly, isentropes), which in turn will remain closely horizontal (e.g., Brighton, 1978). The components u_H, w_H in direction s,n, respectively, of this induced velocity are connected by the relationship

$$u_H\sin\alpha=w_H\cos\alpha. \tag{2.21}$$

Then, more correctly,

$$\int_0^H u\,dn=Uh+u_H H \tag{2.22}$$

in place of (2.10) and U is thus the characteristic flow velocity relative to the induced ambient velocity u_H. But it can be shown using the normalization below (Section 2d) that $u_H\ll U$ except in the limit of very small α or very large s. So, for H not large compared with h the second term on the right of (2.22) may be neglected and relations (2.10) *et seq.* used everywhere except in the buoyancy equation (2.7) where, from (2.21), $u_H H\sin\alpha$ is the same magnitude as $w_H H\cos\alpha$ and so there (2.22) must be explicitly employed.

c. The full model equations

Substitution of (2.10)–(2.14) and (2.18) into (2.6) gives for the momentum equation

$$\frac{\partial}{\partial t}Uh=-\frac{\partial}{\partial s}U^2h-\frac{\partial}{\partial s}(\tfrac{1}{2}S_1\Delta h^2\cos\alpha)+S_2\Delta h\sin\alpha-C_DU^2. \tag{2.23}$$

Upon substitution of (2.12), (2.14), (2.15), (2.19), (2.21) and (2.22) the buoyancy equation (2.7) becomes

$$\frac{\partial}{\partial t}S_2\Delta h+UhN^2(\sin\alpha-S_3E\cos\alpha)+\frac{\partial}{\partial s}U\Delta h=B. \tag{2.24}$$

From (2.8) and (2.19) continuity is

$$\frac{\partial}{\partial s}Uh=EU, \tag{2.25}$$

with

$$E=A/(S_1\,\mathrm{Ri}+K),\quad \mathrm{Ri}=\Delta h\cos\alpha/U^2. \tag{2.26}$$

Eq. (2.25) shows that the flux of fluid in the katabatic flow increases with distance by entrainment of ambient fluid at a rate proportional to the velocity of the layer and approximately inversely proportional to the layer Richardson number. The momentum equation (2.23) expresses the time dependence of this flux in terms of the various accelerations acting. The last term on the right of (2.23) is the turbulent surface shear stress which, from Section 2b1, is likely to be very small; the third term is the acceleration of the layer due to its buoyancy deficit; the second term is the pressure gradient on the layer due to its changing depth. The first term on the right of (2.23) may be written, using (2.25), as $-EU^2-Uh(\partial U/\partial s)$, showing that it expresses both the momentum exchange between environment and the layer (an "interfacial" drag similar to the surface stress) and convection of the layer down the slope. Eq. (2.24) states that the time rate of change of the integrated buoyancy deficit of the layer is determined by the net flux of buoyancy from the layer, given by (2.9), the convergence along the slope of advected buoyancy deficit, and the effect of the ambient stability as the layer flows downslope.

d. Nondimensionalization

A nondimensionalization of the model equations (2.23)–(2.26) which also results in a satisfactory normalization of the equations so long as N and α are nonzero is obtained by the substitutions $x_*=x_Mx$, where henceforth x_* is a parameter, x_M its dimensional magnitude and x its dimensionless equivalent, and

$$\left.\begin{aligned} &U_M{}^2=\frac{S_2{}^{\frac{1}{2}}B}{E_MN}, \qquad \Delta_M{}^2=\frac{BN}{E_MS_2{}^{\frac{1}{2}}} \\ &h_M{}^2=\frac{BE_M}{S_2{}^{\frac{1}{2}}N^3\sin^2\alpha}, \qquad s_M{}^2=\frac{B}{S_2{}^{\frac{1}{2}}E_MN^3\sin^2\alpha} \\ &C=S_1\mathrm{Ri}_M=\left(\frac{S_1A}{S_2\tan\alpha}\right)^{\frac{1}{2}},\quad E_M=A/C=\left(A\frac{S_2}{S_1}\tan\alpha\right)^{\frac{1}{2}} \\ &t_M=1/(S_2{}^{\frac{1}{2}}N\sin\alpha) \end{aligned}\right\} \tag{2.27}$$

Then, the dimensionless model equations become

$$\frac{\partial}{\partial t}Uh+\frac{\partial}{\partial s}U^2h=-\frac{1}{2}C\frac{\partial}{\partial s}\Delta h^2+\Delta h-\frac{C_D}{E_M}U^2, \tag{2.28}$$

$$S_2\frac{\partial}{\partial t}\Delta h+\frac{\partial}{\partial s}U\Delta h=1-Uh(1-S_4CE), \tag{2.29}$$

$$\frac{\partial}{\partial s}Uh=EU, \tag{2.30}$$

$$E=1/(\mathrm{Ri}+K/C), \tag{2.31}$$

where

$$\mathrm{Ri}=\Delta h/U^2 \quad \text{and} \quad S_4=S_3\times S_2/S_1. \tag{2.32, 2.33}$$

If $\alpha=0$ a state of no motion exists so the relations (2.27) to (2.31) are then irrelevant, but if $N=0$ a nondimensionalization and normalization in terms of distance downslope s_* is appropriate. While there is no difficulty in setting down this normalization it proves unnecessary to do so here. (See Section 3b below.)

The following "typical" values give a guide to the magnitude of the various dimensional scaling parameters. Take a slope inclination $\alpha=5°$; an ambient stability $N^2=1\times10^{-4}$ s^{-2}, corresponding to the ICAO Standard Atmosphere; $B=2\times10^{-3}$ m^2 s^{-3}, corresponding to a layer cooling rate of 2°C h^{-1} over 100 m; and $C_D=0.0003$. Then with $S_1=0.5$, $S_2=0.9$, $S_4=1.8$ [from (2.16) and (2.17)] and $A=2\times10^{-3}$, $K=2\times10^{-2}$ as above, $E_M=0.018$, $C_D/E_M=0.017$, $\mathrm{Ri}_M=0.23$, $C=0.11$, $U_M=3.3$ m s^{-1}, $\Delta_M=0.035$ m s^{-2}, $h_M=70$ m, $s_M=4.0$ km and $t_M=20$ min. Note in particular that the length scale s_M for ambient stratification to become important in practice requires a long slope.

3. Steady katabatic flow

Because in most practical situations the time scale t_M is short, and steady conditions are probably achieved within several t_M, the properties of steady flows are of most interest and hence are considered in detail here.

a. Nature of solutions

The steady form of the model equations (2.28)–(2.30) can be rearranged to give an equation for the rate of change of the layer Richardson number along the slope:

$$(1-C\mathrm{Ri})\frac{h}{\mathrm{Ri}}\frac{d\mathrm{Ri}}{ds}=\frac{C_D}{E_M}-\mathrm{Ri}+E(1+\tfrac{1}{2}C\mathrm{Ri})$$
$$+\tfrac{1}{3}[1-Uh(1-S_4CE)](1+\tfrac{1}{2}C\mathrm{Ri})/(U\Delta). \tag{3.1}$$

If the fourth term on the right-hand side of (3.1) is omitted, this equation applies to drainage flow with interfacial entrainment and constant buoyancy flux (Ellison and Turner, 1959). If, in addition, the third term on the right of (3.1) is dropped, the equation applies to two-layer hydraulic flow with constant volume and buoyancy fluxes.

Clearly, the nature of the solutions to the full equations will include properties of the simpler cases mentioned (see, e.g., Turner, 1973, Chaps 3 and 6). Of particular importance is that the nature of the flow is largely determined by the relative size of $C\mathrm{Ri}$ compared to unity. If $C\,\mathrm{Ri}<1$ the flow is called "supercritical" or "shooting." The speed of interfacial waves is lower than the flow speed so the flow may be expected to be insensitive to local perturbations and to be characterized by breaking interfacial waves and strong mixing by entrainment. Any adjustment to changed conditions downstream is likely to be sudden and may be characterized by an internal hydraulic jump as the flow passes from $C\,\mathrm{Ri}<1$ to $C\,\mathrm{Ri}>1$. This property has been applied by Ball (1957) to the slope winds near Adélie Land.

If $C\,\mathrm{Ri}>1$ the flow is called "subcritical" or "tranquil," and with the speed of interfacial waves faster than the flow speed very little wave breaking and spatial growth by entrainment may be expected. The flow may readily adjust to conditions downstream.

b. Steady solution for neutral atmosphere, $N=0$

In this special case the term involving Uh in the buoyancy flux equation (2.29) drops out. This is most easily seen from the dimensional form of this equation, that shown in Eq. (2.21). Then simple power-law solutions satisfy the equations and the boundary conditions $h=0$, $Uh=0$, $\Delta h=0$ at 0:

$$h=\tfrac{3}{4}Es,\quad U^3=(s/\mathrm{Ri}),\quad \Delta=\tfrac{4}{3}E^{-1}(s/\mathrm{Ri})^{-\frac{1}{3}}, \tag{3.2–3.4}$$

with

$$\mathrm{Ri}^2-\left(\frac{5}{8}+\frac{C_D}{A}-\frac{K}{C^2}\right)C\mathrm{Ri}-\left(\frac{5}{4}+C_D\frac{K}{A}\right)=0$$

$$\text{and}\quad E=1/(\mathrm{Ri}+K/C). \tag{3.5, 3.6}$$

[Note that although the scale parameters (2.27) become singular for $N=0$, factors of N cancel when the solutions (3.2)–(3.6) are restored to dimensional form. Thus the value of N used in (2.27) is arbitrary in this case but the scaling still is useful for displaying the solutions.]

The katabatic flow thus grows in thickness linearly with distance downslope, increases in speed at an ever slower rate, and the buoyancy of the flow decreases slowly by entrainment of ambient air with increasing s. The entrainment rate and layer Richardson number are constants, characterized uniquely by the slope angle through C as Ellison and Turner found in their laboratory study of drainage flows. Note that as C is increased, the slope angle is decreased.

Now the boundary conditions imposed at the crest are of course physically unrealistic since the solutions (3.2)–(3.6) imply that the flow acceleration and buoyancy become infinite as $s \to 0$. At some distance downslope, however, the flow behaves as if it started at a "virtual point" upslope and the idea of a starting point where the thickness is zero remains a useful theoretical concept provided the solutions are applied, and are insensitive to the crest conditions, some suitably small distance downslope and beyond.

A characteristic of the solutions (3.2)–(3.6) is that as C is increased the surface stress through C_D becomes more important. However, for C_D values which are likely for katabatic flows (see Section 2b1), the effect of surface stress on the solutions is no more than a few percent for C as large as unity. As may be seen from relation (3.5) C_D would need to be greater than $O(A)$ to dominate the dynamics.

Since the solutions (3.2), etc., imply no restrictions on C let us find the critical slope angle delineating shooting flow and tranquil flow. Then $C\,\mathrm{Ri}=1$, so from Eq. (3.5), noting that $C_D \ll A$, $K \ll 1$,

$$C_{\mathrm{crit}} \approx \sqrt{\frac{8}{15}}\left(1-\frac{4}{15}\frac{C_D}{A}+\frac{1}{2}K\right). \qquad (3.7)$$

The solution (3.2)–(3.6) is thus shooting for $C \lesssim 0.7$ (slopes $>0.1°$) and tranquil for $C \gtrsim 0.7$.

Finally, the sensitivity of the solutions to disturbances is relevant. For $N=0$ and after substitution of (3.2), (3.6) into (3.1), the Richardson number equation reduces to

$$h\frac{d\mathrm{Ri}}{ds}=\frac{\left[\frac{C_D}{A}C\mathrm{Ri}-\mathrm{Ri}^2+\frac{5}{4}\left(1+\frac{1}{2}C\mathrm{Ri}\right)\right]}{(1-C\mathrm{Ri})}. \qquad (3.8)$$

We introduce a small perturbation $\epsilon_0 \exp(\sigma s)$ to Ri and linearize about the perturbation. It is readily found that $\sigma<0$ for $1>C\mathrm{Ri}$, and vice versa. Thus shooting flows, for which $C\mathrm{Ri}<1$, are mathematically stable and insensitive to small perturbations in the boundary conditions. For tranquil flows $C\mathrm{Ri}>1$ and the solutions are mathematically unstable and the response to small perturbations is an exponential growth away from the solutions (3.2)–(3.6). It is found that the flow stagnates with a balance in the model equation (2.28) between the pressure gradient and buoyancy terms. Just as in two-layer hydraulic flow discussed above, the interface becomes horizontal a short distance downstream from the crest, the distance being determined in practice by a downstream control. The situation is similar to that of a flooded internal jump (Wilkinson and Wood, 1971).

This stability result complements work by A. Defant [1933, quoted by F. Defant (1949)]. Defant proposed a model which requires surface stress to be the only retarding force. He found that there then exists a critical value of slope angle $\alpha_{\mathrm{crit}}=4\,C_D$ such that for $\alpha>\alpha_{\mathrm{crit}}$ the flow supports growing interfacial waves. These waves may break causing entrainment and growth of the layer as with the shooting flows discussed above. For $\alpha<\alpha_{\mathrm{crit}}$, interfacial waves do not grow and the flow is tranquil. He took $C_D \approx 0.0025$, so $\alpha_{\mathrm{crit}} \approx 0.6°$ (with $C_D \approx 0.0003$, $\alpha_{\mathrm{crit}} \approx 0.07°$); this is comparable with the critical slope for the present model, but of course obverse problems have been considered. The behavior at such small slope angles in any event is unimportant because in practice the slightest synoptic change would disrupt the flow.

We now return to the example of Section 2d. For $N=0$ we calculate from (3.2)–(3.6) the predicted flow 4.0 km from the slope crest. Since $\alpha=5°>0.1°$ the flow should be stable and shooting in nature. $E_*=0.0155$ and $\mathrm{Ri}_*=0.27$, $U_*=3.1$ m s^{-1}, $h_*=45$ m; $\Delta_*=0.057$ m s^{-2} which implies, if $\theta_{vr}=280$ K, an average temperature deficit in the layer of 1.6°C.

c. Solution for $N>0$

In this case the steady form of the full model equations has no simple analytic solution, but for small s the equations are satisfied by (3.2)–(3.6) since then the term containing Uh in the buoyancy flux equation (2.29) is relatively small. The equations are readily solved numerically with the above results used as starting solutions. The unstable nature of tranquil flow ($C>0.7$) is confirmed and a practically stagnant situation then occurs. For shooting flow conditions the solutions for $N=0$ are reproduced approximately for s as large as unity, while for large s the solutions are strongly influenced by the terms involving the ambient stratification in (2.29).

Figs. 2a–2d illustrate the numerical solutions for $U\Delta h$, Uh, U and Ri, respectively, as functions of s, for a number of values of C. The effect of C_D is only apparent for large C and large s. Also plotted for comparison are the analytic results (3.2)–(3.6). The

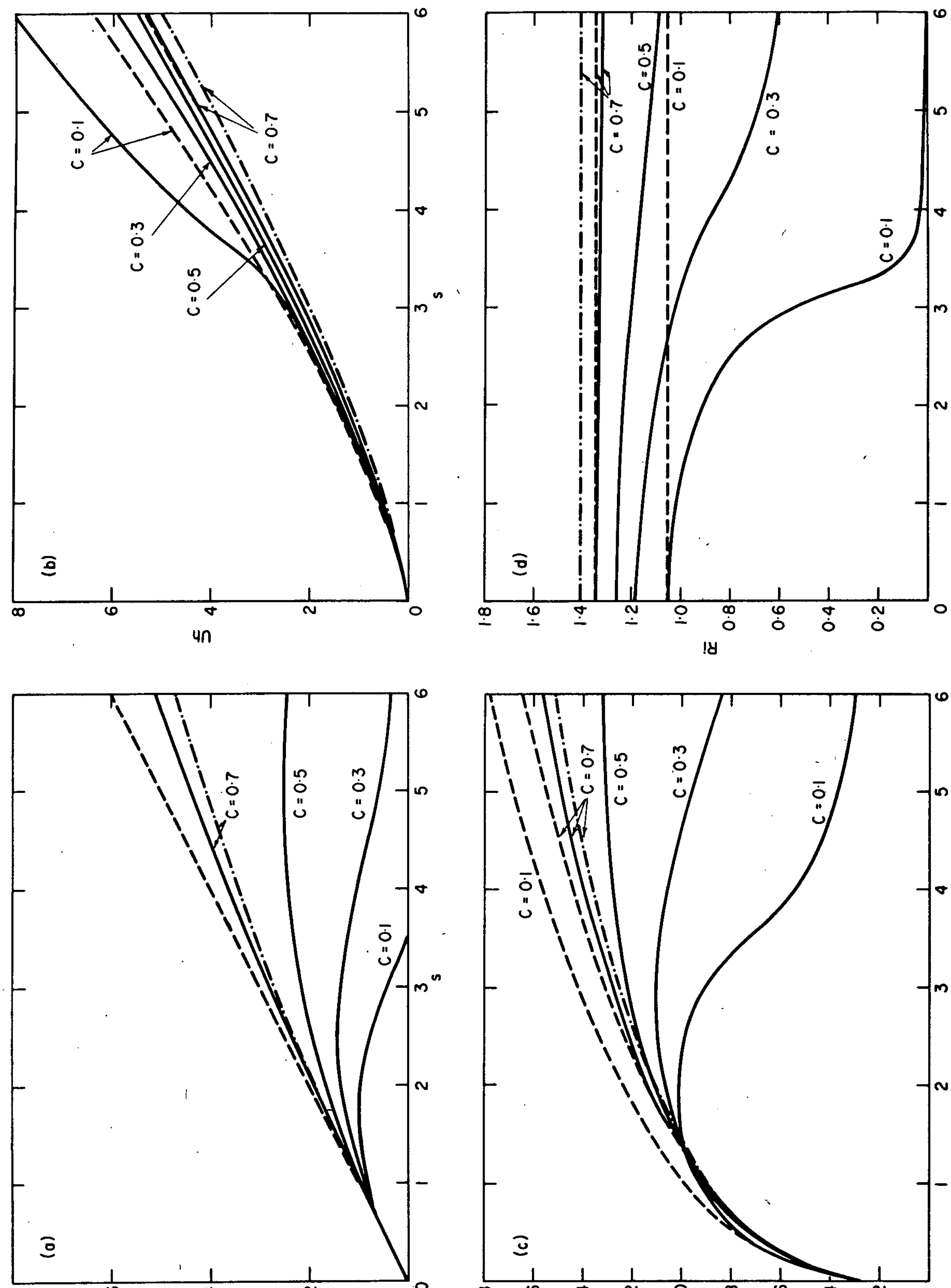

FIG. 2. Dimensionless buoyancy flux $U\Delta h$ (a), volume flux Uh (b), velocity U (c), and layer Richardson number Ri (d) as functions of dimensionless distance s for various values of C; dashed line: $N=0$, $C_D=0$; solid line: $N\neq0$, $C_D=0$; dashed-dotted line: $N\neq0$, $C_D=0.0003$.

scaling (2.27) is adequate and encompasses most of the parametric variation.

It may be observed in Fig. 2b that the volume flux per unit breadth increases more rapidly as the influence of the stable stratification in the environment is felt in the equations of motion. The layer Richardson number decreases (Fig. 2d) and hence the entrainment rate increases as s increases, resulting in this increase in Uh.

The effect of the ambient stratification is most pronounced on the flux of buoyancy deficit of the katabatic flow (Fig. 2a). As the cooled air progresses downslope it encounters denser ambient air and a larger proportion of the cooling is required merely to maintain a buoyancy deficit in the layer. This is expressed by the first part of the term involving Uh in (2.29). The flux of buoyancy deficit, which initially increases by layer cooling, reaches a maximum and thereafter decreases as Uh continues to increase. The smaller is C (steeper slope) the sooner is the flux reduced toward zero. At still larger s the second part of the term involving Uh in (2.29) becomes important. The entrainment of ambient air reduces the demand on the layer cooling to maintain the buoyancy deficit and the result is that $U\Delta h$ tends to zero as $s\rightarrow\infty$. A balance is achieved between the layer cooling and the flux of entrained cool ambient air, so from (2.29)

$$1\sim Uh(1-S_4CE) \quad \text{as} \quad s\rightarrow\infty, \qquad (3.8)$$

and since Uh diverges in this limit

$$CE\sim 1/S_4, \quad \text{or} \quad C\mathrm{Ri}\sim S_4C^2-K \quad \text{as} \quad s\rightarrow\infty. \qquad (3.9)$$

Since Ri is given by (3.5) for small s we see with (3.9) that Ri is in general a function of distance for a stably stratified environment, as is clear from Fig. 2d. Fig. 2c shows that the katabatic flow velocity U reaches a maximum before decreasing as the extra drag due to increased entrainment at larger s becomes effective. U decreases the more slowly the larger is s and this behavior sets in earlier for smaller C.

The details of the large s solution obviously depend on the value taken by S_4 which is not well known, as discussed in Section 2a. However, this is not important since 1) any practical slope may not be long enough for this limit to be of consequence, and 2) in any event the model has other deficiencies for large s, where the assumptions of horizontal ambient thermal field and constant lapse rate require unrealistically low ambient temperatures and where Coriolis forces may no longer be negligible.

We conclude this section by continuing with the example of Section 2d and computing the predicted katabatic flow properties for a stably stratified environment at distances of 4, 8 and 12 km from the crest of the slope. The results are summarized in Table 1.

TABLE 1. Katabatic flow properties for the example of Section 2d.

Parameters	$N=0$	$N=1\times10^{-2}$ s^{-1}		
s_* (km)	4	4	8	12
s	—	1	2	3
C	0.11	0.11	0.11	0.11
U	—	0.92	1.0	0.91
U_* (m s^{-1})	3.1	3.0	3.3	3.0
h	—	0.64	1.4	2.7
h_* (m)	45	45	95	186
Δ	—	1.4	0.71	0.19
Δ_* (m s^{-1})	0.057	0.048	0.025	0.0065
d (°C)	1.6	1.4	0.71	0.19

4. Application of model

a. Data requirements

As a minimum, the following data are required to apply and test the model:

(i) Physical characteristics of the slope including inclination, distance from a well-defined crest to the observation point(s), width and surface properties.

(ii) Ambient stratification and wind in the vicinity of the slope. Preferably steady, the time variation of ambient properties should be slow, i.e., much slower than the time scale t_M, and have little or no component parallel to the contours.

(iii) The radiation divergence in the air on the slope between the ground and a suitable height H in the ambient air.

(iv) The surface energy balance to complete the specification with (iii) of R, the layer cooling rate [Eq. (2.9)].

(v) Vertical profiles of wind velocity and potential virtual temperature as functions of time and/or distance from the crest of the slope.

Data requirement (ii) has been largely ignored in many published experimental studies of katabatic flows while data for requirements (iii) and (iv) do not appear to have been reported. The nearest attempt seems to have been made by Lettau (1966) for a study of the central Antarctic plateau. It is usual to assume that the energy balance at the surface is in equilibrium and that the net radiation flux at the ground is then the same as the divergence of energy flux within the cooled layer (see, e.g., Fleagle, 1950; Lettau, 1966; Petkovsek and Hocevar, 1971; Streten *et al.*, 1974). In view of recent results by Kondo *et al.* (1978) and others, the divergence B is probably strongly overestimated by the latter assumption. An advantage of the present model over previous attempts is that here there is no need to specify the thickness h of the layer which is undergoing diabatic cooling—it is a predicted parameter.

Finally, data sufficient to plot (v) are not available. One or the other of wind velocity or temperature

TABLE 2. Calculations for the McCall Glacier under conditions reported by Streten *et al.* (1974).

McCall Glacier averages for 1900–0600 LT	The night of 16 August 1971	17 August 1971
Slope angle	7°	7°
Distance of site [$(A+B)/2$] to crest (km)	5	5
ΔT_2 (free lapse, $\Delta z=1085$ m) (°C)	9.4	8.6
V_G, relative speed at 2 m, (m s^{-1})	4.1	3.5
R, net radiation at ground (W m^{-2})	68.4	34.8
N^2 [Eq. (2.1)] (s^{-2})	3.6×10^{-5}	6.2×10^{-5}
C†	0.1	0.1
t_M†† (min)	24	18
E_M††	0.021	0.021
$B=gR/(\rho C_p T_0)$ (m^2 s^{-3})	2.0×10^{-3}	1.0×10^{-3}
s_M† (km)	5.6	2.7
h_M† (m)	117	56
U_M† (m s^{-1})	3.9	2.4
U_* [from (3.3)] (m s^{-1})	3.5	
$U_*=U_M U$†† (m s^{-1})		2.3
h_* [from (3.2)] (m)	69	
$h_*=h_M h$†† (m)		82
$f/(S_2^{\frac{1}{2}}N \sin\alpha)$	0.20	0.15

† From relations (2.27).
†† From Figs. 3 and 4.

profiles is the most previously attempted with any resolution (see, e.g., Defant, 1949).

b. A test of the model

The slope winds of Antarctica are perhaps the most intensely studied of all such flows. As a test of the present model the Antarctic situation is generally unsuitable, however. The slope winds which are such a feature there are composed of two parts: 1) drainage flow off the extensive central plateau with no definable flow origin, and 2) local katabatic flow such as that which probably originates at the crest of the Masson Range and flows downslope to the east of Mawson (Streten, 1963; Shaw, 1957). The first part should be treated by a model similar to Ellison and Turner's (1959) with a finite buoyancy source and ambient stratification; the second part could then be treated by the model described in this paper. But there is no clear way of distinguishing between the two parts in the published data.

Now the only observational study which does not appear to be contaminated by drainage flows and which even approaches the data requirements outlined above appears to be that reported by Streten *et al.* (1974) of the McCall Glacier. Wind velocity was measured at an elevation of 2 m at several sites and as a function of time for two favorable nights, 16 and 17 August 1971. These were nights of near-calm ambient wind and clear skies. The quantity V_G, the wind component downslope measured as the average of that at two sites a mean distance of 5 km from the crest, and relative to the same component close to the crest, was reported. In addition, ΔT_2, the temperature difference between two freely exposed sites differing in elevation by 1085 m and away from the influence of katabatic flow, and R, the net radiation measured at the main site, were also reported as functions of time. The reported parameters exhibited strong variations during the two nights with, for example, ΔT_2 varying by ±1°C from its mean value of 9.4°C for the first night (see Table 2). No information is given regarding ambient winds for the two nights in question but there is a suggestion in Fig. 2 of Streten *et al.* (1974) that they were no more than 4 m s^{-1} across-slope, with a weak downslope component.

Now R will be taken as characterizing the layer cooling, and while it is possible that the model could have been solved using the observed variations in R and ΔT_2 as inputs, the lack of information regarding the effects of the ambient wind suggests that it is more reasonable to compare the model with the observed average conditions from say 1900–0600 local time on the two nights reported. Table 2 summarizes the calculations. The layer cooling rate B is computed from the measured net surface radiation flux, and the other parameters are obtained from the relations given in Sections 2 and 3.

For the first night the cooling rate was large and s_M, the distance downslope where ambient stratification must begin to be considered, was greater than 5 km so the analytic solutions [Eqs. (3.2)–(3.6)] have been used to compute U_* and h_*. The layer cooling was less and the ambient stratification greater on the second night so s_M was smaller and the numerical solutions for no ambient wind and $C=0.1$ (Fig. 2) have been used.

The predicted layer thicknesses of 69 and 82 m at 5 km compare well with the estimate of Streten *et al.* (1974) of <100 m. The predicted layer wind speeds, defined in terms of the integrals of Section 2, are 3.5 and 2.3 m s^{-1} and also are in satisfactory agreement with the measured speeds of 4.1 and 3.5 m s^{-1}, respectively, at 2 m elevation. Surface stress was almost certainly negligible ($C_D \ll E_M=0.02$ from Table 2) since the flow was over ice, so the wind at 2 m may indeed characterize the layer speed U_*.

Finally, a check may be made on the importance of Coriolis force on this katabatic flow. A term of order $fU_M h_M$ has been omitted from the model equation (2.28) compared with $S_2\Delta_M h_M \sin\alpha$. The ratio of these terms is $f/(S_2^{\frac{1}{2}}N \sin\alpha)$ and was no more than 0.2 for the two nights (see Table 2, with $f=2\Omega\sin\phi=1.36\times10^{-4}$ s^{-1} for $\phi=69$°N).

c. Effect of ambient winds

The major limitation of this model is perhaps its failure to include finite ambient winds. In particular, cross-slope (i.e., parallel to the height contours) ambient winds always enhance the interfacial shear and may at times play a dominant role in determining whether an observable katabatic wind is present on a slope. Tang (1976) has approached the problem of the disturbance of mountain and valley winds by ambient cross winds as a small perturbation analysis, while Kitabayashi (1977) recently showed experimentally that if an internal Froude number Fr exceeds about 2.3 then all the cooled air may move off the slope in the direction of the ambient wind U_A. Here $\mathrm{Fr}=U_A/(NH)$, where N^2 is a measure of the thermal stratification in the vicinity of the slope and H is the height of the barrier to the wind U_A, i.e., the height of the slope or the height of the lateral ridges to the slope, depending on the direction of U_A. For $\mathrm{Fr}<2.3$ the flow is stagnant or downslope in the cooled air. Consider, for example, a slope rising 600 m above a plain, and having ridges 40 m high on either side. If near the slope the thermal stratification were equivalent to $N^2=10\times10^{-4}$ s^{-2}, then a cross-slope ambient wind >3 m s^{-1} would scour the katabatic flow from the slope, but an upslope ambient wind >44 m s^{-1} would be needed for the same result.

Recent work by Brost and Wyngaard (1978) utilizing second-order turbulence models applied to cooled slopes ($\alpha=0.1°$) have further demonstrated the strong influence ambient wind direction has on the stable boundary layer (see their Fig. 7). Clearly, more work on the effects of ambient wind on the katabatic flow is required.

5. Conclusion

The extended hydraulic approach is a convenient framework in which to develop a model of katabatic flow that includes aspects of the dynamics not considered until now but which influence greatly the observed flow. The model developed here has few strong assumptions and the minimum of set parameters. The most important of these, namely the entrainment relation, is specified by appeal to laboratory and related field experiments.

Mixing between the ambient and cooled air layers has been shown to be of great importance in providing for the first time a consistent explanation of the spatial growth of katabatic flow. The interfacial mixing also generates a retarding stress on the flow and this leads to quite the opposite conclusion to past work. Contrary to models which assume that interfacial stress is negligible or can be treated as merely a component of an increased surface drag coefficient, it is concluded here that for all practical slopes (i.e., for $\alpha>0.1°$) the interfacial stress due to mixing is the dominant retarding stress. Indeed, surface stress may very well then be negligible in the fully developed flow, with the strong stratification close to the ground isolating the katabatic flow from characteristics of the surface.

As shown in Section 4a, much more comprehensive field measurements than hitherto attempted are required for a significant test of models of katabatic flow.

While the model presented here is undoubtedly oversimplified for some applications it is sufficiently general to be applicable in many practical situations, offering as it does the possibility of solutions describing the spatial and temporal development of katabatic flows.

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