A Wind Tunnel Investigation of the Rate of Evaporation of Large Water Drops Falling at Terminal Velocity in Air

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ABSTRACT

An experimental study of the effect of ventilation on the rate of evaporation of millimeter sized water drops falling at terminal velocity in air has been carried out in a wind tunnel where drops were suspended freely in the tunnel air stream. It was found that for drops in the size range 1150 $\mu m \leq a_0 \leq 2500$ μm, the mean ventilation coefficient $\bar{f}_v \approx \bar{f}_h$ could be expressed as $\bar{f} = (0.78 \pm 0.02) + (0.308 \pm 0.010)X$, where $X = N_{Sc,v}^{1/3} N_{Re}^{1/2}$. Previously, we showed that this relation holds for drops in the size range 60 $\mu m \leq a_0 \leq 400$ μm. Taken together, our present and previous data suggest that with reasonable accuracy $\bar{f} = 0.78 + 0.308X$, for $1.4 \leq X \leq 51.4$ (60 $\mu m \leq a_0 \leq 2500$ μm). For $0 \leq X \leq 1.4$ ($0 \leq a \leq 60$ μm), one may use our previous result $\bar{f} = 1.00 + 0.108X^2$. To illustrate how the present data may be applied, we computed the distance which is required for a water drop to travel from cloud base through a NACA Standard Atmosphere of various relative humidities, in order to reach the earth's surface with a given size.

1. Introduction

In precipitation from atmospheric clouds, drops typically have radii between 500 and 2500 μm. In order to know more about the rate of evaporation of such relatively large drops, and thus to contribute toward a better understanding of the effect of drop evaporation in precipitation on the dynamics of clouds, numerous scientists suggested that we extend our previous wind tunnel investigation on the rate of evaporation of drops, ranging between 10 and 400 μm radius (Beard and Pruppacher, 1971), to drops up to 2500 μm in radius. Before following this suggestion we consulted the literature in order to see whether data exist which would fill the present need. Our literature search showed that only the study of Kinzer and Gunn (1951) was carried out for freely falling drops. Unfortunately, however, this study was plagued by inherent inaccuracies in the experimental setup. For drops of radii <400 μm the evaporation rate was derived from inaccurately measured fallspeeds of the drops. Drops of radii between 400 and 2250 μm were suspended in the air stream inside narrow tapered tubes, causing serious wall effects. Drops of radii between 2250 and 2750 μm were supported above the screen at the exit of a small blower-type wind tunnel where the tunnel air mixed with the air of the environment. This made it difficult to control the temperature and humidity.

Other studies on the effects of ventilation on the evaporation of water drops in air were carried out by Froessling (1938), Ranz and Marshall (1952a,b) and Lee and Ryley (1968). However, in these latter studies the water drops were suspended on rigid supports. These supports not only disturbed the flow of the air around the drop, but also disturbed the internal circulation in the drop, and prevented the drop from assuming the shape, the oscillation behavior and the fall pattern it has when freely falling. These studies can therefore not be considered to realistically represent the evaporation of drops in atmospheric clouds and precipitation. In addition, none of the mentioned experimental studies gave the level of turbulence present in the air stream.

In order to improve the results of previous studies, we carried out a new study on the evaporation of large drops in the UCLA Cloud Tunnel.

2. Experimental setup

A detailed description of the UCLA Cloud Tunnel and its calibration have been given by Beard and Pruppacher (1969). The water drops investigated in the present study were formed from doubly distilled, deionized water, filtered through a Milipore filter of 0.01 μm pore size, and were introduced into the tunnel air stream by means of a syringe and hypodermic needle. During the evaporation of a drop in the tunnel air stream the flow control valve was continuously adjusted so that the drop was kept stationary near the tunnel axis and the velocity of the air stream was equal in magnitude

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to the fall velocity of the drop. The variation in the drop size during its evaporation was determined by photographing the drops approximately every 30–60 s by means of a Nikon-F 35 mm still camera, with a suitable lens and extension tubes attached. From the photographs the drop volume was determined which, in turn, yielded the drop's equivalent radius a_0. For photographing, fine-grain Shellburst film was used. Evaporation runs lasted typically for 300 to 1500 s. Drops of equivalent radii between 2500 μm $\leq a_0 \leq$ 1100 μm were investigated. The evaporation rate of these drops as measured for air stream temperatures typically between 0 and 10°C, and air stream relative humidities typically between 10 and 50%. The air stream temperature was monitored by a series of thermocouples which were calibrated against a National Bureau of Standard platinum resistance thermometer. The relative humidity of the air was determined by means of a Cambridge-System dew point hygrometer. The experiment was carried out at sea level, and thus the pressure of the air in the laboratory was near 1000 mb. Since the static pressure difference between tunnel inlet and the air in the observation section was less than 0.7 mb, the pressure in the observation section was assumed to be equal to the static pressure of the air in the laboratory. The air pressure in the laboratory was measured by a precision mercury barometer.

In order to suspend millimeter size drops stably in the tunnel air stream for long periods of time, the drops were floated in the divergent section of the tunnel observation section. This section had a divergence angle of 0.17453 rad (10°). A honeycomb and five screens located upstream of the contraction section kept the tunnel flow laminar at all the vertical velocities at which experiments were performed. In addition, a screen bent slightly concave was mounted at the entrance of the observation section. In this manner drops of $1100 \leq a_0 \leq 2500$ μm were stably suspended in the tunnel air stream for periods up to 30 min. The upper limit of drop radius for this study can be justified by the fact that in atmospheric rains observed at the ground the drops have radii which are mostly smaller than 2.5 mm (Pruppacher and Klett, 1978). Drops of $500 \leq a_0 \leq 1000$ μm could not be suspended in the wind tunnel due to their helical fall mode of large amplitude.

The terminal velocity corresponding to the drop sizes studied ranged between 600 and 900 cm s^{-1}. A careful analysis showed that the upward velocities required to freely suspend these drops in the tunnel air stream corresponded to a Reynolds number in the honeycomb and screen section of the tunnel which guaranteed fully laminar flow in the wake of these section. Thus, even at these relatively high air stream velocities in the tunnel, the honeycomb and screen sections acted to efficiently "smooth" the air entering the tunnel, cutting down the turbulence level to

$$[\overline{(u')^2}]^{1/2}/U < 0.5\%,$$

where U is the mean velocity of the air stream in the wind tunnel. The additional small screen inserted for improving the stability of the large drops raised the turbulence level slightly in the tunnel but still kept it below 0.5%.

3. Data analysis

A detailed discussion of the effect of ventilation on the rate of evaporation of a water drop in air has been given by Beard and Pruppacher (1971) and recently, in a broader context, by Pruppacher and Klett (1978). Briefly, the mean ventilation coefficient $\bar{f}_v$ for evaporation is defined by

$$\frac{dm}{dt} = \bar{f}_v\left(\frac{dm}{dt}\right)_0 , \qquad (1)$$

where (dm/dt) is the rate of evaporation of a drop falling at terminal velocity in air, and $(dm/dt)_0$ is the rate of evaporation of a drop at rest. According to Beard and Pruppacher (1971) the ventilation coefficient is given by

$$\bar{f}_v = \frac{dm/dt}{\dfrac{4\pi a_0 D_{v,a} M_w}{R}\left(\dfrac{e_\infty}{T_f} - \dfrac{e_a}{T_f}\right)} \qquad (2)$$

for the case $e/p \ll 1$, which holds for all conditions studied during the present experiment. In Eq. (2) p is the total air pressure, $T_f = (T_\infty + T_a)/2$, T_a is the temperature at the drop surface, T_∞ the temperature of the air stream, M_w the molecular weight of water, R the universal gas constant, $e_\infty = \mathrm{RH} \times e_{\mathrm{sat}}(T_\infty)$ the water vapor pressure of the tunnel air, $D_{v,a}$ the diffusivity of water vapor in air at drop surface temperature T_a, $e_a = e_{\mathrm{sat}}(T_a)$, and RH is the relative humidity of the ambient air.

The quantities dm/dt and a_0 were determined from our photographic records, e_∞ was determined from the dew point of the air, T_∞ was determined by thermocouples and $D_{v,a}$ was computed from the relation

$$D_{v,a} = 0.211\left(\frac{T_a}{T_0}\right)^{1.94}\left(\frac{p_0}{p}\right) \qquad (3)$$

given by Hall and Pruppacher (1976). The drop surface temperature T_a was determined from the relation

$$\Delta T = T_\infty - T_a = \frac{L_e D_{v,a} M_w (e_a - e_\infty)\bar{f}_v}{kRT_f\bar{f}_h} , \qquad (4)$$

where L_e and k are the latent heat of evaporation and the thermal conductivity of the air at the temperature of the drop surface, respectively. The quantity $\bar{f}_h$ is a measure for the effect of ventilation on the transfer of heat to the drop. Eq. (4) was numerically solved following the method outlined by Beard and Pruppacher (1971). The thermal conductivity of moist air was computed from the following relations for temperatures warmer than 0°C:

$$k_d = (5.69 + 0.017T) \times 10^{-5}, \quad (5)$$

$$k_v = (3.78 + 0.020T) \times 10^{-5}, \quad (6)$$

$$k = k_d\left[1 - (\gamma_1 - \gamma_2 k_v/k_d)\frac{e_\infty}{p}\right], \quad (7)$$

with $\gamma_1 = 1.17$ and $\gamma_2 = 1.02$ (T in °C, and k in cal cm^{-1} s^{-1} °C^{-1}) and where k_d, k_v, k are the thermal conductivity of dry air, water vapor and moist air, respectively. Further, the latent heat of evaporation was computed from

$$L_e = 597.3\left(\frac{273.15}{T}\right)^\gamma,$$

$$\gamma = 0.107 + 3.67 \times 10^{-4}T, \quad (8)$$

where T (in K) and L_e (in cal g^{-1}) apply to temperatures warmer than 0°C. Eqs. (5) to (8) are taken from Pruppacher and Klett (1978).

4. Results and discussion

The variation of the mean ventilation coefficient with $N_{Sc,v}^{1/3}\ N_{Re}^{1/2} = X$ and with a_0, experimentally determined via Eq. (2), has been displayed in Fig. 1. Combining the present results for the drop size range 1100 μm $\leq a_0 \leq$ 2500 μm ($29.7 \leq X \leq 51.4$) with those obtained by Beard and Pruppacher (1971) for the drop size range 60 μm $\leq a_0 \leq$ 400 μm ($1.4 \leq 12.3$), shows that the same ventilation law applies to both size ranges. This can be expressed by the relation

$$\bar{f} = (0.78 \pm 0.02) + (0.308 \pm 0.010)X, \quad (9)$$

applying to $1.4 \leq X \leq 51.4$ (60 μm $\leq a_0 \leq$ 2500 μm). For $0 \leq X \leq 1.4$ ($0 \leq a_0 \leq 60\,\mu$m) the relation

$$\bar{f} = 1.00 + 0.108X^2 \quad (10)$$

found by Beard and Pruppacher (1971) still holds. Notice that the rate of evaporation of a water drop of $a_0 = 2$ mm, freely falling at its terminal velocity in air, is about 15 times larger than the rate of evaporation of the same size drop at rest. Even for considerably smaller drops the ventilation effect is still quite large. Thus, the rate of evaporation of a freely falling drop of $a_0 = 200$ μm is enhanced by a factor of about 2.5 over the evaporation rate of the same size drop at rest.

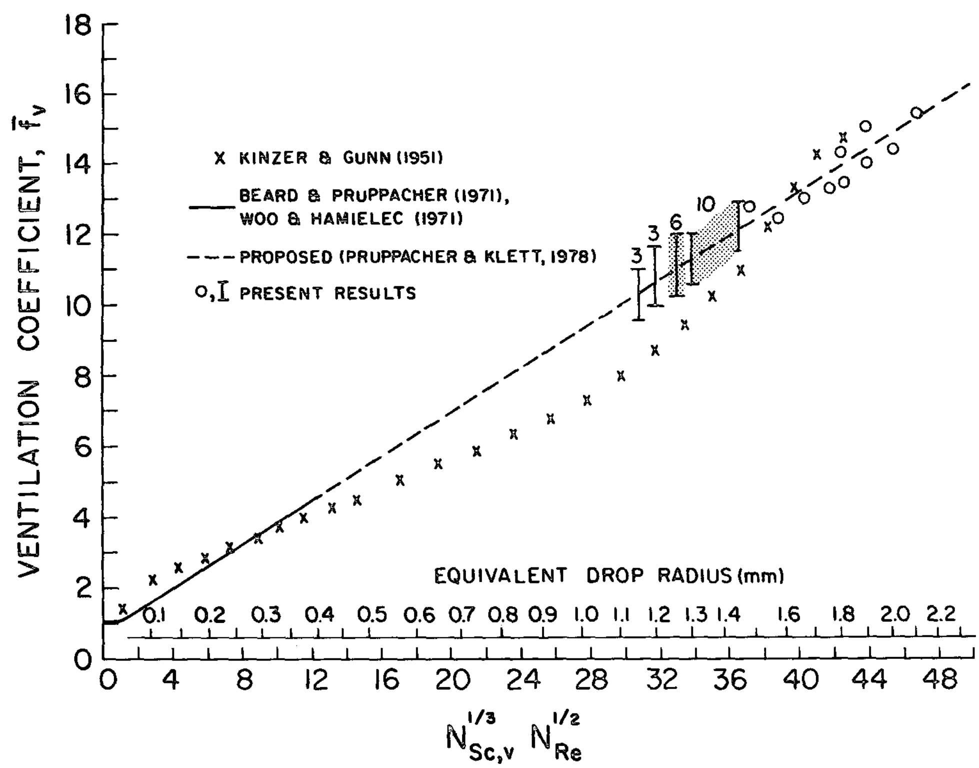


FIG. 1. Variation of the ventilation coefficient $\bar{f}$ with $X = N_{Re}^{1/2}\ N_{Sc,v}^{1/3}$ and equivalent drop radius a_0.

Also, note that the dip in the variation of $\bar{f}$ with a_0, implied by the data of Kinzer and Gunn (1951), has not been verified by our present results. It is likely that the experimental inaccuracies inherent in the set-up of Kinzer and Gunn (see Section 1) are responsible for this discrepancy.

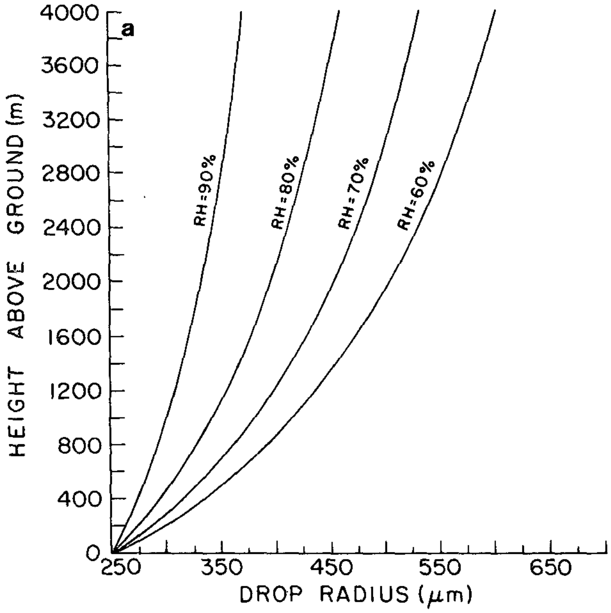
We recall now that a drop of $a_0 \lesssim 140$ μm ($N_{Re} \lesssim 20$) is spherical in shape while a drop of $140 \lesssim a_0 \lesssim 500$ μm ($20 \lesssim N_{Re} \lesssim 260$) is oblate spheroidally deformed, and drops of $a_0 > 500$ μm ($N_{Re} > 260$) have the shape of oblate spheroids with a flat base in the center of which there exists an increasingly pronounced concave depression (Pruppacher and Beard, 1970; Pruppacher and Pitter, 1971; see also Pruppacher and Klett, 1978). We also recall that water drops falling in air exhibit a pronounced, organized internal circulation. This circulation becomes broken up by turbulent mixing if a drop reaches a size $\gtrsim 500$ μm radius since at this size the drop begins to oscillate (Pruppacher and Beard, 1970, Le Clair *et al.*, 1972; see also Pruppacher and Klett, 1978). One would have expected that the mentioned changes in shape, internal circulation and oscillation, as well as the drop's helical fall motion beginning at a radius near 500 μm, would reflected themselves in changes in the drop's evaporation behavior. Such changes, in fact, have not been found in our experiment. Rather, it appears that the effect of ventilation on drop evaporation can be expressed by a *single* law in the drop size range $60 \leq a_0 \leq 2500$ μm. This behavior can be understood, at least qualitatively, if one considers that two opposing effects are at work. On the one hand, ventilation enhances mass and heat transport to and from a body to a degree which is smaller the stronger the body is deformed. This was demonstrated by Pitter *et al.* (1974) who computed the ventillation coefficient for a thin, oblate spheroid of ice in air. Pitter *et al.* showed that the ventilation coefficient of such a spheroid is considerably smaller than that of an ice sphere of same Reynolds number. On the other hand, Rose and Kintner (1966), Brunson and Wellek (1970) and Yao and Schrock (1976) showed that the mass transfer from a liquid body which oscillates between an oblate and prolate spheroidal shape is enhanced over and above the mass transfer from a body that is oblate spheroidally deformed and does not oscillate. The reason for this behavior lies in the fact that the ventilation effect is larger for a prolate spheroid than for a sphere; and that for a sphere is in turn larger than that for an oblate spheroid. This result is borne out by the theoretical computations of Pitter *et al.* (1974) and Masliyah and Epstein (1972), and by the experiments of Hsu *et al.* (1954); see also Pruppacher and Klett (1978). In the light of our experimental results it appears then that within the accuracy of our experiment the effects of drop vibration and drop deformation, as they are typical for freely falling drops of $a_0 > 500$ μm, approximately compensate each other.

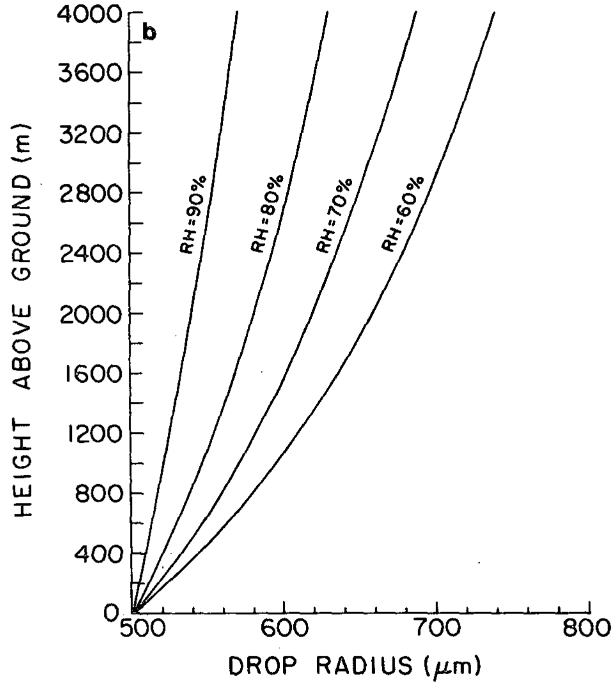
As an example for how to apply the present results to problems of interest in meteorology, we computed by an iterative method the variation in radius of single, isolated water drops falling from cloud base through subsaturated air of various relative humidities in a NACA Standard Atmosphere, and arriving at the earth's surface with a radius of 250, 500 and 1000 μm. The results of these computations are summarized in Fig. 2. We note from this figure that 1) the lower the relative humidity of the air beneath the cloud the larger the drop has to be at cloud base in order to arrive at the ground with a given final radius; 2) the larger the final radius of the drop arriving at the ground after having fallen through an air layer of given conditions and given thickness the smaller is the change in size experienced by the drop; and 3) the higher the relative humidity of the air beneath a cloud the higher the cloud base can be for a drop of given initial size to arrive at the ground with a given final size.

We finally want to stress that the results presented here for the effects of forced convection on mass and heat transfer are applicable only to an environment of low turbulence, i.e., for

$$\overline{[(u')^2]}^{1/2}/U \lesssim 0.005 \ (0.5\%).$$

Unfortunately, little is known on the effect of turbulence on forced convective mass and heat transfer to and from water drops in air. However, one may estimate the effect from studies on rigid spheres. A comprehensive study on the effect of turbulence on mass and heat transport to rigid spheres has been made by Galloway and Sage (1964). A later more quantitative study is due to Lavender and Pei (1967). Although both studies suffer from an incorrect description of the N_{Sh} and N_{Nu} as $N_{Re} \rightarrow 0$, the study of Lavender and Pei suggests that for $1 \times 10^3 \leq N_{Re} \leq 8 \times 10^4$ the slope of Eq. (9) increases as $N_{Re,T}^{0.035}$, where $N_{Re,T} = N_{Re} \times \alpha_T$ and where $\alpha_T = \overline{[(u')^2]}^{1/2}U$. This result indicates that at $N_{Re} = 4000$ and $\alpha_T = 0.05$ (5%) the slope increases by a factor of 1.2 over the slope it has when no turbulence is present in the air. This result is in good agreement with the experimental results of Joss and Aufdermauer (1969) who found for the same Reynolds number that the slope is raised by a factor of about 1.3 as the turbulence level of the environment is raised to 5%. Assuming that the effect of free-stream turbulence on heat and mass transfer from rigid spheres in air also applies to water drops in air, we find that for a drop of $N_{Re} = 3000$ and a free-stream turbulence level of 0.5% the slope of the curve for the ventilation coefficient is raised by a factor of 1.09 over the slope turbulence exists in the flow. At $N_{Re} = 1000$ this factor has





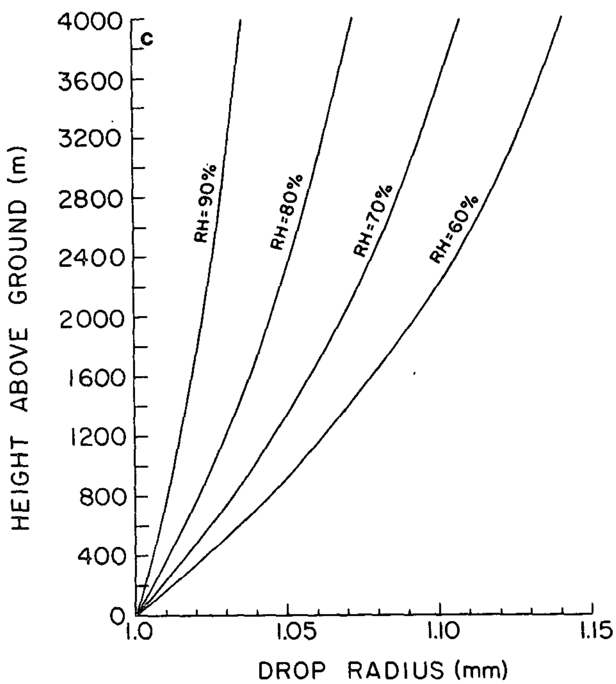


FIG. 2a. Variation in radius of a water drop falling from cloud base through subsaturated air of various relative humidities in a NACA Standard Atmosphere, and arriving at the ground with a radius of 250 μm. b). As Fig. 2a except for a final radius at the ground of 500 μm. c) As Fig. 2a except for a final radius at the ground of 1000 μm (1 mm).

decreased to 1.05. This latter value is within the experimental error of our experiment. It is further of interest to note from the work of Lavender and Pei that only the *intensity* but not the *scale* of the free-stream turbulence affect heat and mass transfer.

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