Dynamics of Closed Systems of Resonantly Interacting Equatorial Waves

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(Manuscript received 5 January 1979, in final form 3 April 1979)

ABSTRACT

Nonlinear dynamics of equatorial waves, interacting resonantly in coupled triad configurations which form closed systems, are investigated in the context of the divergent β-plane model. Closure is attained by demanding that spatial structures of the modes obey the atmospheric constraints. At larger fluid depths the wave systems are relatively small and concentrated at the smaller wavenumbers; at small depths the systems are larger and spread more widely in the wavenumber domain. Strong energy transfers in a system are consistently associated with modes characterized by the maximum frequency in individual triads. The lower frequency modes are energetically less active, especially when their frequencies are much less than and amplitudes greater than those of the maximum frequency modes in the same triads.

1. Introduction

In an attempt to improve our understanding of the nonlinear processes in equatorial atmosphere, this study focuses on the finite-amplitude dynamics of systems of resonantly interacting discrete equatorial waves, within the context of the divergent β-plane model. In a previous study (Domaracki and Loesch, 1977, hereafter referred to as DL) it was established that the wave types permitted by the model (east and west propagating inertia-gravity, Kelvin, mixed Rossby-gravity and Rossby waves) interact resonantly in triad configurations; individual triads may be composed of the same as well as differing wave types. Energy solutions obtained in DL for *isolated* triads show that the triad member having maximum frequency always loses (or gains) energy at the expense of the other triad members.

The Hermite polynomial meridional structure of the equatorial modes yields a relaxation of the kinematic resonance conditions and, therefore, enhancement in interactions, in the equatorial model as compared to the midlatitude case. A given equatorial mode is capable of simultaneously interacting with many wave pairs and it may participate in second harmonic resonance. Examples of this occurrence are given in Tables 1 and 2 in DL. Because of the abundance of intertriad interactions, and the fact that they can involve different wave types, in the present study the interaction concept is generalized to closed systems of interacting equatorial waves. The closed systems are composed of *all possible coupled triads* of waves, at a given fluid depth, whose members have their zonal wavenumbers quantized (or nearly so) and meridional structures restricted to the lowest three integers. Such a choice of modes guarantees that only those spatial structures are allowed which occur in the equatorial atmosphere. From a theoretical point of view, these restrictions aid in more efficiently closing the interacting wave systems.

Our main goal in this paper is to investigate the effectiveness with which resonant interactions can redistribute energy generated at selective scales to various other wave scales in the equatorial atmosphere. In particular, we want to assess the role of the maximum frequency triad members and the lower frequency modes in the dynamics of the closed wave systems.

2. Theoretical development

The model employed here is identical to that used in DL and introduced by Matsuno (1966). We consider a single layer of homogeneous hydrostatic fluid bounded from below by a rigid horizontal surface and with a free upper surface on an equatorial β-plane. In the absence of dissipation, the equations and boundary conditions governing large-scale perturbations may be written as

$$u_t + \epsilon(uu_x + uu_y) - yu + \phi_x = 0, \qquad (1a)$$

$$v_t + \epsilon(uv_x + vv_y) + yu + \phi_y = 0, \qquad (1b)$$

$$\phi_t + \epsilon(u\phi_x + v\phi_y) + u_x + v_y = 0, \qquad (1c)$$

$$u, v, \phi \rightarrow 0 \quad \text{as} \quad y \rightarrow \pm\infty. \qquad (1d)$$

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[3] The National Center for Atmospheric Research is sponsored by the National Science Foundation.

0022-4928/79/081490-08$06.00

The variables in (1) have been nondimensionalized using the following scheme:

horizontal velocities	$(u',v') \sim U(u,v)$
horizontal coordinates	$(x',y') \sim L(x,y)$
time	$t' \sim t/\beta L$
geopotential height	$\phi' \sim \beta L^2 U\phi$

$$L = \left(\frac{\sqrt{gH}}{\beta}\right)^{1/2}$$

$$\epsilon = \frac{U}{\beta L^2}\,.$$

Finite-amplitude treatment of (1) for $\epsilon \ll 1$ [in the atmosphere $\epsilon \sim O(10^{-2})$], employing asymptotic expansions in ϵ for the dependent variables (u,v,ϕ) and the technique of multiple scales in (x,t), is given in detail in DL and will not be repeated here. It suffices to say that the $O(1)$ solutions to the problem, which physically represent inertia-gravity, Kelvin, Rossby and mixed Rossby-gravity waves, interact resonantly at $O(\epsilon)$ in triad configurations, through the quadratic terms in (1), whenever the following side conditions are satisfied:

$$\sum_{j=1}^{3} k_j = 0, \tag{2a}$$

$$\sum_{j=1}^{3} n_j \text{ is odd}, \tag{2b}$$

$$\sum_{j=1}^{3} w_j = 0, \tag{2c}$$

$$w_j^2 - k_j^2 - \frac{k_j}{w_j} = 2n_j + 1. \tag{2d}$$

In (2), k_j is the zonal wavenumber, w_j the frequency and n_j an integer representing the index of the Hermite polynomial meridional structure of the jth resonantly interacting mode in the system. In general, a given mode participates in infinitely many interactions; the resulting triads may be composed of the same as well as differing wave types. Imposition of atmospheric constraints on (2), such as quantization of the zonal wavenumber and restriction of meridional structure, limits the number of possible interactions to a small finite number. A detailed discussion of the solutions to (2) under the above constraints and of the resulting closed systems of interacting waves is presented in the next section.

Equations governing the time evolution of the amplitudes of the interacting modes are obtained from (1) at $O(\epsilon)$ by removing the secular contributions. These are generated, among the linear terms in (1), by the introduction of the multiple scales and, among the nonlinear terms in (1), by the imposition of the resonance conditions (2). The technique used for their removal is presented in DL. When (2) is satisfied by a *single* triad of *discrete* waves the resulting amplitude equations derived in DL are

$$\frac{dA_j}{dT} = iC_{kl}^{(j)}A_k^*A_l^*, \quad j, k, l = 1, 2, 3;\ j \neq k \neq l. \tag{3}$$

In (3), A_j is the nondimensional complex amplitude of the jth mode, T the nondimensional long time related to the dimensional time t' by

$$T = \frac{\epsilon}{\beta L}t',$$

and $C_{kl}^{(j)}$, denoted by CC_j in DL and stated explicitly there in the Appendix, represents the nonlinear coefficient coupling mode j to the pair (k,l). When the given mode j participates simultaneously in a *finite* member of triads (rather than a single triad), Eq. (3), for that mode, generalizes to

$$\frac{dA_j}{dT} = i\sum_{k,l} C_{kl}^{(j)}A_k^*A_l^*, \tag{4}$$

where the summation is over all the possible pairs (k,l) which *directly* interact with the mode j.

3. Determination of closed systems of interacting waves

In this section we discuss the solutions to the kinematic resonance conditions (2). To establish some physical correspondence between the model and the atmosphere we require cyclic continuity in the zonal direction and allow only those meridional structures which are observed. The mathematical manifestations of these physical constraints are, respectively, quantization of the zonal wavenumber and restriction to the lowest three meridional structures

$$n_j = 0, 1, 2. \tag{5a}$$

Quantization of the zonal wavenumber is carried out by requiring that a given nondimensional wavenumber k_j be, approximately, a multiple of the fundamental nondimensional wavenumber k, i.e.,

$$\left.\begin{aligned} k_j &= m_jk \pm \delta_jk \\ m_j &= 1, 2, \ldots \\ |\delta_j| &\leq \delta \end{aligned}\right\}, \tag{5b}$$

where $\delta = O(\epsilon)$ represents the upper bound on the tolerance to within which the resonance conditions (2) must be satisfied among three quantized zonal wavenumbers for a significant interaction to take place at $O(\epsilon)$ [for a discussion on this point see Bretherton (1964)]. Solutions of (2) subject to the wave structure constraints (5a,b) yield tractable closed wave systems. The actual size and composition of these systems depends on the numerical

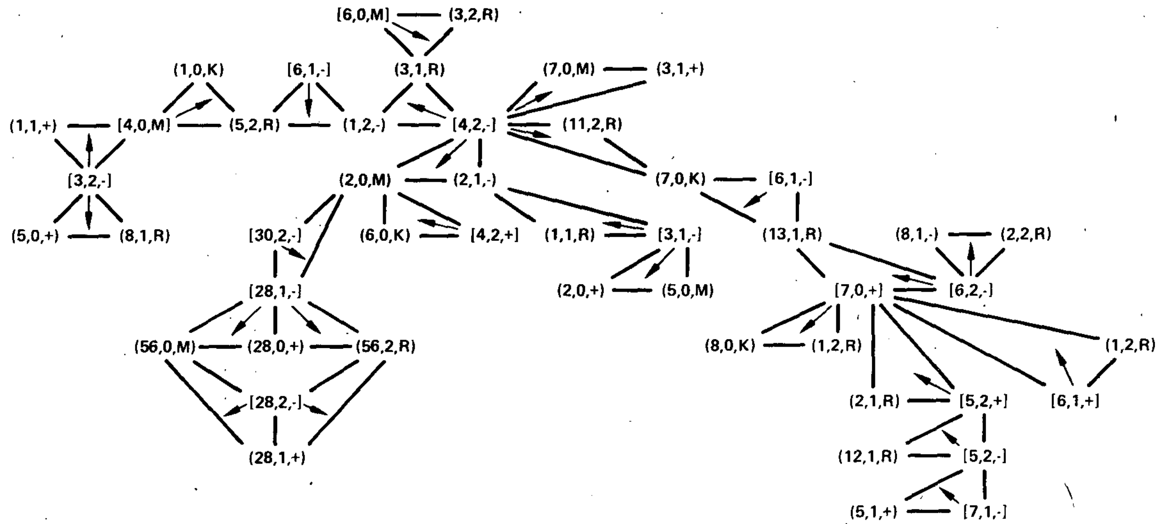

FIG. 1. A closed system of resonantly interacting equatorial waves at $H = 1$ km. The triangles represent individual triads and the bracketed expressions individual modes. Within the parentheses and brackets, the first integer gives the zonal structure m_j, the second integer gives the meridional structure n_j and the last symbol identifies the wave type. R, K, M, + and − represent, respectively, Rossby, Kelvin, mixed, and east and west propagating inertia-gravity waves.

choice of δ and, through the fundamental wavenumber

$$k = k'L = \left(\frac{\sqrt{gH}}{\beta}\right)^{1/2} \frac{1}{R}$$

(R is the earth's radius) on the depth of the fluid H.

Closed wave systems, satisfying (2) and (5), have been determined numerically, assuming $\delta \leq 0.1$, for the following depth scales: $H = 10$ m [the wave-CISK scale (Lindzen, 1974)], $H = 1$ km (the equivalent depth scale) and $H = 10$ and 18 km (the tropospheric heights). The starting point for the determination of each system was a mode j considered relevant to equatorial dynamics (e.g., Rossby wave, wavenumber 3; mixed wave, wavenumber 6). After all the wave pairs resonantly interacting with this mode were determined, only those pairs (k,l) were retained whose spatial structures satisfy constraints (5a,b). As the next step in the calculations all additional interactions of modes k and l were found and, again, only those satisfying (5a,b) retained. This procedure was continued for all the retained modes until, within the limits of constraints (5a,b), no new interactions could be found for *any* number of the wave system, i.e., until the wave system closed on itself.

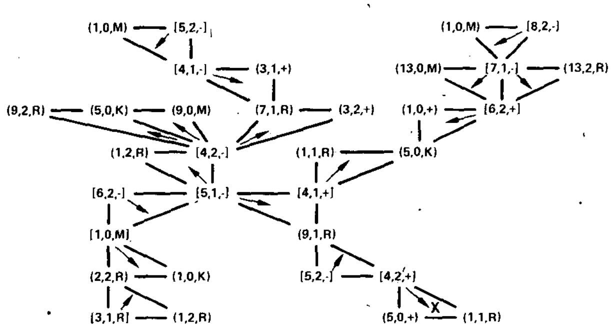

FIG. 2. As in Fig. 1 except at $H = 10$ km.

In general, the closed wave systems involve all the equatorial wave types. They are strongly dominated by triads composed of one low-frequency mode (Rossby or mixed Rossby-gravity) and two high-frequency modes (inertia-gravity or Kelvin). Some strictly low-frequency and strictly high-frequency triads also exist, but no triads involving solely the inertia-gravity modes, satisfying constraints (5a,b) could be found. [If constraints (5a,b) are relaxed, then, as shown in Table 3 in DL, such triads can exist.]

The size of the wave systems and their zonal wavenumber composition depend on the tolerance parameter δ and the fluid depth H. At larger fluid depths ($H = 10$ km and 18 km) the systems are generally limited to fewer waves, whose zonal wavenumbers tend to be grouped together and most strongly concentrated at the smallest integers ($m = 1, 2, 3$). In contrast, at smaller fluid depths ($H = 10$ m to 1 km) the systems are larger and spread more widely in the wavenumber domain. There is, therefore, a tendency at greater fluid depths for the smallest wavenumbers to contain a larger fraction of the allowable modes.

The above properties are illustrated with examples of wave systems at fluid depths $H = 1$ km and $H = 10$ km for $\delta = 0.1$ in Fig. 1 and Fig. 2, respectively. Wave systems at $H = 10$ m exhibit an even greater spread in the zonal wavenumber and contain spectra excessively large for illustrative purposes (usually in excess of 100 waves). Wave systems at $H = 18$ km resemble those at $H = 10$ km.

Irrespective of the fluid depth, a wave system reduces in size upon decreasing the tolerance parameter δ. For example, the 27-wave system given in Fig. 2 reduces to 13 waves if we decrease δ from 0.1 to 0.08. This is a reflection of the fact that, as we more severely constrain the zonal structure (5b) of the modes, we further limit their interactions.

The sensitive dependence of the wave systems on δ reflects a certain degree of artificiality inherent in our procedure for finding the closed systems. This artificiality, we feel, is not serious since the systems at each depth scale are highly self-consistent, i.e., exhibit a remarkably similar composition, irrespective of which wave is taken as the starting point for the calculations. Thus, at each H, there appears to be, at most, a small number of closed systems important for a dynamical study. Also, with the present model, which neglects the energy-generating mechanisms, stratification, dissipation, etc., we can only hope to reproduce the qualitative aspects of the dynamics of interacting waves in the equatorial atmosphere. The results of the next section show that the dynamics we obtain here are *not* critically dependent on the size of the wave spectra and, therefore, on the numerical choice of δ.

4. Dynamics of the closed wave systems

We have shown in Section 2 that, when an equatorial wave is involved in a finite number of resonant triads, the time evolution of its amplitude is described by Eq. (4). In the case of a closed system of N equatorial waves, interacting in triad configurations (e.g., the wave system given in Fig. 1 or Fig. 2), the time evolution of the wave amplitudes is given simply by a *coupled* set of N equations identical in form to (4), i.e.,

$$\frac{dA_j}{dT} = i \sum_{k,l} C_{kl}^{(j)} A_k^* A_l^*, \quad j = 1, 2, \ldots N, \tag{6}$$

provided the summation is over only those pairs (k,l) in the system which form closed triads with the mode j.

If we write the complex wave amplitudes in the polar form

$$A_j(T) = R_j(T) e^{i\theta_j(T)},$$

it may be shown by a direct substitution into (6), that an exchange of energy among any three interacting modes in the system, say (j,k,l), exists provided $\theta_j + \theta_k + \theta_l \neq \pi$, and is maximized when $\theta_j + \theta_k + \theta_l = \pi/2$. In order to examine the potential for energy exchanges within the closed wave system, and to substantially simplify the analysis, we shall henceforth restrict the phase relationships within individual triads to

$$\theta_j + \theta_k + \theta_l = \pi/2. \tag{7}$$

Under constraint (7), the phases θ_j become constant, and the dynamics of the closed systems are fully described by the following set of equations for the real wave amplitudes $R_j(T)$:

$$\frac{dR_j}{dT} = \sum_{k,l} C_{kl}^{(j)} R_k R_l; \quad j = 1, 2, \ldots N. \tag{8}$$

Total energy per unit meridional strip contained in the jth mode (see DL, Section 2c) is related to its amplitude by the relation

$$E_j(T) = \pi^{1/2} 2^{n_j} n_j! [(w_j^2 - k_j^2) + (n_j + 1)(w_j + k_j)^2 + n_j(w_j - k_j)^2] R_j^2(T). \tag{9}$$

Thus, with the help of (9), the solutions to (8) can easily be recast in terms of the wave energies.

Analytical solutions to (8) can be obtained only for wave systems consisting of one or two triads. In more realistic situations, such as those illustrated in Figs. 1 or 2, solutions have to be found by numerical methods.

Numerical solutions to (8) have been obtained for a number of representative closed wave systems at $H = 10$ m, 1 km, 10 km and 18 km. In the case of each system the following initial conditions were used:

- IC1: 90% of the energy in a given mode and the remaining 10% spread equally among the other modes.
- IC2: 90% of the energy spread equally among modes in a selective group (e.g., the modes making up a given wavenumber; the modes characterized by a maximum frequency in individual triads; etc.) and the remaining 10% spread equally among the other modes.

As a check on the numerical calculations, total wavefield energy was compared at each time step against its initial value. If necessary the time step was adjusted until the percentage difference between the two was less than 1%.

Initial conditions IC1 allow us to study each wave system as a stability problem for the individual modes in the system. Taking this point of view, we ask which modes are stable (i.e., retain their energy at or near their initial values) and which are unstable (i.e., lose all or a large portion of their initial energy sometime within a vacillation period) to resonant perturbations represented by the remaining $(N - 1)$ waves in the system. Interestingly, the answer to this question does not depend on the choice of H or δ (although, we recall, the composition of wave systems is dictated by these parameters) but on the *frequency status* of the modes. We found, for all the systems considered, that if the mode characterized by a large initial energy does not, in any of its triads, attain the maximum frequency (MF) status then it is

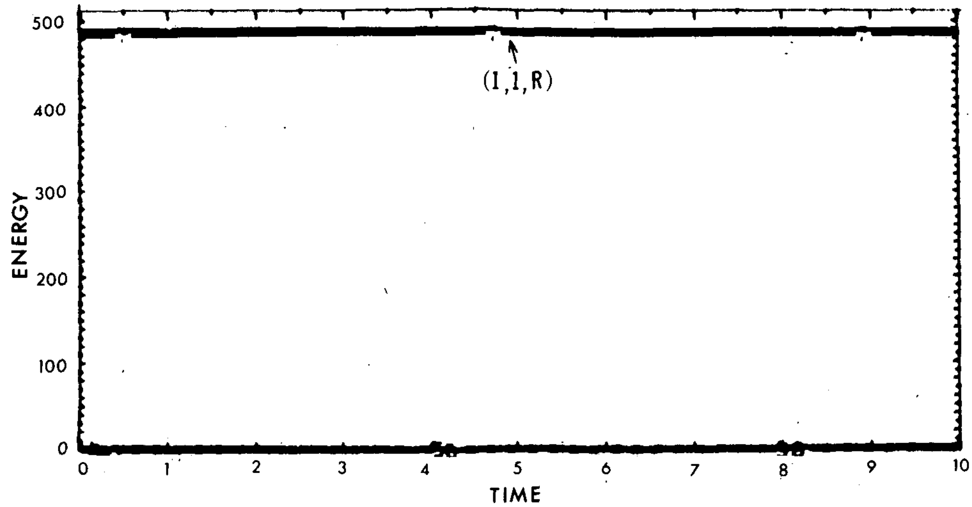

FIG. 3. Energy evolution of the mode (1,1,R) participating in the wave system given in Fig. 2 when initially 90% of the system's energy is placed in that mode.

stable. On the other hand, any mode which attains the maximum frequency status in at least one triad is unstable. As shown in DL, a mode qualifying for the MF status in a given triad (j,k,l) has its nonlinear coupling coefficient $C_{kl}^{(j)}$ of an opposite sign to the coupling coefficients $C_{jl}^{(k)}$ and $C_{jk}^{(l)}$ of the other triad members. Such a mode need not necessarily be a high-frequency mode since triads composed solely of low-frequency modes are possible.

In the wave systems illustrated in Figs. 1 and 2 the MF modes are designated by square brackets, the non-MF modes by parentheses, and the arrows indicate the triads in which the MF modes qualify for that status. Figs. 3 and 4, respectively, illustrate the stable behavior of modes (1,1,R) and (5,0,+) (for notation refer to the legend in Fig. 1) which are the non-MF modes in the triad marked × in Fig. 2. Fig. 5 illustrates the unstable behavior of mode [4,2,+], which is the MF mode in the same triad.

Having established that strong energy transfers *originate* with the MF modes, we ask next: how many modes in a system can gain a significant fraction of the energy stored initially in an MF mode? As Fig. 5 illustrates, this energy is transferred, first of all, to the non-MF pair(s) *within* the triad(s) in which the MF mode qualifies for the maximum frequency status. Relative energy gain by each non-MF member of such triad(s) is directly proportional to the initial energy difference and inversely proportional to the frequency difference between this mode and the MF mode. For a sufficiently large frequency difference, the role of the low-frequency

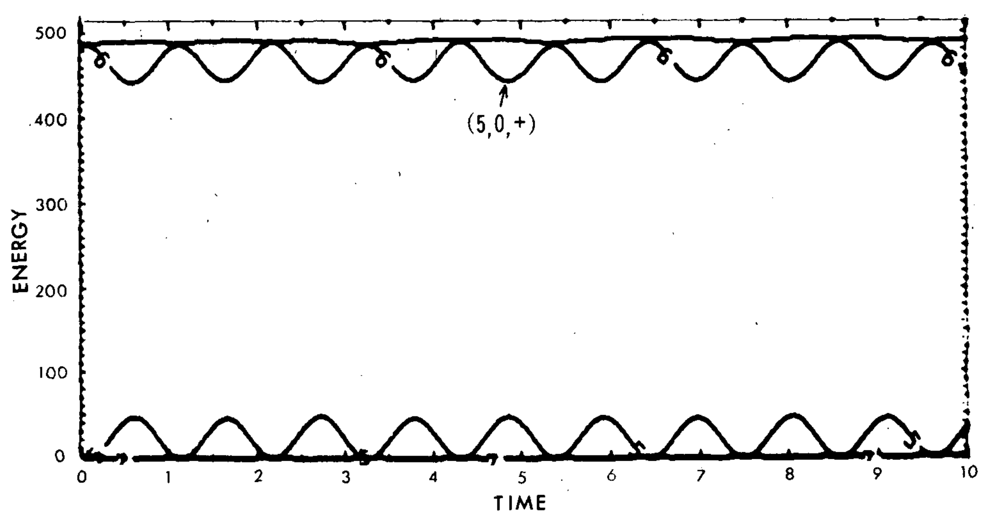

FIG. 4. As in Fig. 3 except for the mode (5,0,+).

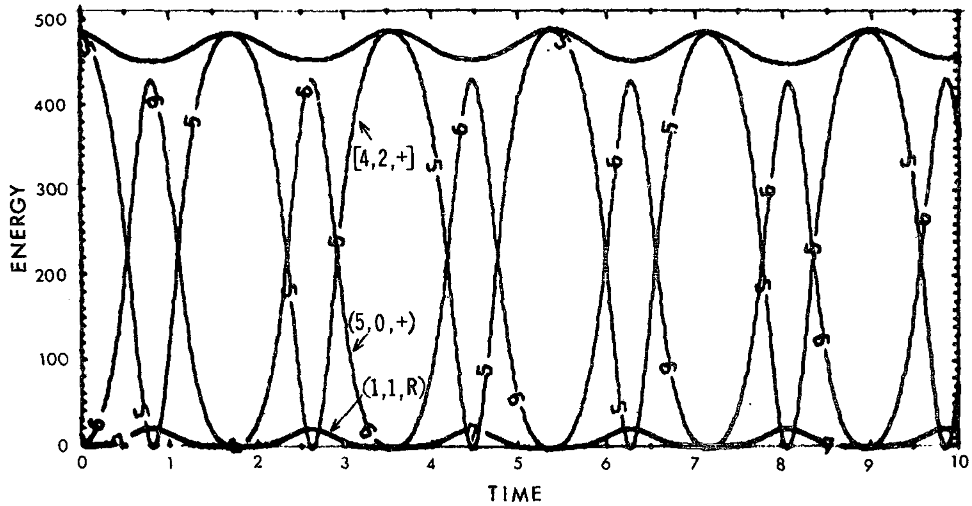

FIG. 5. As in Fig. 3 except for the mode [4,2,+].

mode may reduce solely to that of a catalyst for the energy exchange between the MF mode and the other non-MF member of the triad. Energy transfers *outside* the triad(s) of the MF mode depend on the frequency status of the non-MF members of the triad(s) in other triads, if any, in which they participate. If none qualify *elsewhere* in the system for the MF status, energy remains contained in the original triad(s). If any do qualify, transfer continues on to the appropriate triad(s). The flow of energy, in this manner, can continue until only strictly non-MF modes are encountered. For example, in the wave system shown in Fig. 2, the MF mode [4,2,+] can exchange significant energy only with the modes (1,1,R) and (5,0,+), thus confining the energetic activity solely to the triad marked ×; the MF mode [5,2,−], on the other hand, can exchange energy directly with the modes (9,1,R), [4,2,+], (1,0,M) and [4,1,−] and indirectly through [4,2,+] with the modes (5,0,+) and (1,1,R) and through [4,1,−] with the modes (3,1,+) and (7,1,R), confining the energetic activity in this case to four triads. Therefore, within a closed system, energy exchanges are possible only among those triads whose MF modes are linked through direct interactions, thus forming a continuous path for the flow of energy.

We now turn to the dynamics generated by Eq. (8) for the initial conditions IC2, i.e., when a select group of waves in a system possesses large energy initially. As expected, based on the discussion of IC1 such a group will efficiently exchange energy with other modes through its MF members. The fraction of the energy exchanged depends on the relative number of the MF modes within a group. The extent to which this energy reaches the rest of the system depends on the number and length of the continuous paths of energy flow associated with the MF modes within the group. The group generated energetics are *MF mode additive*, i.e., the energy exchanges produced by the group are the same as those obtained by combining the contributions produced by each MF member of the group separately. A given wavenumber, for example, exchanges energy with other wavenumbers directly through the triads containing its MF modes and also indirectly through those triads whose MF modes are linked, through a sequence of direct interactions, to any MF mode of the wavenumber in question. It's complete energetics could also be reconstructed by combining the individual energetics of its MF members.

In Fig. 6 we illustrate the energetics associated with wavenumber 3 at $H = 10$ km for the system given in Fig. 2. Of the three modes making up this wavenumber: [3,1,R], (3,1,+) and (3,2,+) only [3,1,R] qualifies for the MF status, and is, therefore, responsible for most of the energy exchange between wavenumber 3 and other wavenumbers. Since neither mode (1,2,R) nor (2,2,R) which [3,1,R] interacts with directly (see Fig. 2), qualifies for the MF status elsewhere in the system, energy remains contained in that triad. Thus, as Fig. 6 shows, energy loses in wavenumber 3 (about 35%, i.e., the amount contained in [3,1,R]) are associated with simultaneous gains in wavenumber 1 and 2. Maximum energy gain of wavenumber 2 exceeds that of 1, because the frequency of (2,2,R) exceeds that of (1,2,R).

The discussion thus far demonstrates the energetically active character of the MF modes. The question arises, then, if circumstances exist under which efficient energy exchanges in a system can originate with its non-MF members? This possibility exists for IC2, if the non-MF group chosen to have large energy initially contains at least one

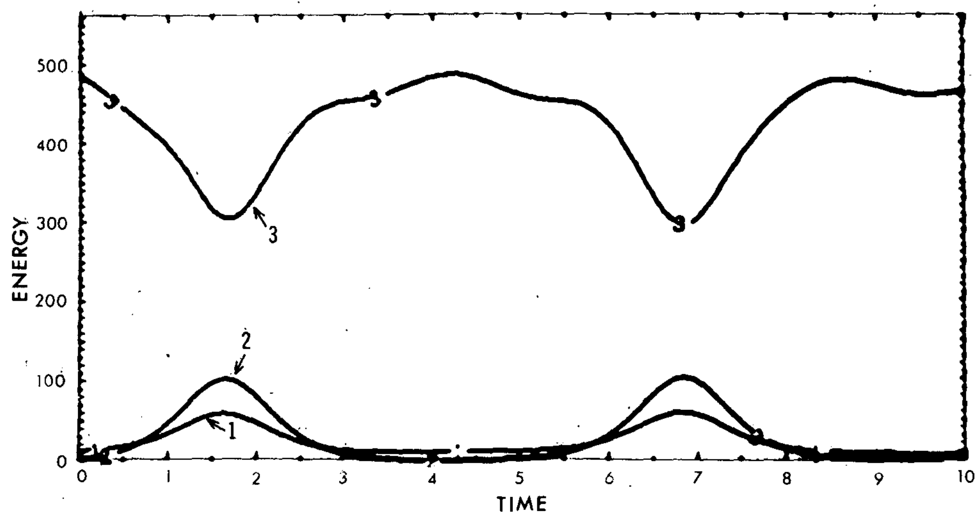

FIG. 6. Energy evolution of wavenumber 3 in the wave system given in Fig. 2, when initially 90% of the system's energy is placed in that wavenumber.

pair of modes belonging to the *same* triad. As required by the integral energy constraints [see DL, Eq. (3.2)] such a pair will efficiently exchange energy with the MF member of the triad which, in the presence of further interactions, can pass this energy outside the original triad. This kind of energy transfer, however, is unlikely to occur in the real atmosphere since the probability of exciting *both* non-MF modes in a single triad by the tropical energy generating mechanisms is small.

We conclude that the energy transfers within closed systems of resonantly interacting waves are initiated mainly by their MF members. Each MF mode involves in energy exchanges, a group of waves consisting of the members of those triads in which it has the MF status and those triads whose MF modes are linked, through a sequence of direct interactions, to it. Within the group, modes closest in frequency to the MF mode undergo largest energy gains. Usually such a group consists of 5 or less triads. Therefore, for the energy to be redistributed *throughout* a system, it must be placed initially in at least those of its MF modes whose combined energetics include *all* the triads in the system. For example, for the system given in Fig. 2, energy must initially be placed in at least these five MF modes: [4,2,−], [5,2,−], [6,2,−], [8,2,−], [3,1,R], in order to excite that entire system.

The wave systems, we recall, are strongly dominated by triads composed of two high-frequency modes and one low-frequency mode. As a result of this pronounced frequency separation within triads, inertia gravity and high-wavenumber Kelvin waves tend to be energetically most active. Rossby and mixed Rossby-gravity waves tend to be energetically least active; the only exception occurs in the case of triads composed solely of low-frequency modes. The participation of the low-frequency modes in the energy exchanges is somewhat enhanced at greater fluid depths, where smaller frequency differences exist among the modes due to strong wavenumber concentration at the lowest integers. At small fluid depths, the wide spread in the zonal wavenumbers can create such large frequency differences that the low-frequency modes (especially those corresponding to higher wavenumbers) become merely catalysts for energy exchanges between the high-frequency modes. Also, as a result of the variations in the zonal wavenumber concentration with depth, at greater fluid depths accumulation of energy is favored at the longest wavelengths, whereas at smaller fluid depths the energy can be spread more widely in the wavenumber domain.

5. Summary and conclusions

Resonant interactions in the context of the divergent equatorial β-plane were investigated for closed systems of discrete equatorial waves. The systems were determined iteratively starting with a wave of geophysical importance. The iteration was continued until self-consistency of the wave system was obtained for discrete waves with meridional structures observed in the atmosphere. These systems, nearly identical at a given fluid depth, involve all the equatorial wave types but are very strongly dominated by triads composed of one mode mainly due to rotational effects (low frequency) and two modes mainly due to gravitational effects (high frequency). The systems exhibit dependence on the depth of the fluid through the fundamental wavenumber such that at smaller fluid depth the systems generally become larger, more spread in the wavenumber domain, and hence, less concentrated at low wavenumbers.

The energy exchanges in a system, which were studied via the systematic varying of initial conditions, allowing treatment of the problem as a stability problem, were found to be governed by the frequency spectrum of the individual triads. The individual modes which do not attain maximum frequency (MF) status in any triad of which they are a member were found to be stable to their resonant perturbations. Those modes which do reach MF status in at least one triad were found to be unstable. Energetic activity of a given non-MF mode depends upon the frequency difference between it and the MF mode of the triad in question.

When energy was placed in a group of waves energetics were found to be MF mode additive. As a consequence, significant energy cannot be transferred further through a system once an appreciable amount of energy does not reach the MF modes of other triads. Thus, we have a dynamical separation within the wave system. This leads us to conclude that the wave interaction mechanism alone cannot account for the wave spectrum of the equatorial atmosphere, especially in the presence of friction (absent in this study), given a very scale-selective excitation mechanism. Therefore, we might speculate that the excitation of the tropical atmosphere occurs over broad scales and/or there may be a Rayleigh type wave instability present under which the non-MF modes might also transfer an appreciable amount of energy to other scales. However, resonant interactions may still play a major role in the ocean and at larger wavenumbers in the atmosphere where the wave spectrum can be taken as continuous, rendering the methods of this study intractable.

Acknowledgment. This research was supported in part by the National Science Foundation under Grant ATM 74-23439A01. During the completion of this research, Richard C. Deininger was supported by a UCAR Fellowship, which is gratefully acknowledged.

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