Thunderstorm Vertical Velocities Estimated from Satellite Data

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ABSTRACT

Infrared geosynchronous satellite data with an interval of 5 min between images are used to estimate thunderstorm top ascent rates on two case study days. A mean vertical velocity of 3.4 m s^{-1} for 23 clouds is calculated at a height of 8.7 km. This upward motion is representative of an area of approximately 10 km on a side. Thunderstorm mass flux of $\sim 2 \times 10^8$ kg s^{-1} is calculated, which compares favorably with previous estimates. There is a significant difference in the mean calculated vertical velocity between elements associated with severe weather reports ($\bar{w}$ = 4.9 m s^{-1}) and those with no such reports (2.4 m s^{-1}).

Calculations were made using a velocity profile for an axially symmetric jet to estimate the peak updraft velocity. For the largest observed w value of 7.8 m s^{-1} the calculation indicates a peak updraft of $\sim$50 m s^{-1}.

1. Introduction

Vertical velocity is a basic parameter in meteorological analysis and forecasting. However, unlike horizontal velocity, it usually cannot be observed directly. The presence of upward air motion can often be inferred by the occurrence of clouds and precipitation, but quantitative evaluation of vertical motion must usually be made through the omega equation, the adiabatic method or a similar indirect technique. An exception to this is the recent work with multiple-Doppler radar data.

In the current paper we present the results of using a simple method to estimate thunderstorm cloud top vertical velocity from SMS/GOES rapid-scan (5 min interval) window channel infrared (IR) data. Time rate of change of cloud-top minimum equivalent blackbody temperature T_{BB} is converted to vertical velocity w by

$$w = \left(\frac{\partial T}{\partial z}\right)^{-1} \frac{dT_{BB}}{dt}, \quad (1)$$

where the lapse rate is determined from rawinsonde data. An example calculation using SMS data and Eq. (1) was given by Adler and Fenn (1976). In the following sections the calculated vertical velocities are compared for clouds with associated severe weather reports and for those with no such reports, and the computed values are also compared with previous estimates of thunderstorm vertical velocities.

2. Description of analysis technique

The analysis of the digital satellite data was performed on the Atmospheric and Oceanic Information Processing System (AOIPS), an interactive image analysis system described by Billingsley (1976). Sequences of images are enhanced, thunderstorm elements are isolated and identified by locating relative minima of T_{BB} in the IR images. Their maximum gray level (minimum T_{BB}) are then recorded. The analysis of the thunderstorms used in this study (on 24 April 1975 and 6 May 1975) was part of a larger effort (see Adler and Fenn, 1979). On each of these study days an area of convection was monitored during a set period of time ($\sim$4 h). Thunderstorm elements were defined and time histories of each element were determined.

The emphasis in the earlier work (Adler and Fenn, 1979) was on convective elements after they had reached $\sim$10 km in height (as estimated by the satellite IR data). This paper highlights observations of thunderstorms earlier in their growth cycle as they penetrate upward through the middle troposphere. It should be emphasized that not all thunderstorms can be observed at middle tropospheric heights. This is because they are often hidden by dense cirrus clouds produced by previous convection. In the two case studies that will be described, of the thunderstorm elements defined above 10 km (T_{BB} = 226 K), only about 25–30% could be detected at lower heights. On typical con-

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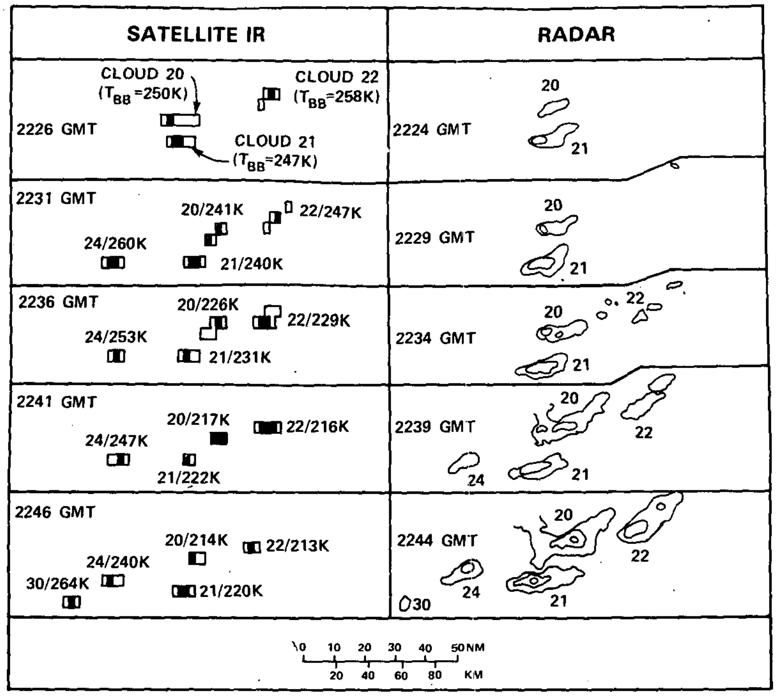

FIG. 1. Evolution of 24 April 1975 thunderstorms in southern Oklahoma as observed by satellite IR and radar measurements.

vection days this percentage will vary with time, being large early in the day as convection begins and decreasing as the buildup of thunderstorm-produced high clouds obscures younger convective elements. The results described in this paper concern that relatively small group of thunderstorms that could be identified at lower levels.

A representation of the thunderstorm features seen in the satellite IR is given in Fig. 1 for a region of southern Oklahoma on 24 April 1975. Also shown are radar echo intensity contours from the National Weather Service radar at Oklahoma City. The storms are located ~90 km to the south and southeast of the radar site. On the satellite side of the diagram the cloud identification number and minimum T_{BB} are noted in each panel. The shaded blocks are the areas covered by the satellite data points with the minimum temperature. The smallest possible area is exemplified by the shaded area in cloud 22 at 2226 GMT. The additional outlines are T_{BB} isotherms at higher temperatures that are associated with that particular cloud element.

Fig. 1 illustrates that the cloud elements being identified in the satellite data correspond to individual thunderstorms as evident in the radar presentation. Cloud 22 has a very rapid decrease of T_{BB} with time, and is associated with reports of large hail. Cloud 20 also has a hail report associated with it, while the other defined clouds in Fig. 1 do not.

3. Vertical velocity estimates

a. Sources of error

The vertical velocity estimates presented in this paper are subject to error because of uncertainties in the satellite radiance measurements and possible unrepresentativeness of the data. The first question is: How are cloud top ascent rate and vertical velocity related? Even if the satellite measurements are without errors and are representative, the w calculated using Eq. (1) is actually the cloud top ascent rate. Due to mixing at the cloud top this may not be identical to the vertical air motion at the cloud top. For example, Woodward (1959) shows laboratory results indicating that the center of an isolated thermal rises at approximately twice the speed of the thermal cap. However, this is the case for a small, decelerating thermal, with extensive mixing through the sides. In addition, for a decelerating thermal the center of the thermal, with a vertical velocity of w_1 at time t_1, will show a decrease of w with time so that when (at t_2) it reaches an altitude equal to that of the thermal cap at the earlier time t_1 its velocity will have decreased significantly to w_2, much closer in magnitude to the rate of rise of the thermal cap as it passed that altitude. Thus for the scale of observation in the current analysis, the cloud top ascent rate is probably nearly equal to the magnitude of the vertical air flow at that level.

Another possible source of error is in the cloud emissivity. For the thick water clouds that were part of this study, the emissivity (ϵ) is very close to unity. In order to use T_{BB} in place of T in Eq. (1), $\epsilon = 1$ must be assumed. For ice clouds, especially thin cirrus clouds, the emissivity can be substantially less than 1.0. As the thunderstorm top glaciates, the change from water to ice may reduce the cloud top emissivity and T_{BB} will be larger than T, the cloud-top temperature. This effect, however, appears to be small and will not affect the vertical velocity calculations. This conclusion is based on calculations made with the help of tables presented by Hunt (1973). For a water cloud with liquid water content (LWC) of 4×10^{-4} kg m^{-3}, 95% of the emitted radiation at the 11 μm wavelength comes from the top 84 m of the cloud. For an ice cloud with the same LWC value, the depth of extinction (to the 95% level) is 150 m. Therefore, the water-ice variation in cloud-top structure will cause only a very small (<1 K) variation in T_{BB}. Calculations based on coefficients presented by Cox (1977) also support this conclusion.

The most serious source of error stems from the satellite data itself. The IR channel has an instantaneous field of view (IFOV) of 8 km on a side at the subsatellite point and approximately 10 km at 40°N. As noted by Negri *et al.* (1976) the use of SMS/GOES IR temperatures to determine thunderstorm height results in underestimates, especially for small elements. In comparison with radar measurement of thunderstorm tops, the satellite underestimation is ~2 km. This effect is related to two factors. First, the satellite, because of its rather large IFOV, is averaging over an area of ~100 km^2 compared with the radar observation, which is applicable to a much smaller area. Thus the radar will be identifying smaller, higher features because of its better resolution. The second factor is inadequate sensor response when going from a warm (low) to a cold (high) target (Negri *et al.*, 1976). In general, the bias in the estimation of storm height will not significantly affect the vertical velocity calculation. However, it may affect the height to which the velocities are assigned.

Other errors might arise from the use of inaccurate lapse rates in Eq. (1). In the calculations to follow we use a smooth profile which is a mean of the ambient (determined from rawinsonde data) and the moist adiabatic lapse rate. This compromise was chosen because it was thought that, although the air in updraft cores of large thunderstorms is believed to ascend moist adiabatically, the large area represented by the satellite data implies a large environmental effect. The calculations are also rather insensitive to variations in the lapse rate. Using either the moist adiabatic or the ambient lapse rate instead of the average of the two produces only 10% differences in the calculated velocities.

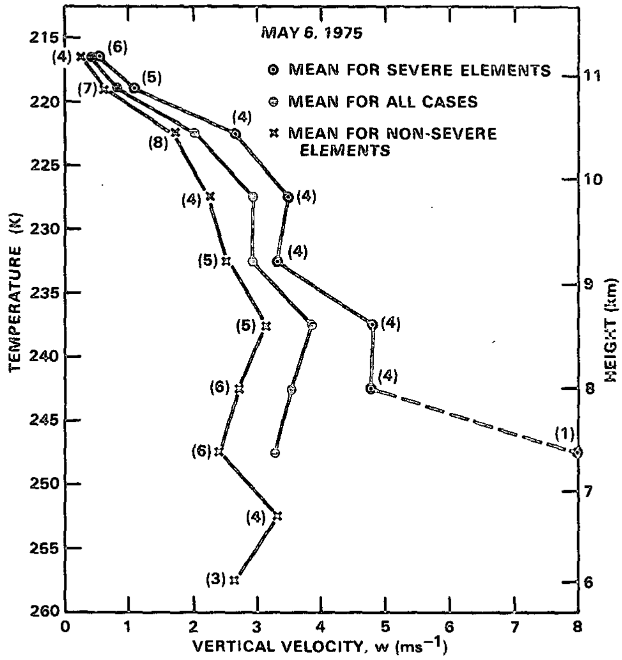

FIG. 2. Composite vertical velocity profiles for 6 May, 1975. Numbers in parentheses are the number of cases.

The final validation of the calculated vertical velocities must come through a careful comparison with radar, aircraft or satellite stereo observations. Some of the work toward these comparisons is underway. Based on the previous discussion of errors, however, the authors believe the values calculated from (1) are accurate to within 20%, for the size area they represent. Comparison with other estimates and calculations will be made in the following sections.

b. Vertical velocity results

Fourteen elements on 6 May 1975 and 14 on 24 April 1975 were analyzed. Minimum T_{BB} as a function of time for each element was plotted and dT_{BB}/dt values were calculated. The warmest or lowest of the monitored elements were at T_{BB} = 260 K (~6 km). Not all thunderstorms could be observed from that point upward through the remainder of the troposphere. Some elements were obscured by other storms; other elements were not detected until they penetrated middle level cloud fields.

Vertical velocities were calculated every 5 K in the vertical for each cloud element or thunderstorm using Eq. (1) and a lapse rate halfway between ambient and moist adiabatic. The mean w was then calculated for various categories of clouds to produce composite profiles. The results for the 6 May case are shown in Fig. 2. Profiles are shown for thunderstorms associated with severe weather reports (based on National Severe Storms Forecast

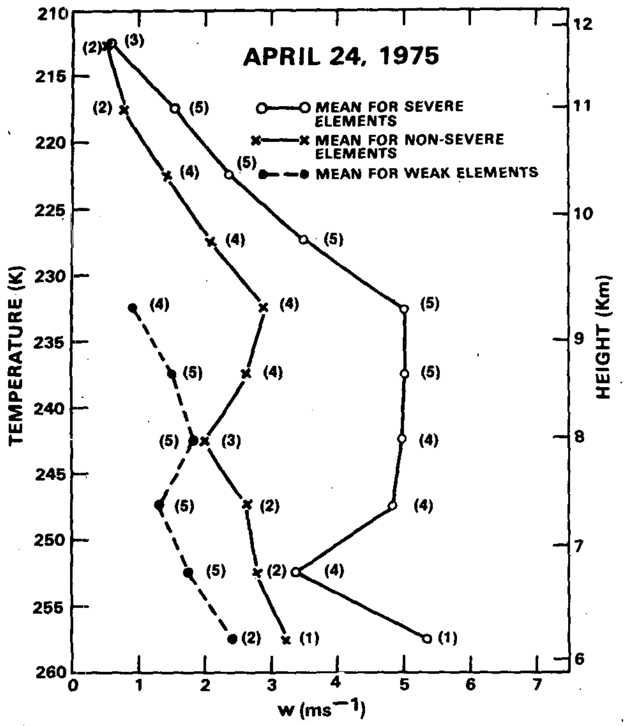

FIG. 3. Composite vertical velocity profiles for 24 April, 1975.

Center logs) and those with no accompanying reports. The numbers in parentheses indicate the number of cases constituting each mean or composite vertical velocity. The mean profile of all cases is also shown. All the elements defined in the 6 May case were intense thunderstorms reaching heights >10.5 km as determined from the SMS/GOES T_{BB}'s. These heights are probably low compared to radar echo top heights. The 6 May storms were located in a relatively narrow band from eastern Nebraska to Texas along a north–south oriented cold front.

The profile in Fig. 2 for all of the 6 May cases indicates a mean magnitude for w of ~3.5 m s^{-1} at 8 km. Above that level there is a gradual decrease in the upward flow and a more rapid decrease above 10 km. Between 10 and 11 km the horizontal divergence can be estimated through the continuity equation, assuming incompressibility, so that

$$-\frac{\partial w}{\partial z} = \nabla \cdot \mathbf{V} = 2 \times 10^{-3}\ \mathrm{s}^{-1}. \qquad (2)$$

The vertical velocities in Fig. 2 and the calculated divergence noted above are applicable to an area of about 10 km on a side. The calculated vertical velocities *do not* represent updraft core velocities, which could be an order of magnitude larger when measured on a horizontal scale of 1 km (see Section 5).

The results displayed in Fig. 2 indicate that on the average the elements with associated severe weather reports have larger vertical velocities. This is not surprising since intensity of convection has always been closely associated with severe weather.

A similar diagram for 24 April 1975 is given in Fig. 3. The convection of interest on this day was centered in southwestern Missouri. The composite w profiles for three categories are displayed. The additional category is for weak elements which did not reach a height of 10 km (as determined by the T_{BB} values). In the layer from 7 to 9 km the average w is ~1.5 m s^{-1} for those storms. This is significantly lower than the composite for "non-severe" elements in either Figs. 2 or 3. Part of the difference in w values may be due to a size difference and therefore a difference in the fraction of the IFOV filled by the ascending cloud. The severe elements have a mean w of 4–5 m s^{-1} in the 6–9 km height range, similar to that found in the 6 May case (Fig. 2).

The 235–240 K level (~8.7 km) is representative of the layer of relatively large vertical velocities on both days. Fig. 4 shows the frequency distribution of the 23 elements or clouds for these two days. The hatched portion of the histogram contains the values for the severe weather elements. The average w for all 23 cases is 3.4 m s^{-1}. The severe and non-severe elements have average values of 4.9 and 2.4 m s^{-1}, respectively. The severe thunderstorms dominate the high end of the distribution where six out of seven cases with w > 4 m s^{-1} are associated with severe weather reports. The difference in mean w between severe and non-severe clouds is significant at the 1% level using the t-distribution test (Panofsky and Brier, 1963).

c. *Mass flux calculation*

Because the values of w calculated in the last section are representative of an area larger than a typical thunderstorm updraft, vertical volume or mass flux calculations can be simply made from

$$F_m = \rho A w, \qquad (3)$$

where F_m is the vertical mass flux, ρ the density and A the area. For the 235–240 K layer (~8.7 km) ρ is assumed to be 5×10^{-1} kg m^{-3}, and A is assigned a value of 100 km^2 for the area of the satellite IFOV.

With the given values for ρ and A the mean w for all storms of 3.4 m s^{-1} is converted to a mass flux of 1.7×10^8 kg s^{-1}. The mean w of severe elements (4.9 m s^{-1}) is equivalent to a mass flux of 2.4 $\times 10^8$ kg s^{-1}. These magnitudes are for the mass flux through a given layer associated with a growing thunderstorm top. The calculated values compare favorably with results presented by other investigators. Kropfli and Miller (1976) calculate a

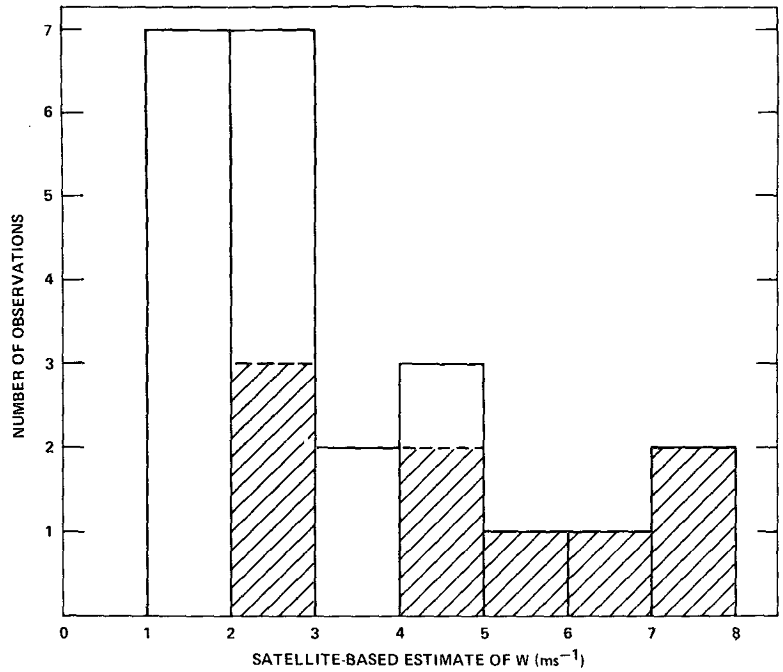

FIG. 4. Frequency distributions of estimated vertical velocities at 8.7 km for clouds on 24 April and 6 May, 1975. Hatched portion of histogram indicates thunderstorms with accompanying severe weather reports.

value of 1.9–2.0 $\times$ 10^8 kg s^{-1} between 8 and 9 km for a northeast Colorado storm calculated using vertical velocity inferred from dual-Doppler radar data. Auer and Marwitz (1968) present results of the cloud-base mass flux into 18 hailstorms on the high plains deduced from aircraft measurements. Their average value is 2.3 $\times$ 10^8 kg s^{-1}. Therefore, it appears that the upward flow deduced from the satellite observations is of a reasonable magnitude when compared to calculations and observations on approximately the same scale. Inferences about the magnitude of the maximum updraft are presented in Section 5.

4. Example of intense thunderstorm

On 24 April 1975 a severe thunderstorm complex developed over extreme northeastern Oklahoma in the late afternoon and moved into southwestern Missouri around sunset. The most significant severe weather associated with the system was the Neosho, Missouri, tornado which touched down at approximately 0040 GMT 25 April. By following the evolution of the storm system backward in time, the initial intense convection can be detected and its associated rapid cloud-top growth calculated.

The Neosho cloud system was designated cloud 18 as part of a larger study of this day. Fig. 5 exhibits minimum cloud-top T_{BB} as a function of time for cloud 18 in its early stages. The temperature drops precipitously between 2200 and 2220 GMT with a maximum calculated rate of 4 K min^{-1}. The drop in temperature between 260 and 220 K takes only a little more than 15 min. This type of rapid change emphasizes the importance of short-interval data to study and monitor thunderstorm activity.

Using a lapse rate varying from 7.8 K km^{-1} at 260 K, to 8.6 K km^{-1} at 240 K, to 8.0 K km^{-1} at 215 K the vertical velocity was calculated using Eq. (1). The results for cloud 18 are shown in Fig. 6. The maximum w is 7.8 m s^{-1} at about 9 km. Above that height there is a rapid decrease of w with height, with a calculated divergence [using Eq. (2)] of 4.2 $\times$ 10^{-3} s^{-1} over nearly a 2 km deep layer. It must be remembered that the vertical velocities and divergences are applicable for an area of $\sim$10 km on a side.

The temperature curve in Fig. 5 flattens out and reaches a plateau at 214 K, which is equivalent to a height of $\sim$11.5 km. Figs. 5 and 6 are indicative of the convective surge of the first thunderstorm cell in the system. After 2320 GMT the organization of the system (as viewed by the satellite) becomes more complex, with three cold centers appearing. The main center (designated cloud 18c) undergoes some short-lived growth associated with the Blue Jacket, Oklahoma, tornado, but then remains quiet, although definable, until a new cell evidently penetrates the already cold cirrus canopy. With this new cell the T_{BB} values again fall, this time to 206 K. During this period of decreasing T_{BB} values, the Neosho, Missouri, tornado touched down. A further

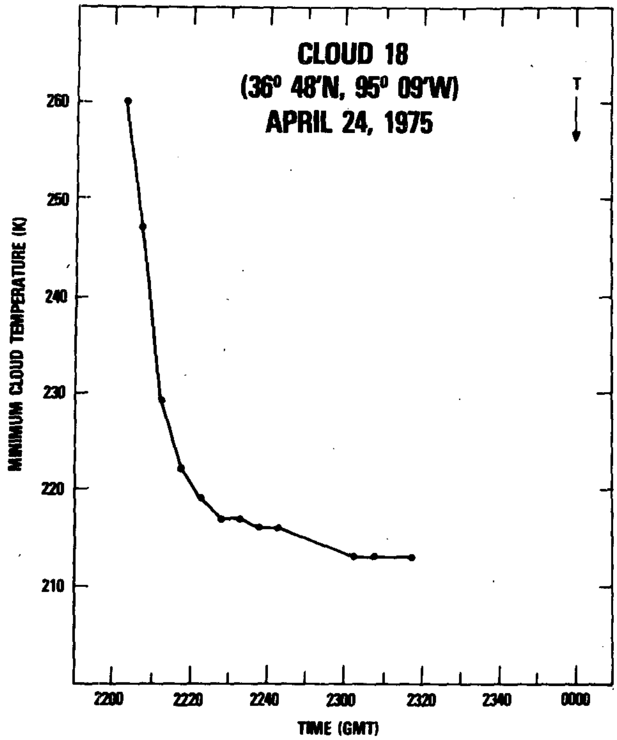

FIG. 5. Minimum equivalent blackbody temperature T_{BB} as a function of time for cloud 18 on 24 April 1975.

drop in temperature to 201 K occurs 20 min later. The minimum temperature on the nearby Monett, Missouri sounding was 210 K, indicating that the storm had penetrated well into the stratosphere. Although the initial disturbance (Fig. 5) leveled out at 214 K, it was the starting point for convection that eventually reached 201 K, about 13 km in height, 1 km above the tropopause.

5. Interpretation of calculated vertical velocities in terms of maximum updraft

The vertical velocities presented in the previous sections of this paper are mean velocities over an area equivalent to the satellite instantaneous field of view (IFOV), which in this case is ~100 km². In the temperature range 235–240 K (~8.7 km) the 23 observed w's based on the satellite data ranged from 1.2 to 7.8 m s^{-1}, with a mean of 3.4 m s^{-1}. Although these are very large values when compared to typical synoptic-scale w's, they are small when compared to maximum thunderstorm updraft magnitudes. Thunderstorm updrafts can reach magnitudes of 10 m s^{-1} very easily and are typically 30 m s^{-1} in supercell thunderstorms (Browning, 1977; Davies-Jones, 1974). These large updraft values are probably representative of an area ~1 km². Thus there are two orders of magnitude difference in the area covered by the estimated w's obtained from the satellite data in this study and the area covered by the peak updraft velocity.

Assuming axial symmetry and a knowledge of the shape and size of the radial profile of vertical velocity, one can make an estimate of the maximum updraft magnitude. Kyle *et al.* (1976) have investigated updraft profiles determined from penetrating aircraft and have fitted mathematical expressions to the observations. One of the expressions tested, with good results, is the profile for an axially symmetric jet (Schlichting, 1968). The formula is

$$w = w_0 e^{-a(r/R)^2}, \qquad (4)$$

where w_0 is the peak w, r is the radial distance, R the radius of the updraft, and the constant $a = 2.3$. The constant was chosen in the present work so that $w = 0.1\, w_0$ at $r = R$. That is, the updraft radius R is defined so that the vertical velocity is not zero at the updraft edge, but one-tenth the maximum value.

Integrating Eq. (4) over a circular area of radius r_1 and dividing the result by the area of the integration produces an expression for the mean w over the area, i.e.,

$$\bar{w} = \frac{w_0}{a}\left(\frac{R}{r_1}\right)^2 [1 - e^{-a(r_1/R)^2}]. \qquad (5)$$

For values of $r_1 > R$, the term in brackets approaches a value of 1.

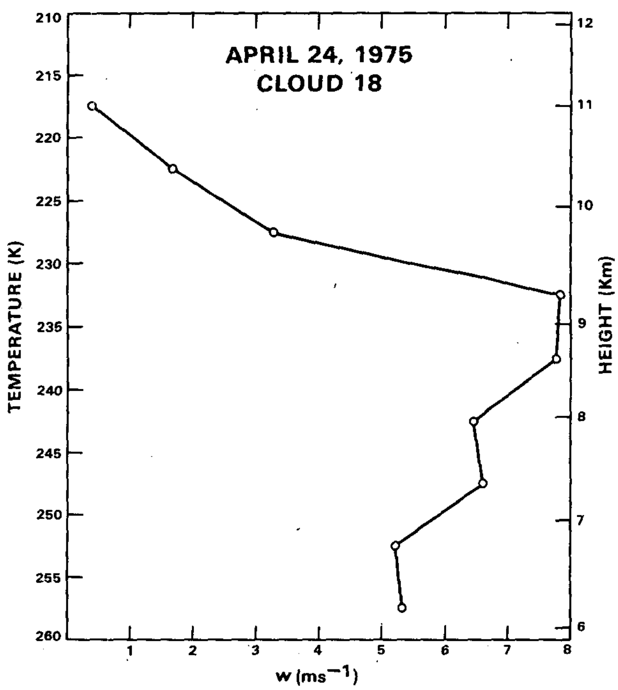

FIG. 6. Estimated vertical velocity profile for cloud 18.

If the radius of integration is equal to the updraft radius ($r_1 = R$),

$$\bar{w} = 0.39 w_0. \quad (6)$$

Therefore, if the updraft, as defined by R, exactly fills the satellite IFOV, the maximum updraft will still be 2.5 times the satellite-based estimate of $\bar{w}$.

The size of thunderstorm updrafts is highly variable (Browning, 1977). Based on the discussion of updraft sizes by Browning (1977) and an examination of cross sections by Kropfli and Miller (1976) and Ray (1976) of thunderstorm vertical motions deduced from Doppler radar data, an updraft radius of 3 km is reasonable.

For an area of 100 km² (approximately equal to the IFOV), the equivalent radius of integration is 5.6 km. Thus with $R = 3$ km and $r_1 = 5.6$ km,

$$\bar{w} \approx 0.12 w_0. \quad (7)$$

Thus, with all the assumptions as to profile shape and updraft size, Eq. (7) indicates that the mean $\bar{w}$ of 3.4 m s^{-1} is equivalent to a w_0 of 28 m s^{-1} and the 7.8 m s^{-1} value from the Neosho storm (Section 4) is equivalent to a w_0 of 65 m s^{-1}.

If the updraft radius is larger than 3 km, the w_0 values would be smaller than just calculated. For an R of 3.5 km, instead of 3 km, the 65 m s^{-1} value in the last paragraph would drop to 46 m s^{-1}. It is obvious that the calculated w_0 is sensitive to the size of the updraft. Observations summarized by Kyle *et al.* (1976) indicate that R is a function of w_0, at least for small radii. An accurate expression for R as a function of w_0 would allow Eq. (5) to become a statement of the relation between $\bar{w}$ and w_0.

Despite the variability and sensitivity of the w_0 calculations, it is evident that the satellite observations on a scale of 10 km are producing $\bar{w}$ of up to approximately 8 m s^{-1}, and this can be interpreted as being roughly equivalent to a maximum updraft of 50 m s^{-1} in intense thunderstorms.

6. Summary

Rapid-scan (5 min interval) SMS/GOES IR data have been used to estimate thunderstorm top ascent rates for severe and non-severe thunderstorms on two case study days. The vertical velocities are calculated by converting the time rate of change of minimum T_{BB} to vertical velocity w through use of a lapse rate. On both days examined (6 May and 24 April 1975) the thunderstorm elements with associated severe weather reports have larger average w's. At a temperature level (235–240 K) equivalent to a height of ~8.7 km, the 23 elements monitored had a mean w of 3.4 m s^{-1}. The severe and non-severe elements had mean w's of 4.9 and 2.4 m s^{-1}. Intensity of convection appears to be correlated with the occurrence of severe weather, and the satellite data appear to be capable of quantifying the convection intensity.

The calculated vertical velocities are representative of an area (100 km²) roughly equivalent to the satellite instantaneous field of view (IFOV). Mass flux estimates of ~2 × 10⁸ kg s^{-1} are calculated, which are reasonable in comparison with other estimates.

The largest w, calculated directly from the satellite data (at the 8.7 km height), was 7.8 m s^{-1} occurring with the initial convection associated eventually with the Neosho, Missouri, tornado. This vertical velocity corresponds to a rate of cloud top temperature decrease of 4 K min^{-1}, which indicates the need for high time resolution geosynchronous satellite data even with the relatively coarse spatial resolution of the IR measurements.

Calculations were performed to estimate the peak updraft velocity from the satellite-based values (averages over 100 km² areas). Using the velocity profile formula for an axially symmetric jet to represent the thunderstorm updraft, we derived an expression relating $\bar{w}$ (the satellite-based w) to w_0 (peak updraft velocity) and R (updraft radius). With a reasonable value of R (3–3.5 km), the $\bar{w}$ of 7.8 m s^{-1} for the Neosho storm produces an estimate of approximately 50 m s^{-1} for w_0.

The calculations and examples of this paper reveal that the SMS/GOES rapid-scan data can provide useful quantitative information on thunderstorm vertical motions and therefore convection intensity. Although the vertical motion calculations must undergo closer scrutiny through comparison with Doppler radar estimates and possibly with vertical growth rates determined from stereo geosynchronous satellite images, they provide a starting point for the quantitative use of the satellite data in the study of thunderstorms and provide the possibility for eventual use in the monitoring of thunderstorm activity.

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