Conditional Instability of the First Kind Upside-Down

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ABSTRACT

It is shown that entrainment leads to the generation of turbulence kinetic energy in a stratocumulus layer when the virtual temperature jump at the cloud top is weaker than a critical value. The critical value increases as the relative humidity of the air above cloud top decreases. This result is interpreted as a criterion for the instability of the layer cloud to penetrative downdrafts. The role of the instability in determining the subtropical and tropical distributions of boundary-layer cloudiness is assessed.

1. Introduction

Turbulent entrainment ordinarily tends to destroy turbulence kinetic energy, because the buoyancy force acts to oppose vertical motions in the statically stable layer which usually marks the edge of the turbulence. Under dry adiabatic processes, only the extraordinary circumstance of a statically unstable edge layer can permit entrainment to generate turbulence kinetic energy through the buoyancy force. But Lilly (1968) pointed out that the evaporative cooling of unsaturated air which has been entrained into a cloud can, under some conditions, cause the entrained air to sink unstably as a convective downdraft. Because this instability is somewhat similar to conditional instability of the first kind (cumulus instability), we call it conditional instability of the first kind upside-down (CIFKU). The purposes of this paper are to determine the criterion for the onset of CIFKU, to suggest a simple theory of the effects of CIFKU on a stratocumulus layer, and to assess the role of CIFKU in determining the global distribution of boundary-layer cloudiness.

2. The stability criterion

We measure the relative buoyancy of fluid particles using the virtual dry static energy

\[ s_v = c_p T_v + g z, \quad (2.1) \]

where the virtual temperature

\[ T_v = T (1 + \delta q - l) \quad (2.2) \]

is defined in such a way as to take into account the effects of both vapor (with mixing ratio \( q \)) and liquid (with mixing ratio \( l \)). Here \( \delta = 0.608 \). As moist (and dry) conservative variables, we use the total water mixing ratio \( (q + l) \) and the moist static energy

\[ h \equiv c_p T + g z + L q. \quad (2.3) \]

Also, we define the saturation moist static energy

\[ h^* \equiv c_p T + g z + L q^*, \quad (2.4) \]

where \( q^* \) is the saturation mixing ratio, and the following useful thermodynamic coefficients:

\[ \epsilon = \frac{c_p T}{L}, \quad (2.5) \]

\[ \gamma = \frac{L}{c_p} \left( \frac{\partial q^*}{\partial T} \right)_p, \quad (2.6) \]

\[ \beta = \frac{1 + (1 + \delta) \gamma \epsilon}{1 + \gamma}. \quad (2.7) \]

For convenience, the variations of \( \gamma \) and \( \beta \) with \( T \) and \( p \) are plotted in Fig. 2. We can use (2.1)–(2.3) and (2.5) to write

\[ s_v = h - [1 - (1 + \delta) \epsilon] L q - \epsilon L (q + l). \quad (2.8) \]

We let \( \partial_p \) represent a differential at constant pressure, so that

\[ L \partial_p q^* = \gamma \partial_p (c_p T) = \frac{\gamma \partial_p h^*}{1 + \gamma}. \quad (2.9) \]
With this preparation, we can now write

\[ \partial_p s_v = \beta \partial_p h + [1 - (1 + \delta)\varepsilon]\left(\frac{\gamma \partial_p h}{1 + \gamma} - L \partial_p q\right) - \epsilon L \partial_p (q + l) \]

\[ = \beta \partial_p h + [1 - (1 + \delta)\varepsilon]\left[\frac{\gamma}{1 + \gamma} \partial_p (h - h^*) + \dot{L} \partial_p (q^* - q)\right] - \epsilon L \partial_p (q + l) \]

\[ = \beta \partial_p h + \left[\frac{1 - (1 + \delta)\varepsilon}{1 + \gamma}\right] L \partial_p (q^* - q) - \epsilon L \partial_p (q + l), \hspace{1cm} (2.10) \]

from which it follows that the turbulent fluxes of \( s_v, q^* - q \) and \( q + l \) across isobaric surfaces are related by

\[ F_s = \beta F_h + \left[\frac{1 - (1 + \delta)\varepsilon}{1 + \gamma}\right] LF_{q^* - q} - \epsilon LF_{q + l}. \hspace{1cm} (2.11) \]

In uniformly saturated air \( q^* \) and \( q \) are everywhere equal, so that (2.11) simply reduces to

\[ F_s = \beta F_h - \epsilon LF_{q + l}. \hspace{1cm} (2.12) \]

We now ask how entrainment influences \( F_s \). As indicated in Fig. 1, we let the subscripts \( B \) and \( B^+ \) denote levels just below and just above the cloud top level, respectively. Assuming that the layer between \( B \) and \( B^+ \) is too thin to accommodate significant storage of \( h \) or \( (q + l) \), and using the fact that the turbulent fluxes vanish at level \( B^+ \), we can express the conservation laws for \( h \) and \( (q + l) \) as

\[ (F_h)_B = -E \Delta h + \Delta R, \hspace{1cm} (2.13) \]

\[ (F_{q + l})_B = -E \Delta (q + l), \hspace{1cm} (2.14) \]

respectively, where \( E \) is the entrainment mass flux, \( R \) the radiation flux, and

\[ \Delta ( ) = ( )_{B^+} - ( )_B. \hspace{1cm} (2.15) \]

From (2.12)–(2.14), we find that in uniformly cloudy air,

\[ (F_s)_B = -E [\beta \Delta h - \epsilon L \Delta (q + l)]. \hspace{1cm} (2.16) \]

We first note that, as pointed out by Lilly (1968), cloud top radiative cooling tends to drive convection in the cloud layer (see also Randall, 1980). This largely explains why cloud layers are almost always turbulent. But our purpose here is to determine under what conditions entrainment itself promotes convection [i.e., tends to increase \( (F_s)_B \)]. According to (2.16), the requirement is

\[ \beta \Delta h - \epsilon L \Delta (q + l) < 0; \hspace{1cm} (2.17) \]

this is the criterion for the onset of CIFKU. Assuming that the layer between \( B \) and \( B^+ \) is infinitesimally thin, we can apply (2.10) to relate the various

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**Fig. 1.** Schematic diagram of the distributions of temperature and mixing ratio near cloud top. The stippled area represents cloudy air.

**Fig. 2.** Distributions of (a) \( \gamma(T, p) \) and (b) \( \beta(T, p) \).
discontinuities across the cloud top. In this way, we find that

$$\beta \Delta h - \epsilon L \Delta(q + l) = \Delta s_v - (\Delta s_v)_{\text{crit}},$$

(2.18)

where

$$(\Delta s_v)_{\text{crit}} = \frac{1 - (1 + \delta)c}{1 + \gamma} L(q_B^+ - q_B^+).$$

(2.19)

In view of (2.18), we can rewrite (2.17), the criterion for the onset of CIFKU, as

$$\Delta s_v < (\Delta s_v)_{\text{crit}}.$$  

(2.20)

This shows clearly that CIFKU occurs when $\Delta s_v$ is less than a positive critical value which increases as the relative humidity of the air above cloud top decreases. Fig. 3 shows the distribution of $(\Delta s_v)_{\text{crit}}/c_p$ for average summer conditions over the eastern North Pacific, based on the temperature, pressure and relative humidity distributions at the inversion top, as reported by Neiburger (1960). Values in excess of 5 K occur, particularly near the California coast.

We can also give the following “parcel theory” interpretation of CIFKU. When unsaturated air (for which $(\Delta s_v)_{\text{crit}} > 0$) is entrained into a cloud, it is cooled and moistened by the evaporation of cloud droplets. As a result, its virtual dry static energy is decreased from $(s_v)_B^+$ to $(s_v)_B^+ - (\Delta s_v)_{\text{crit}}$. If the latter value is less than $(s_v)_B^+$, i.e., if (2.18) is satisfied, then the entrained air is negatively buoyant with respect to its cloudy environment, and so it sinks unstably through the cloud. This process is schematically illustrated in Fig. 4.

Lilly (1968) assumed that the criterion for CIFKU is that the wet bulb potential temperature decreases upward at cloud top; this is equivalent to the condition $\Delta h < 0$. But from (2.17) we see that $\Delta s_v - (\Delta s_v)_{\text{crit}} < 0$ means $\Delta h < \epsilon L \Delta(q + l)/\beta < 0$, so that CIFKU actually does not begin until $\Delta h$ is appreciably negative. For $T = 273$ K and $p = 1000$ hPa, we find that $\Delta h/c_p \approx 0.4 \Delta(q + l)$ K, where $\Delta(q + l)$ is expressed in g kg$^{-1}$. Since $\Delta(q + l) \lesssim -5$ g kg$^{-1}$ can occur, our instability criterion is significantly more stringent than Lilly’s. If we repeat our derivation but ignore the virtual temperature correction or ignore the liquid water component of the virtual temperature correction, our stability criterion reduces to $\Delta h > 0$. This illustrates the importance of including the liquid water correction in the definition of virtual temperature.

This importance may seem strange, since $l \ll q$. But what matters is that $F_l$ can be as large as or even larger than $F_q$. Imagine, for example, that fluctuations in $(q + l)$ occur on an isobaric surface where $T$ is homogeneous. Then, since there can be no fluctuations of $q^+(T, p)$, $q'$ must be zero, and so $|l'|$ is “infinitely larger” than $|q'|$. On the other hand, if fluctuations in $T$ occur on an isobaric surface where $(q + l)$ is homogeneous, then fluctuations of
$q^*$ (and $q$) must occur, so that $l' = -q'$, or $|l'| = |q'|$. In real situations, in which fluctuations of both $T$ and $(q + l)$ occur, we therefore anticipate that $|l'|$ will often be at least as large as $|q'|$.

As mentioned above, the stability criterion reduces to $\Delta h > 0$ if we ignore the virtual temperature correction or its liquid water component. This implies that the water vapor component of the virtual temperature correction does not matter for the criterion. The reason is that, as shown by (2.9), inside the cloud the fluxes of vapor, sensible heat and moist static energy are related to each other by positive factors of proportionality, so that they must all have the same sign, and increase or decrease together.

3. The nature of the convective downdrafts

It is interesting to compare the convective downdrafts of CIFKU with the convective updrafts of CIFK (cumulus updrafts). The most obvious differences are in the directions of the buoyancy forces and the resulting motions. While cumulus updrafts are driven by condensation heating, CIFKU downdrafts are maintained by evaporative cooling. The updraft heating extends upward from the PBL top throughout the conditionally unstable layer, which may fill the troposphere. But downdraft cooling is confined to the cloud layer, which in many cases is only a few hundred meters deep. As is well known, the entrainment of environmental air into cumulus updrafts has a debilitating effect, through dilution of the cloud mass with dry, low-energy air and also through the frictional effects of momentum exchange. Although entrainment into CIFKU downdrafts also tends to retard their motion through momentum exchange, it is essential, nevertheless, to their existence, because it supplies the liquid water which, by evaporation, maintains their negative buoyancy. This suggests that the downdrafts will prefer a small scale, perhaps on the order of a few hundred meters, so that mixing can be highly effective.

4. The roles of CIFKU in the marine boundary layer

The ability of CIFKU sharply to reduce the fractional cloudiness within a cloud layer suggests that the instability is responsible for the breakup of the subtropical marine stratocumulus regimes at their westward and equatorward boundaries. The marine stratocumulus regimes have been discussed by Lilly (1978) and Schubert et al. (1979a,b), while the neighboring tradewind cumulus regimes have recently been studied by Albrecht et al. (1979) and Albrecht (1979).

Fig. 5 summarizes the envisioned roles of the marine stratocumulus layers and CIFKU in the Hadley circulation. The right side of the figure represents a marine stratocumulus regime, and the left side represents tradewind cumulus and ITCZ regimes. From right to left, the sea surface temperature increases and the large-scale subsidence decreases. Separating the marine stratocumulus regime of large fractional cloudiness from the tradewind cumulus regime of small fractional cloudiness is a "transition zone." By definition, the poleward boundary of the transition zone marks the onset of CIFKU; the stratocumulus deck is therefore destroyed as it is advected into the transition zone, in the sense that the fractional cloudiness is reduced below 100%.

Some cloudy fragments of the stratus layer may escape evaporation, and may persist as the upper portions of moist plumes rising through the marine layer. Are these cumulus clouds? They are certainly associated with buoyant convection, since CIFKU is by its nature a convective instability. But in order for the cloud fragments to qualify as cumulus clouds, they must derive their kinetic energy primarily from CIFK, i.e., from buoyancy maintained by the release of latent heat in saturated updrafts embedded.

Fig. 5. A schematic illustration of the role of CIFKU in determining the tropical and subtropical distributions of cloudiness. Details are given in the text.
in a dry adiabatically subsiding environment. The convective thermals of the (pre-CIFKU) strato-
cumulus layer certainly do not qualify as cumulus
clouds, and since (2.20) is not the criterion for the
onset of CIFK, neither do the (post-CIFKU) cloud
fragments.

Instead, the cloud fragments can be described as
cumulus humilis clouds, which generate a significant
fraction of their kinetic energy during their rise as
unsaturated thermals in the subcloud layer. Esbensen
(1978) has termed these “boundary-layer cumuli,”
and has shown that their impact on the large-scale
environment is very different from that of deeper
cumuli. The eventual release of latent heat, with its
attendant cloudiness, does not dominate the dy-
namics of these thermals; it only serves to make
them visible near the upper limits of their trajecto-
ries. For this reason, boundary-layer cumuli are
more closely related to dry convective turbulence
than to deeper cumuli.

As argued by Betts (1973), nonprecipitating satu-
rated thermals which overshoot their level of neutral
buoyancy and then sink back, experiencing dilution
with their environment at all times, produce a net
downward transport of thermodynamic energy, and
so cool the upper portion of the cloud layer. In some
cases this cooling near cloud top may be accelerated
radiatively. The resulting destabilization favors an
increase in the depth of the cloud layer with time.
In this way, cumulus humilis clouds may significantly
intensify the process of mass entrainment at the top
of the transition zone’s convective layer.

Further downstream, warming, moistening and
deepening of the cumulus humilis layer will lead to
the onset of trade cumulus convection. The cumulus
humilis clouds may continue to exist, but they will
be inconspicuous among their towering neighbors.
We define the equatorward boundary of the transit-
ion zone as the location of the onset of CIFK.

The cloud base level of the trade cumulus layer
can be regarded as the equatorward extension of
the midlatitude stratocumulus cloud base. Similarly,
the trade inversion, which generally marks the top
of the tradewind cumulus layer, can be regarded
as the equatorward extension of the inversion at the
top of the midlatitude stratus deck. We thus have a
continuous evolution of the cloud layer from mid-
lattitudes into the tropics. But a conceptual problem
nevertheless arises, since the top of the PBL is
usually taken to coincide with the cloud top in the
marine stratocumulus regime, and with the cloud
base in the trade cumulus regime. Is it possible that
the PBL top descends through the cloud layer, from
the stratocumulus cloud-top level in midlatitudes to
the cumulus cloud base level in the trades?

In my opinion, the cleanest solution to this prob-
lem is to consider cumulus humilis clouds as lying
within the PBL, and to define the PBL top as the

highest level reached by the cumulus humilis con-
vection, when such convection is active. When
dereper cumuli coexist with the cumulus humilis
clouds, the bases of the deeper clouds will lie
within the PBL, but in any case these deep clouds
have subcloud inflow layers which can extend to the
earth’s surface, and so it is impossible to define the
PBL top in such a way that the circulations of deep
cumuli lie entirely above it.

Since the equatorward boundary of the transition
zone marks the onset of CIFK, it also marks the
separation of the PBL top from the cloud top level.
Further toward the equator, the PBL top lies at the
top of the cumulus humilis layer. We have thus con-
cluded that the PBL top and the cloud top coincide
throughout the transition zone, but that the cloud
top lifts above the PBL top further toward the equa-
tor. This is the situation summarized in Fig. 5.

Although an extensive observational study is
needed to verify this picture of the interrelations be-
tween the PBL stratus regime, the transition zone
and the trade cumulus regime, we can obtain some
preliminary observational support from the data of
Neiburger (1960). The data were obtained during a
series of cruises in the eastern North Pacific, during
the summer months. In Fig. 6, the shaded regions
are those in which the average relative humidity at
the inversion base (which, we assume, coincided
with the PBL top) exceeded 90%. The observed
average values of $\Delta S_6 - (\Delta S_6)_{net}$, also shown in the
figure, were mainly positive in the humid regions
where stratus layers must have occurred frequently.
This is consistent with our analysis, which shows

Fig. 6. The distribution of $[\Delta S_6 - (\Delta S_6)_{net}] / c_p [K]$, off the
coast of California, from the data of Neiburger (1960). Stippled
regions are those in which the relative humidity at the inver-
sion base exceeded 90%.
that such stratus layers would have been stable. In those regions where the low average relative humidity suggests that the PBL was usually cloud-free, the figure shows that $\Delta s_v = (\Delta s_v)_{\text{crit}}$ was mainly negative. This is also consistent with our analysis, which shows that stratus layers could not have been stable in these regions.

5. Conclusions and discussion

The central points of this study are that cloud masses can be unstable with respect to penetration by the overlying air, that the instability can lead to the partial destruction of the cloud, and that the distribution of cloudiness in the subtropical planetary boundary layer is partly determined by this mechanism. The first priority of further research along these lines should be more complete verification of these results, by both observational studies and detailed numerical simulations.

Several questions must be raised. First, given that CIFKU occurs, how can its effects be parameterized for numerical models? The fractional cloudiness, the rate of entrainment at cloud top and the turbulent exchanges associated with CIFKU must all be determined in an adequate parameterization theory. Is it possible for CIFKU to continue indefinitely without greatly reducing the fractional cloudiness, under some synoptic conditions?

We must also ask whether CIFKU influences the mesoscale circulations which are so prominent in satellite photos of stratocumulus layers. Closed cellular convection (e.g., Agee et al., 1973) seems particularly likely to be modified by CIFKU.

The role of CIFKU in determining the distributions of upper level cloudiness, including altostratus and cirrus, must be assessed. For this purpose, it will be necessary to generalize the theory to include the ice phase.

Finally, it seems likely that CIFKU is partly responsible for the rapid entrainment and intense turbulence near the tops of cumulus clouds (see, e.g., Warner, 1970). The study of CIFKU may therefore improve our understanding of CIFKU.

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REFERENCES


Neuberger, M., 1960: The relation of air mass structure to the field of motion over the eastern North Pacific Ocean in summer. Tellus, 12, 31–40.


