A Stochastic Model of Cumulus Clumping

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ABSTRACT
Observations show that cumulus clouds often occur in long-lived mesoscale groups, or clumps. Five possible explanations of clumping are surveyed. The "mutual protection hypothesis," that clumps occur because cumulus clouds create and maintain, in their near environments, relatively favorable conditions for the development of succeeding clouds, is examined at length. This idea is tested through the use of a simple time-dependent model in which clouds, triggered at randomly selected locations, tend to stabilize their environment in the face of a prescribed constant forcing. Results show that clumping occurs when the cloud-induced stabilization rate is strongest at an intermediate distance from a cloud, and that it does not occur when the stabilization rate decreases monotonically away from a cloud.

1. Introduction
There exist in cumulus cloud fields a great variety of features with spatial scales in the range 10–100 km, and temporal scales in the range 2–10 h. The most familiar of these mesoscale structures are squall lines and non-squall bands, and open cells and cloud streets. Their sheer ubiquity, revealed by satellite photographs, suggests that they act as important mediators of the interactions between the cumulus and synoptic scales. Cumulus parameterization theories should therefore take their influence into account, but to date none does. The prerequisite theoretical understanding of the mesoscale has, so far, eluded us; in mesoscale meteorology, observations are far ahead of theory. This may be largely due to the baffling variety of mesoscale phenomena. It seems that, at this point, theoretical progress may best be achieved by focusing on a single, particularly simple form of well-observed mesoscale structure.

2. Review of observations
In an interesting study of GATE radar data, López (1978) investigated local groups of cumulus cells, which he called composite echoes. He reported that "... at the start of a composite echo, a few cells are observed to occur together. With time, new cells form next to the old ones. The old ones may decay while the newer ones grow and develop. During the lifetime of the composite echo, many cells can go through their cycle of growth and decay." The individual cells which were members of composite echoes lasted longer, and rained more copiously, than isolated cells of the same cross-sectional area. They also achieved larger cross-sectional areas, on the average, than their isolated counterparts. We refer to these local cloud groups as cumulus "clumps." More precisely, we define a clump as a group of cumulus clouds whose members are much more closely spaced than the average spacing over the population, and which maintains its identity over many cloud lifetimes. This is somewhat similar, but not identical, to Cho's (1978) concept of the independent cloud group.

A search of the observational literature reveals that clumps have been reported by many investigators. In an aircraft photographic survey of summertime Florida cumulus populations, Plank (also see Plank, 1969) found that clumps almost always developed, whether or not the clouds were organized into streets or bands. In a typical diurnal cycle, clumping was most noticeable between 1030 and 1300 EST; it was also especially pronounced in disturbed weather. Plank (1969) reported that

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1 We prefer "clump" to "cluster" because the term "cloud cluster" has been widely used to refer to the satellite-observed cloud features, of small synoptic scale, which are associated with tropical waves.

the onset of clumping was associated with a shift in the population from a unimodal distribution (morning) to a bimodal distribution (afternoon). The largest clouds of the populations were found in clumps. Many clumps occurred within a population, and Plank found that their distribution and spacing was "geometrically regular and seemingly systematic." Their sizes ranged from roughly that of the largest individual member cloud to one order of magnitude larger. The largest clumps occurred on disturbed days. Large, clear "holes" in the cloud field were sometimes observed in the vicinity of the clumps.

Browning and Harrold (1969), Kreitzberg and Brown (1970), Harrold and Browning (1971) and Hill and Browning (1979) observed long-lived mesoscale areas of heavy precipitation in the convective regions of extratropical cyclones. They reported that these features included numerous cumulus congestus elements, were typically 50 km across, persisted for up to 8 h, appeared to be advected by the mean wind in the convective layer, and were often aligned in bands. Similar observations were reported by Austin and Houze (1972).

Although chance groupings will occur even in a completely random cloud field, both the observed persistence of clumps over many individual-cloud lifetimes and the observed tendency for the most vigorous clouds to occur preferentially in clumps strongly suggest that there exists a dynamical mechanism which favors the formation of clumps and tends to maintain them through time. Since clumping seems to occur in a wide variety of synoptic contexts (as long as cumulus clouds are present), the mechanism must be rather fundamental, in the sense that no special circumstances are required to trigger it.

3. A survey of hypotheses

In this section, we discuss five possible explanations of clumping. These hypotheses are by no means mutually exclusive; each of them may explain some observations of clumping, and probably no one of them can explain all of the observations.

Perhaps the simplest possibility is that clumps are directly forced by hot spots, orography, or other surface features. Malkus (1957) reported that trade cumulus cloud groups were associated with mesoscale sea surface temperature anomalies. But although surface features can undoubtedly force the formation of clumps, such geographically fixed forcing cannot account for those clumps which persist as they drift with the wind over synoptic distances.

A second possibility is that latent heat release in a clump forces a mesoscale circulation which, in turn, tends to support the clump. Zipser (1969, 1977), Zipser and Gauthier (1978), Houze (1977) and Leary and Houze (1979) have presented ample observational evidence for the existence of mesoscale circulations in large cloud groups, but it is not known whether such circulations can or do cause or maintain features which we might identify as clumps. Although we do not discount this hypothesis, we shall not discuss it further in this paper; other possibilities exist.

Two of these involve direct mechanical interaction between clouds. The first, discussed by López (1978), is that clumping occurs because downdrafts from raining clouds trigger the formation of new clouds nearby. The second is that cloud-induced circulations, like vortex rings, tend to organize themselves into groups, by mutual advection. Hill (1974) noticed such a tendency in his numerical results. But it is not clear how either of these two ideas can explain López's observation that clouds in clumps are the largest, longest lasting, and most heavily precipitating members of the population.

Finally, the possibility which we shall examine most carefully in this paper is the "mutual protection hypothesis": that clumps occur because cumulus clouds tend to create, in their near environments, relatively favorable conditions for the development of succeeding clouds. The clouds can do this by moistening the near environment (even while drying the distant environment), and by steepening the near environmental lapse rate (even while stabilizing the distant environmental lapse rate). The relatively moist, unstable near environment then serves to "protect" succeeding clouds by sheltering them from the relatively dry, stable air of the distant environment. Hill (1974) found that moist anomalies were left behind by the decay of his numerically simulated clouds. Through a stochastic process, clumps of vigorous clouds might then arise spontaneously, even with a random spatial distribution of cloud-triggering events. This is actually an old idea, first advanced by Scorer and Ludlam (1953), and resurrected by López (1978).

Although modern studies (e.g., Arakawa and Schubert, 1974) have emphasized that cumulus clouds are self-stabilizing, in the sense that they tend to relieve the conditional instability of their environment, the distribution of the rate of stabilization over the volume around an isolated cloud has not been discussed in the literature.

4. Model formulation

In this section we present a model designed to test the hypothesis that clumps spontaneously develop and persist through time because the rate of stabilization has a local minimum on the cloud.
The same model will also be used to show that persistent clumps do not occur if the rate of stabilization decreases monotonically away from the cloud. A discussion of the physical processes which tend to produce a local minimum of the rate of stabilization on the cloud is postponed until Section 6. Our model is designed only to show the possible consequences of such a minimum. We cannot use the model itself to explain why the stabilization profile has or does not have any particular shape.

Consider a square grid of points. In practice, we use a grid 50 points on a side. Each point represents a vertical column which may or may not contain a cloud. We define over the grid a prognostic variable \( J \), which represents the degree of conditional instability, i.e., the positive area on a tephigram; clouds are permitted if \( J \) is positive. The prognostic equation for \( J \) is

\[
J_{ij} = \alpha_{ij} - \sum_{i'j'} C_{i'j',j'} I_{i'j'},
\]

(1)

where the subscripts \( i \) and \( j \) denote a particular column, \( \alpha_{ij} \) is the forcing due to all non-cloud processes, and the summation term represents the tendency of nearby clouds to reduce \( J_{ij} \), i.e., to "stabilize." The rate at which column \( ij \) is stabilized by cloud activity in column \( i'j' \) is determined by the stabilization profile \( C_{i'j',j'} \), which satisfies

\[
\sum_{ij} C_{i'j',j'} = 1, \quad \text{for all} \quad i'j',
\]

(2)

and by \( I_{i'j'} \), which is a measure of the intensity of convection in column \( i'j' \). In order to evaluate the summation in (1) for columns near the grid boundary, we use cyclic boundary conditions in the \( i \) and \( j \) directions. The value of \( I_{ij} \) is determined from

\[
I_{ij} = \begin{cases} 
0, & \text{if} \quad J_{ij} \leq 0 \quad \text{or} \quad r_{ij} > \tau \\
\gamma J_{ij}, & \text{otherwise}
\end{cases}
\]

(3)

Here \( r_{ij} \) is a random number between zero and one, \( \tau \) a prescribed "seeding rate," and \( \gamma \) a prescribed constant. The seeding rate represents the area-averaged rate of occurrence of "seeds," which we define as subcloud-layer perturbations suitable for the formation of a cloud. In our model, the spatial distribution of seeds is random and is not influenced at all by the simulated cloud field. We do not claim that the distribution of subcloud-layer perturbations is unrelated to the distribution of clouds in the real world, but we wish to demonstrate that no special distribution of seeds is needed to produce a clumped cloud field. For this reason, we deliberately choose a random seeding distribution for our model.

In writing (1), we have assumed that the net stabilization of a given column, by clouds in two or more neighboring columns, can be obtained simply by adding the effects that the clouds would produce if they acted in isolation. We have also assumed, in writing (3), that the local intensity of convection is independent of the activity of neighboring clouds (except as that activity is reflected, through time, in the \( J \) field).

The model works as follows: The \( J \) field is initialized to zero for all columns. The forcing \( \alpha \) is given a spatially uniform and time-independent positive value, so that \( J \) tends to increase uniformly with time. On each iteration, for each column, \( I \) is positive only if the random number generator selects the column to receive a seed, i.e., if \( r_{ij} \leq \tau \), and if the column is conditionally unstable, i.e., if \( J \) is positive. Once the \( I \) field has been determined, the new \( J \) field is predicted using (1). Since \( J = 0 \) when \( J \leq 0 \), the model responds nonlinearly to its random forcing whenever the minimum value of \( J \) is negative; although seeds where \( J > 0 \) produce positive values of \( I \), seeds where \( J < 0 \) are not permitted to produce negative values of \( I \).

If we ignore the stochastic forcing of the model, i.e., if \( \tau = 1 \), then (1) and (3) have the steady, horizontally homogeneous solution

\[
J = \alpha/\gamma, \quad I = \alpha.
\]

(4)

This shows that increasing the forcing increases both the degree of conditional instability and the level of cloud activity. For \( \tau < 1 \), we expect the area-averaged values of \( J \) and \( I \), denoted by \( \bar{J} \) and \( \bar{I} \), respectively, to approximately satisfy

\[
\bar{J} = \alpha/(\gamma\tau), \quad \bar{I} = \alpha.
\]

(5)

5. Results

We have performed experiments using the axially symmetric stabilization profiles shown in Fig. 1. The "peak" stabilization profile decreases monotonically with radius, while the "dip" profile has a local minimum at zero radius. For the reasons given in Section 6, we believe that (for an isolated cloud) the dip profile is more realistic. For both profiles, a given column is influenced only by those neighboring columns which lie within a circle of radius of ten columns, and the normalization condition (2) holds. We have performed experiments with the peak and dip stabilization profiles of Fig. 1. Unless otherwise stated, all results presented here are for \( \alpha = 5.0, \gamma = 1.6 \), and \( \tau = 0.10 \). The values of \( \alpha \) and \( \gamma \) have been chosen simply to obtain convenient values of \( J \) and \( I \) [see Eq. (4)]. The value of \( \tau \) has been chosen with a view to observations of the fractional area covered by active cumulus cells.

Figs. 2a and 2b show the predicted \( I \) field, averaged over time steps 110–124, for the peak and dip profiles, respectively. The apparent disorder for the peak is in clear contrast with the distinct, regular clumped pattern for the dip. For the latter profile,
the largest values of $I$ occur within the regularly spaced maxima. We shall now confirm these subjective impressions by objective methods, and we shall also show that the temporal structure for the dip is much more orderly than for the peak.

Figs. 3a–3d show the predicted $J$ field, for the peak, for time steps 110, 125 and 150, and also the time average over steps 110–124. Figs. 3e–3h show the corresponding results for the dip. The spatial structures of the peak and dip results are markedly different. First, for the dip there is more power at small scales. This simply reflects the introduction of the scale defined by the radius of maximum stabilization. A second, more spectacular, difference is that the range of values of $J$ for the dip greatly exceeds that for the peak. The dip maxima exceed the peak maxima by about a factor of 2 and, whereas the peak minima are always positive, some of the dip minima are negative.

Figs. 4a and 4b show the lagged spatial autocorrelation of $J$, for the peak, for steps 110 and 125, respectively, and Figs. 4c and 4d show the corresponding results for the dip. The spatial distribution of $J$ is quite regular for the dip; we see (imperfect) concentric rings of alternating positive and negative correlation. Negative features tend to be separated from neighboring positive features by about five columns—the assumed radius of maximum stabilization. In contrast, the peak results show no evidence of spatial regularity.

**FIG. 1.** "Peak" and "dip" stabilization profiles tested in this study. The two curves have the same area-averaged values (over the disk). The constant stabilization case is shown for comparison only.

**FIG. 2.** The predicted $I$ field averaged over steps 110–124: (a) peak, (b) dip. The contour interval is 8, with the zero line omitted; particularly intense regions are stippled.

The temporal structures of the two stabilization profiles also differ greatly. Inspection of Figs. 3a–3h and 4a–4d shows that, for the dip, both individual features and the overall spatial pattern are remarkably persistent, while, for the peak, features appear and disappear sporadically. This is made even more evident by Fig. 5, which shows the lagged time autocorrelation of $J$ for both stabilization profiles. The past history of the $J$ field is quickly forgotten for the peak, but is long remembered for the dip.
Fig. 3. The predicted $J$ field: (a) peak, step 110; (b) peak, step 125; (c) peak, step 150; (d) peak; average over steps 110–124; (e) dip, step 110; (f) dip, step 125; (g) dip, step 150; (h) dip, average over steps 110–124. The contour interval is 15, with negative values shaded.

Whereas the features of the dip results tend to become more intense with time, the maxima and minima of the peak results have reached a statistical equilibrium by step 110. This can be seen from Figs. 3a–3h, and it is also apparent in Fig. 6, which depicts the time evolution of the maximum, minimum and area-averaged values of $J$ for both the peak and dip. In addition to the very evident time dependence of $J_{\text{max}}$ and $J_{\text{min}}$, for the dip, we also see that $J_{\text{min}}$ becomes negative. This is significant, because wherever $J < 0$ clouds cannot occur [see Eq. (3)]. The persistent regions of negative $J$ in Figs. 3e–3h are holes in the cloud field. Some of the holes are ringed by active clouds (cf. Fig. 2b); the pattern is somewhat similar to open cellular convection (Agee and Dowell, 1974).

Other experiments (not shown) reveal that, regardless of the choice of stabilization profile, the time- and domain-averaged values of $J$ and $I$ agree well with the values given by (5). Also, it is of
some interest that as the seeding rate is decreased [by decreasing $\tau$; see Eq. (3)] the average of $I$ over those columns where $I \neq 0$ increases, i.e., the clouds become more vigorous. But because the number of active columns is smaller, the area-averaged value of $I$ is not changed. This suggests that the rate at which subcloud-layer perturbations trigger new clouds influences the intensity of individual clouds more than it influences the overall level of convective activity.

In summary, our results show no evidence of clumping for the peak profile, but for the dip profile we find that the largest values of $I$ occur in persistent, regularly spaced patches on the scale of the radius of maximum stabilization, and that cloud-free holes form and persist in the vicinity of these patches of intense convection.

6. The stabilization profile

Since our simple model predicts that the assumed shape of the stabilization profile strongly influences
the clumpiness of the model cloud field, we now discuss the physical process of stabilization, and argue that the dip profile is more realistic than the peak profile.

As is well-known, clouds tend to modify their environments by the three very different mechanisms of induced subsidence, detrainment and radiative cooling (Fig. 7). Subsidence tends strongly
to stabilize by warming and drying, but detrainment actually tends to destabilize by moistening, and both detrainment and radiation tend to destabilize by cooling near the cloud top.

However, recent observational and theoretical studies show that the small-scale temperature anomaly associated with a cloud or small group of clouds has only a short lifetime. In an analysis
of the GATE data, Grube\textsuperscript{3} found that cumulus-induced subsidence does indeed produce mesoscale warming events, which are characteristically displaced from the associated rain events by distances on the order of 30 km. She also reported that these warm anomalies are quickly smeared out from the mesoscale to the synoptic scale. This observation may be explained by the argument of Schubert \textit{et al.} (1980), who point out that whenever cumulus convection tends to create a temperature anomaly whose scale is less than the Rossby radius of deformation, the well-known geostrophic adjust-

\textsuperscript{3} Grube, P. G., 1979: Convection induced temperature change in GATE. Atmos. Sci. Pap. No. 305, Colorado State University, 128 pp. [NTIS PB-298 130/6GA].
ment process [see the review by Blumen, (1972)] acts to remove the temperature anomaly through the dispersive propagation of inertia gravity waves.

These considerations suggest that during and immediately after the period of active cloud growth cloud-induced temperature and moisture anomalies both occur within the mesoscale environment, while the longer term impact of a cloud on its mesoscale environment is felt mainly in the moisture field, which is relatively undisturbed by the geostrophic adjustment process. On the short time scale, the upper level cooling and moistening produced by the evaporation of detrained liquid water and by radiation tend to destabilize the column in the immediate vicinity of the cloud, while subsidence warming and drying tend to stabilize not only near the cloud but also in the more distant environment. On the longer time scale, the cloud-induced temperature anomaly may have disappeared (on the mesoscale), but the near-field moistening and far-field drying will remain, and will continue to represent a net stabilization which is relatively weak near the former position of the cloud. On either time scale, the stabilization profile will have a local minimum on the cloud.

As discussed by Cho (1978), numerical studies of cumulus convection show that simulated clouds are insensitive to lateral boundaries placed more than about five cloud radii away from the cloud. This suggests that the radius of maximum stabilization must lie within five cloud radii, i.e., within the mesoscale.

7. Discussion and conclusions

We have demonstrated that the spatial and temporal structure of cumulus activity is strongly influenced by the distribution of the rate of stabilization over the volume around each individual cloud. In our model, clumping occurs if the stabilization profile has a maximum at an intermediate distance from the cloud, but not otherwise. Although the model studied here is obviously too schematic to provide conclusive proof that the mutual protection mechanism accounts for clumping, our results are highly suggestive, and certainly warrant further research.

Perhaps the first step should be the use of a time-dependent, axisymmetric cloud model, of good radial resolution, to study the stabilization process in some detail, including the stabilization profile. Next, we must clarify the way in which the stabilization profiles of neighboring clouds interact with each other; the superposition principle adopted here is probably an oversimplification. It would be interesting to study the stabilization surface set up in the presence of shear.

Further development of the model should begin with the introduction of the third dimension, and the use of predicted temperature and moisture fields with a more explicit cloud model. The present model includes no smoothing processes, such as gravity wave dispersion, to oppose the formation of unreasonably sharp features; this deficiency must be corrected. As the model is generalized, it will become possible to study some or all of the hypotheses listed in Section 3 but not included in the present model. One of the most interesting possibilities is that the spatial distribution of seeds could be allowed to depend on the distribution of clouds. Such a generalization would permit us to study the impact on cloud population statistics of the triggering of new clouds by neighboring clouds. A similar dynamic has been introduced into a stochastic model of star population statistics by Gerola and Seiden (1978) and Seiden and Gerola (1979). It will be a challenge to generalize the model in these ways without making it prohibitively complicated and expensive.

Observational studies of clumping should focus on the distributions of warming and drying around both individual clouds and clumps.

Finally, analogies between cloud populations and biological populations, some of which are implicit in this study, should be exploited, not ignored. We do not suggest that one can fruitfully argue by such analogies, but their pursuit can suggest ideas and/or analytical techniques. For example, Hubbell (1979) has discussed clumping in tropical tree populations, and the statistical methods employed in his and similar studies could well prove useful in the study of cumulus clumping.

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