Local Multiple Equilibria and Regional Atmospheric Blocking

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ABSTRACT

Stationary flow of a barotropic fluid in a β channel has been shown by Charney and De Vore (1979) to possess multiple-equilibrium solutions when sinusoidal topographic forcing is exerted within the region of resonance near the wavenumber of stationary Rossby waves, and nonlinear effects are taken into account. Charney and De Vore associate the different solutions with zonal and blocking states of global circulation. However, real topography is non-sinusoidal and, most of the time, observed blocking configurations display a pronounced regional character. On the other hand, the problem of superimposing different harmonics is made difficult here by the essential role played by nonlinearity in the theory of multiple equilibria.

In this paper, the mathematical problem of determining the stationary states of flow of barotropic fluid in a β plane when topography is nonsinusoidal is analyzed with the help of the perturbative assumptions that the latitudinal scale of the flow is very large and topography has the form of a slowly modulated sinusoid.

The multiple states of stationary flow described by Charney and De Vore are found to exist simultaneously in different regions of the β plane. Theoretical solutions corresponding to different kinds of resonant forcing are analyzed.

The theoretical solutions are discussed in relationship to the problem of blocking as a "regional" phenomenon and are shown to have several different features in common with observed persistent blocking patterns.

1. Introduction

The process of amplification of ultralong (zonal wavenumbers 1–3) and long (zonal wavenumbers 4–7) planetary waves, taking place during blocking and stratospheric warming events, has recently been the object of several very innovative theoretical investigations.

Tung and Lindzen (1979a,b) and Tung (1979) have examined in great detail the physical properties (time growth, vertical and horizontal confinement) of planetary waves in the spectral region around the well-known resonance situated at the stationary wavenumber of the linear propagation equations. They find, among other things, that the zonal wind velocities required to make planetary waves stationary are easily achieved for wavenumbers above 4 and, less easily, for ultra waves. Even if frictional dissipation is taken into account, the stationary response to nonsymmetric forcing is quite strong. In fact, negative interference of orographic and thermal asymmetrics must be advocated in order to explain the amplitudes of stationary waves observed during winter. This circumstance may also be taken as an indication of the weakness of the linear approach in describing the large-amplitude regime of forcing of planetary waves.

The basic effects of nonlinearity on topographically forced waves have been explored by Charney and De Vore (1979) with the aid of barotropic spectral equations of the kind used by Lorenz (1960) at the beginning of his pioneering work on the nonlinear dynamics of severely truncated models of atmospheric flow. Even if, due to the particular type of truncation choosen, the stationary response found by Charney and De Vore is still linear, the presence of nonlinear coupling with an extra mode (different from the one that is directly forced by topography) limits the amplitude of the quasi-resonant equilibrium states and gives rise to multiple equilibria of the kind mathematically discussed by Vickroy and Dutton (1979): in a spectral region of near-resonant wavenumbers three different stationary wave amplitudes appear as mathematically possible. Among these, only two (corresponding to the smallest and largest amplitudes of the forced wave) can be stable, while the third one is unstable.

This instability is found to be of new type ("form-drag instability"), different from the classical resonant and shear barotropic instabilities, whose growth depends essentially on the presence of bottom

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topography. The two stable states are associated by Charney and De Vore with the zonal and blocked configurations, respectively of atmospheric flow.

The same multiplicity of stationary solutions is found by Wiin-Nielsen (1979) in the study of barotropic forced equations on the sphere. The existence of multiple equilibria is therefore not a consequence of geometry.

On the other hand, the fact that multiple-equilibrium states are not a mere consequence of severe truncation either has been proved by Hart (1980) and Trevisan and Buzzi (1980) by means of different perturbative formulations of the continuous barotropic equations in wide and narrow β channels. Trevisan and Buzzi demonstrate (with evidence) the fact that the existence of multiple states can be explained in terms of nonlinear bending of the linear resonance curve of the type well known in classical mechanics (see Landau and Lifchitz, 1969; Bogoliubov and Mitropolsky, 1961; Nayfeh and Mook, 1979). Extension of the barotropic theory of multiple states to more realistic situations (simultaneous orographic and thermal forcing on the sphere) has been investigated by Källen (1980) who finds that by adjusting the two nonsymmetric forcings in specific relative phase relationships it is possible to obtain multiple states that somehow resemble the observed zonal and blocked states of atmospheric flow.

In all of the abovementioned studied the nonsymmetric forcing is considered as strictly sinusoidal. Since in nonlinear problems, solutions cannot be superimposed, the results obtained are relevant only in relationship to blocking as a global, wavelike phenomenon. However, blocking is also (and most of the time) a local, regional phenomenon [see, e.g., Austin (1980) for a distinction between these two aspects of the phenomenology of blocking]. Even local, regional equilibria can arise in nonlinear system, as Charney himself seems to imply in the conclusions of his and De Vore's paper:

> We have refrained from commenting on the geometrical sizes of the flow system under consideration since we do not necessarily regard the equilibria as global in extent. The existence of large-amplitude meanders of the zonal flow in some regions simultaneously with small-amplitude meanders, or none at all, in others, i.e., high index and low index flows existing side by side, suggests that regional stable or metastable equilibria can exist. Such flows would be stable or nearly so to variations of the upstream flow, although the weak coupling through the zonal flows associated with other regional circulations could well be one of the factors that eventually destroy a quasi-equilibrium circulation. Our studies also suggest that the large-scale forced disturbances may themselves be unstable, leading to regular or irregular fluctuations.

In addition, we must consider that the earth's topography is far from being sinusoidal and several different harmonic components are simultaneously needed for its realistic representation.

The theory of regional multiple equilibria is dealt with in this paper where the mathematical difficulties of the dynamic equations are overcome with the help of some heuristic considerations arising out of a large body of experience in dealing with nonlinear problems, together with extensive use of perturbative analysis. The physical picture that emerges is more or less that which is predicted in the paper by Charney and De Vore:

When nonsymmetric forcing is exerted at different wavenumbers within the spectral region of nonlinear resonance, adjustment to the value of the wavenumber typical of the local orographic forcing can take place, so that the solution can "jump" in space from one to the other of the stable solutions that, therefore, simultaneously exist in the β plane.

The streamfunction patterns corresponding to such multiply-connected solutions have many features in common with typical blocking configurations: they exhibit jet splitting, simultaneous zonal and wave character at different locations, and even the dipolar, high-low features that appear so frequently over the eastern Atlantic and western Europe.

2. Nonlinear resonance in the β plane

In view of the mathematical complexity of the problem, we have tried to reduce its formulation to the simplest form that can still describe in a physically meaningful way the nonlinear mechanism that leads to multiple states near resonance.

Consider the equation for quasi-geostrophic, barotropic flow over small topography in a β plane:

$$\frac{Dq}{Dt} = 0; \quad q = \nabla^2\psi + \beta y + \frac{f_0 h}{H}, \qquad (2.1)$$

where $D/Dt = \partial/\partial t + u(\partial/\partial x) + v(\partial/\partial y)$ is the total time derivative, ∇^2 is the Laplacian operator, ψ the horizontal streamfunction, β the latitudinal derivative, f_0 the local value of Coriolis parameter, H the depth of the barotropic atmosphere and h the height of topography.

In its stationary form this equation reduces to

$$J(\psi, q) = 0 \qquad (2.2)$$

that requires the potential vorticity q to be a function only of the streamfunction, i.e., $q = F(\psi)$.

The steady-state streamfunction is then the solution of the differential equation

$$\nabla^2\psi + \beta y + \frac{f_0 h}{H} = F(\psi). \qquad (2.3)$$

Eq. (2.3) in general, is a nonlinear partial differential equation that must be further manipulated in order to be made analytically tractable. An initial simplification can be performed by reducing Eq. (2.3) to a ordinary differential equation by means of the wide (Hart, 1979) or narrow (Trevisan and Buzzi, 1980) channel approximation. We have chosen the first approximation. With the introduction of nondimensional variables $\zeta = x/l$, $\eta = y/L$, and the substitutions $\psi \rightarrow \psi/UL$, $h \rightarrow h/h_m$, $F \rightarrow Fl^2/UL$ (where l is a longitudinal and L a latitudinal scale, U a typical zone velocity and h_m the maximum topographic height), and defining the horizontal aspect ratio $\delta = l/L$, the Rossby number $\mathrm{Ro} = U/f_0l$, the vertical aspect ratio $\tilde{h}_m = h_m/H$, the stationary wavenumbers $k_s^2 = \beta l^2/U$, we obtain the nondimensional form of Eq. (2.3):

$$\partial_{\xi\xi}\psi + \delta^2\partial_{\eta\eta}\psi + k_s^2\eta + \frac{\tilde{h}_m\delta h}{\mathrm{Ro}} = F(\psi). \quad (2.4)$$

For a wide channel $\delta \ll 1$, Eq. (2.4) reduces to the total differential form

$$d_{\xi\xi}\psi + k_s^2\eta + \tilde{h}_m \frac{\delta h}{\mathrm{Ro}} = F(\psi). \quad (2.5)$$

In order to keep the problem analytically simple, we have choosen from the start to deal with the inviscid, adiabatic equations (2.1). This choice reflects itself into the lack of definition of the stationary problem (2.2) that admits solutions by any form of the indeterminate function $F(\psi)$. This is a well-known problem in fluid mechanics. There are several ways of eliminating such ambiguity. One procedure would be to formulate the search for stationary states as an initial value problem; alternatively, we could take the limit for vanishing values of some parameterization of thermal forcing and dissipation [typically, $k(\nabla^2\psi^* - \nabla^2\psi)$ as in Charney and De Vore (1979)]. Both of these procedures would determine a specific form of the function $F(\psi)$. Otherwise, within the context of stationary inviscid theory, such function, in principle, can be deduced from a knowledge of the streamfunction along a cross section of the flow. However, leaving aside uniqueness problems, we prefer here to consider $F(\psi)$ as specified in the weakly nonlinear form

$$F(\psi) = -k_s^2\psi - a\psi^2 - b\psi^3, \quad (2.6)$$

where a and b are constants depending on the shear of the flow. The reason for this choice will appear clear from the results of this section. In any case the form (2.6), that can be considered as a Taylor expansion of the real $F(\psi)$, leads, as we shall see, to a stability condition identical to that found by Jacobs (1980) in his treatment of the nonlinear problem (2.2) with a method not based on the specification of $F(\psi)$. Eq. (2.4) then becomes

$$(d_{\xi\xi} + k_s^2)\psi + k_s^2\eta + a\psi^2 + b\psi^3 + \frac{\tilde{h}_m\delta h}{\mathrm{Ro}} = 0. \quad (2.7)$$

We will further assume that the nonlinearity coefficients a and b and the mountain height h_m are small[2] and perform the substitutions

$$\frac{\tilde{h}_m\delta}{\mathrm{Ro}} \rightarrow \frac{\tilde{h}_m\delta}{\mathrm{Ro}}\gamma; \quad a \rightarrow \gamma a; \quad b \rightarrow \gamma b,$$

where $\gamma \ll 1$, $\delta^2 \ll \gamma$.

In physical terms Eq. (2.7) describes the stationary flow of an inviscid barotropic fluid over small topography in a wide β channel. Eq. (2.7) is the simplest formulation of nonlinear resonance.

In order to show that (2.7) has solutions comparable with the ones discussed by Charney and De Vore (1979), we assume now that the bottom topography is sinusoidal, i.e.,

$$h(\xi) = \tfrac{1}{2}[e^{ik_m\xi} + (*)],$$

where k_m is a nondimensional wavenumber characterizing the horizontal variation of the mountain and the symbol (*) indicates the complex conjugate of the preceding term.

The wavenumber k_m is further assumed to be quasi-resonant

$$k_m - k_s = \gamma\Delta k, \quad \text{where} \quad \Delta k = O(1) \quad \text{and} \quad \gamma \ll 1.$$

Eq. (2.7) then becomes

$$(d_{\xi\xi} + k_s^2)\psi + \gamma a\psi^2 + \gamma b\psi^3 + k_s^2\eta + \gamma\frac{\tilde{h}_m\delta}{2\mathrm{Ro}}[e^{ik_m\xi} + (*)] = 0, \quad (2.8)$$

representing a spatial nonlinear oscillation with external forcing (β effect and topography). The latitudinal variation comes into the problem only parametrically. Eq (2.8) can be integrated by means of perturbation expansion in terms of the small parameter γ. In fact, having defined

$$\psi = \psi^{(0)} + \gamma\psi^{(1)} + \gamma^2\psi^{(2)} + \cdots,$$

the zero and first order equations are

$$(d_{\xi\xi} + k_s^2)\psi^{(0)} = -k_s^2\eta, \quad (2.9)$$

$$(d_{\xi\xi} + k_s^2)\psi^{(1)} = -a\psi^{(0)2} - b\psi^{(0)3} - \frac{\tilde{h}_m\delta}{2\mathrm{Ro}}e^{ik_m\xi} + (*). \quad (2.10)$$

Eq. (2.9) has a solution of the form

$$\psi^{(0)}(\xi, \eta) = \Psi(\eta)e^{ik_s\xi} + (*) - \eta, \quad (2.11)$$

[2] This last assumption is not strictly necessary since it can be shown (see Bogoliubov and Mitropolsky, 1961) that the case of finite forcing can be reduced to that of small forcing.

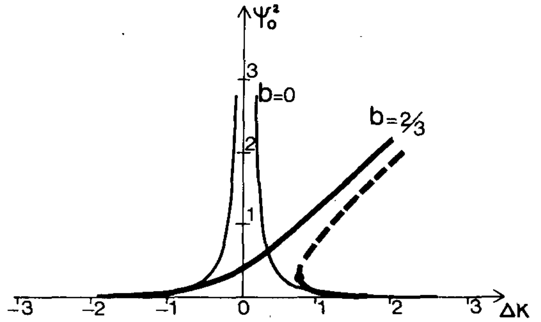

FIG. 1. Bending of the resonant part of the inviscid, stationary response curve due to a cubic nonlinearity. The two curves, corresponding respectively to $b = 0$ (linear case) and $b = 2/3$, are calculated at the central latitude $\eta = 0$ for $\delta/\text{Ro} = 1$, $K_s = 1$, $h_m = 1$. The dashed line represents the unstable branch of the stationary response curve.

that when substituted into (2.10) gives rise to secular excitation of the linear operator $(d_{\xi\xi} + k_s^2)$.

Such a secularity can be eliminated assuming that the amplitude $\Psi(\eta)$ of the zero-order solution (2.11) is slowly modulated in the ξ direction (a procedure that goes under the name of two-scale analysis).

Introducing the long scale $X = \gamma\xi$ and substituting formally $\partial/\partial\xi \to \partial/\partial\xi + \gamma(\partial/\partial X)$ into the first-order equation (2.10), we obtain

$$(d_{\xi\xi} + k_s^2)\psi^{(1)} = -2d_{\xi X}\psi^{(0)} - a\psi^{(0)2} - b\psi^{(0)3} - \frac{\tilde{h}_m\delta}{2\text{Ro}} e^{ik_m\xi} + (*), \quad (2.12)$$

$$\left[2ik_s \frac{d\Psi}{dX} - 2a\eta\Psi + 3b(|\Psi|^2\Psi + \eta^2\Psi)\right]e^{ik_s\xi} + \frac{\tilde{h}_m\delta}{2\text{Ro}} e^{ik_m\xi} = 0, \quad (2.13)$$

that has a solution of the form

$$\Psi(X, \eta) = \Psi_0(\eta)e^{i(k_m-k_s)\xi} = \Psi_0(\eta)e^{i\Delta kX},$$

provided the amplitude $\Psi_0(\eta)$ satisfied the algebraic cubic condition

$$\Psi_0^3 + \Psi_0\left(\eta^2 - \frac{2a\eta}{3b} - \frac{2\Delta kk_s}{3b}\right) + \frac{\tilde{h}_m\delta}{6b\text{Ro}} = 0. \quad (2.14)$$

In some region of the Δk domain, Eq. (2.14) has three real roots that correspond, of course, to the multiple states of Charney and De Vore. The mechanism producing multiple equilibria is the same as that in classical mechanics [see Landau and Lifchitz (1969) for a vivid description of this physical process] sometimes characterizing the near-resonant behavior of nonlinear oscillators: The zero-order, linear resonant curve (see Fig. 1) is "bent" by nonlinearity so that in some range of values of detuning Δk large-amplitude states (due to nonlinear equilibration of resonant, excited waves) appear together with the usual, small-amplitude ones.

From previous analysis [see, for example, the general demonstration by Bogoliubov and Mitropolsky, (1961)], it is known that the two extreme states (in amplitude) are always stable,[3] while the intermediate one is unstable.

Two of the states (the ones of small and intermediate amplitude) have opposite phase with respect to the third one. This property is rather unlucky from a meteorological point of view, since it is known that blocking perturbations tend to be in phase with respect to quasi-stationary (long-term time-average) waves. Another unpleasant aspect of the solution of (2.14) is the multiple connection of the stationary response curve that makes it difficult to study the transition from one to the other of the multiple-equilibrium states.

The easiest way of removing both of these undesirable features without altering the general structure of the problem is to introduce a "frictional" term[4] of the form $\gamma k^*\delta_\xi\psi$ into Eq. (2.8).

The resulting curves of stationary response to topographic excitations are shown in Fig. 2, where it can be seen how the damped resonance curve is bent by nonlinearity, again giving rise to multiple equilibria within limited regions of the wavenumber spectrum of forcing. Analysis of the phase shows that the phase opposition is now only asymptotic (for $|\Delta k| \to \infty$) in wavenumber.

The solutions that we are here discussing are waves, whose amplitude is slowly modulated by nonlinearity, superimposed on a zonal flow. The typical behavior of the streamfunction field is shown in Fig. 3 where the wide-channel approximation is masked by the use of the stretched coordinates ξ and η. Notice how the positive or negative bending of the nonlinear resonance (corresponding to different nonlinearity coefficients a and b and therefore to different shears of zonal flow) determines opposite latitudinal curvatures of the wave profiles.

All the multiple-equilibrium flows discussed in the papers mentioned in the Introduction belong to this class of near-resonant, nonlinear wave solutions. The physical mechanism that permits the existence

[3] It is important to notice that we are referring here to the stability of Eq. (2.8) with respect to fluctuations involving only space variation and not to the stability of the stationary solution with respect to time-dependent perturbations described by barotropic equations as discussed by Charney and De Vore (1979).

[4] It is clear that such a "vorticity preserving" friction is only an artifice to damp the resonant excitation due to the mountain and is not intended to model in any realistic way the effects of dissipation in the real atmosphere.

of the large-amplitude state is the equilibrium of resonant forcing with nonlinearity.

3. Nonsinusoidal forcing and local multiple equilibria

From a phenomenological point of view, the theoretical solutions discussed in section 2 are of limited use since they are essentially "global" (consisting of a single harmonic wave superimposed on a zonal flow) while the atmospheric phenomenon they should model, i.e., blocking, is often strongly localized in space.

Moreover, earth topography is not even approximately sinusoidal and, within the context of a nonlinear theory, single sinuisoidal states cannot be superimposed.

Previous experience in dealing with weakly nonlinear equations (see, e.g., Zaslavsky and Chirikov, 1972) gives some general indications about the behavior of solutions when forcing is not sinusoidal. Heuristic knowledge is concentrated in the so-called theory of "overlapping resonances": if in an equation of the form of (2.5) two or more harmonics of forcing do not operate simultaneously in the region of nonlinear resonance, the solution is still periodic and nonlinearity introduces only "smooth" corrections to the linear theory; if there is simultaneous forcing at different values of detuning within the region of nonlinear resonance, the solution becomes nonperiodic and discontinuities develop in the solution itself. In order to understand the physical nature of the mechanism that leads to this nonperiodicity, we again reduce the problem to its simplest mathematical form assuming that the forcing is exerted by a weakly nonsinusoidal, slowly modulated mountain, i.e.,

$$h(\xi, X) = \gamma \frac{\delta}{\mathrm{Ro}} \tilde{h}_m(X) \cos[k_s\xi + \Phi(X)],$$

with amplitude and phase changing on the long scale $X = \gamma\xi$. This corresponds to a WKB approximation in which topography is still locally characterized by a specific amplitude $h_m(X)$ and wavenumber $k(X) = k_s + \gamma[d\Phi(X)/dX] = k_s + \gamma\Delta k$, but both slowly vary in space.

If we move then along one of the branches of the nonlinear resonance curve that are stable, we can expect the solution to stay near this equilibrium branch until the unstable branch of the curve is reached; here the solution is expected to depart from the unstable equilibrium and eventually approach another stable branch. This process of transition between multiple stable sites is called the "jump" phenomenon.

The formulation of the stationary problem with the new topography $h(\xi, X)$ gives the same zero-order solution as in the case of sinusoidal topography

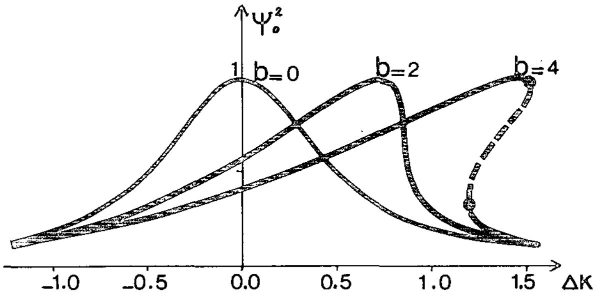

FIG. 2. Bending the resonant part of the frictional (see Section 2) stationary response curve due to a cubic nonlinearity. The friction coefficient is $K^* = 1$. The remaining parameters are as in Fig. 1.

$$\psi^{(0)}(\xi, \eta, X) = \Psi(\eta, X)e^{ik_s\xi} + (*) - \eta. \quad (3.1)$$

However, removing secularity appearing at first order yields

$$2ik_s d_X\Psi - 2a\eta\Psi + b(3\Psi^2\Psi^* + 3\eta^2\Psi) + \frac{\tilde{h}_m(X)\delta}{2\mathrm{Ro}} e^{i\Phi(X)} + ik^*k_s\Psi = 0, \quad (3.2)$$

which is an extension of Eq. (2.13) for weakly nonsinusoidal topography (with inclusion of friction).

Representing the complex amplitude $\Psi(\eta, X)$ in polar form $\Psi = \frac{1}{2}|\Psi|e^{i\theta}$, the real and imaginary part of Eq. (3.2) becomes respectively,

$$\frac{d|\Psi|}{dX} = -\frac{1}{2}|\Psi|k^* - \frac{\tilde{h}_m(X)\delta \sin\alpha}{2\,\mathrm{Ro}k_s}, \quad (3.3)$$

$$\frac{d\alpha}{dX} = \Delta k(X) + a\eta k_s^{-1} - \frac{3}{2}b\eta^2 k_s^{-1} - \frac{3}{8}b|\Psi|^2 k_s^{-1} - \frac{\tilde{h}_m\delta \cos\alpha}{2\,\mathrm{Ro}|\Psi|k_s}, \quad (3.4)$$

where

$$\alpha = \Phi - \theta; \quad \frac{d\alpha}{dX} = \frac{d\Phi}{dX} - \frac{d\theta}{dX} = \Delta k - \frac{d\theta}{dX}.$$

We have not yet specified the exact dependence of the amplitude and the wavenumber of topography on the long space coordinate.

Let us consider the case in which only the wavenumber is modulated while the amplitude h_m is kept constant. We introduce here a linear variation $\Delta k = \sigma_0 + rX$ that can always be considered as a local approximation to the real law of slow modulation. In such a way, we obtain

$$\Phi(X) = \Phi_0 + \sigma_0 X + \frac{1}{2}rX^2. \quad (3.5)$$

Integration of the system (3.3), (3.4) gives solutions of the kind plotted in Fig. 4 for some specific values of the wavenumber "drift velocity" r. The heavy line represents the stationary response of the system (3.3), (3.4) obtained for $r = 0$ (which obviously cor-

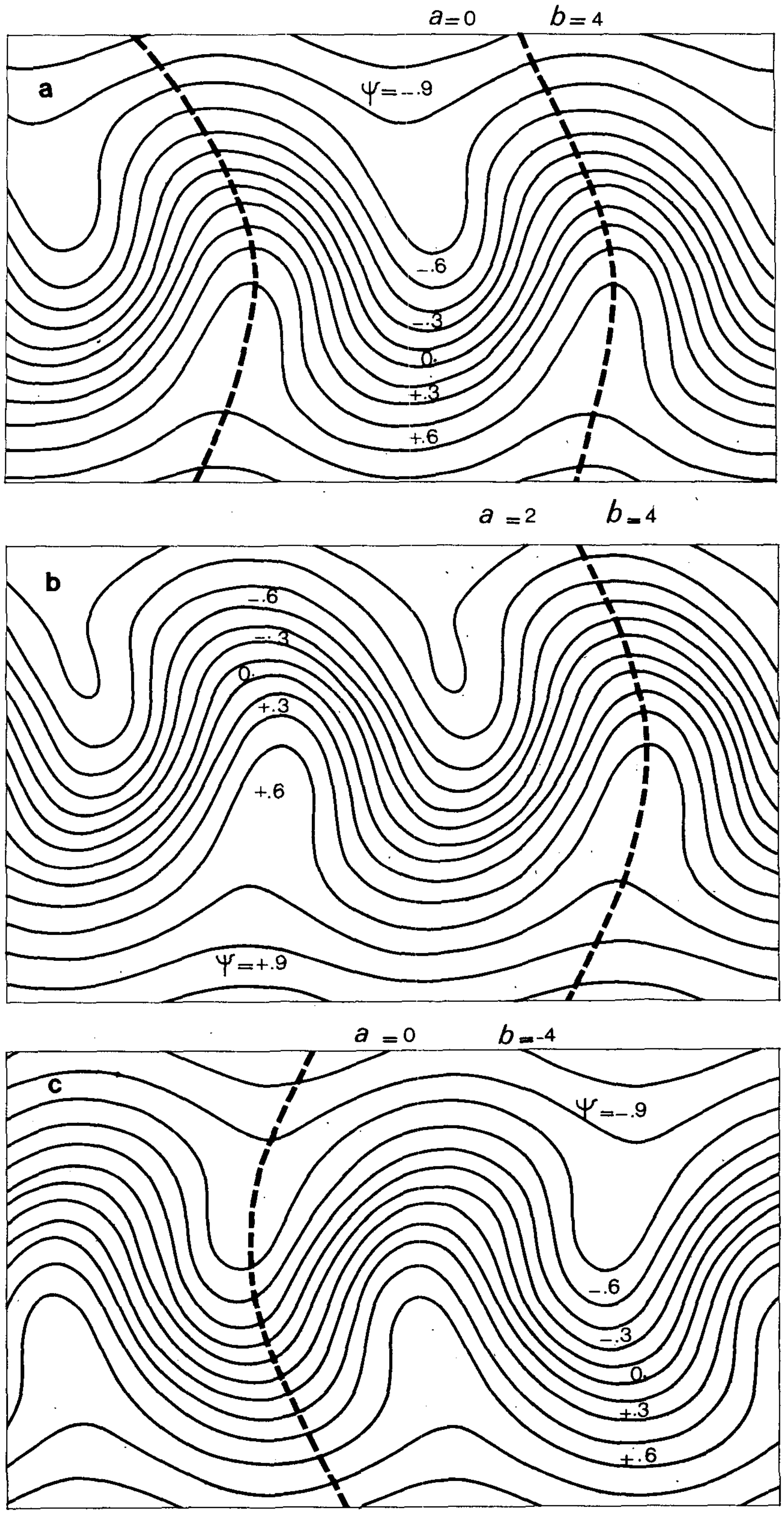

FIG. 3. Streamline patterns corresponding to large-amplitude wave solution of Eq. (2.8) for different values of nonlinearity coefficients a and b. The values of the parameters are as in Fig. 1.

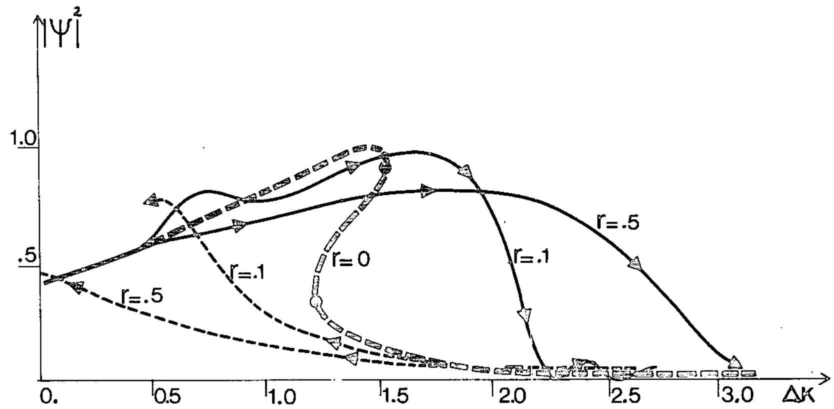

FIG. 4. The jump phenomenon for different values of the parameter r determining the velocity of spatial drift in wavenumber in Eq. (3.5). The heavy line, corresponding to $r = 0$, is the stationary response curve shown in Fig. 2. The thin continuous and thin dashed lines represent respectively forward and backward jumps (as indicated by the arrows).

responds to the solution for pure sinusoidal forcing shown in Fig. 3).

We now consider orbits starting from infra-resonant (left side of Fig. 4, negative r) values of the wavenumber. As the wavenumber increases, corresponding to different local shapes of the mountain, the solution moves, with minor oscillations, along the upper branch of the stationary state curve.

As the solution reaches the region of connection with the intermediate, unstable branch, the solution jumps down to the lower stable state. The length of such a jump increases with the space-derivative of the wavenumber r.

If the orbits start from super-resonant (right side of Fig. 4, positive r) values of the local wavenumber, they again move (with smaller oscillations than in the preceding case) along the stable branch of the stationary curve and suddenly jump up to the high-amplitude solution in the proximity of the connection with the intermediate, unstable branch of the stationary curve.

This unstable transition is usually smoother than the preceding one. Again, the length of the jump increases with the speed of variation of the local wavenumber. Similar jump transitions take place also when the amplitude is modulated instead of the wavenumber.

The streamfunction configurations corresponding to such jumps are rather typical and physically suggestive. A couple of examples are represented in Fig. 5 for different nonlinearities corresponding to different latitudinal shears. The transition takes place from the excited state to the small-amplitude one (moving from west to east). Due to β effect, the jump occurs at different longitudes across the channel. This gives rise to a jet-splitting, with the two branches of the jet contouring the region in which the small-amplitude wave solution dominates. There is also a tendency to create high-low dipoles through the latitudinal "bending" caused by nonlinearity. Analysis of the solutions reveals that dipoles are due to the change of phase in the passage through the resonance. The analogy with the same features of the observed blocking is evident and will be further discussed in the next section. Finally, we would just like to draw attention to the similarity between this theory and the one (based on the hydraulic jump analogy) put forward by Rossby (1950). In both theories, two different solutions, matched through a jump region, can coexist in the atmosphere and the regional character of blocking is a consequence of such a multiple connectivity of the global solution. However, the specific mechanism of jump is completely different in the two theories.

4. Extrapolation to realistic topography

In the preceding sections, we have described some mathematical aspects of the nonlinear resonance theory in order to elucidate the essence of the mechanism of "jump" between equilibrium states. We want now to discuss some physical implications of the approximations used and the nature of the solutions that are obtained once the theory is extended to more realistic situations.

The earth's topography at middle latitudes in the Northern Hemisphere is characterized by a spectrum with most of the amplitude of the zonal wavenumbers between 1 and 5. These wavenumbers are obviously too small for the approximation of slow variation ($\Delta K \ll K_s$) introduced in the Section 3 to apply rigorously. However, it is interesting to see what kind of features appear in the streamfunction field if the discussed balance of nonlinearity and topographic forcing is assumed to hold true for realistic topography. The resonant wavenumber of the real

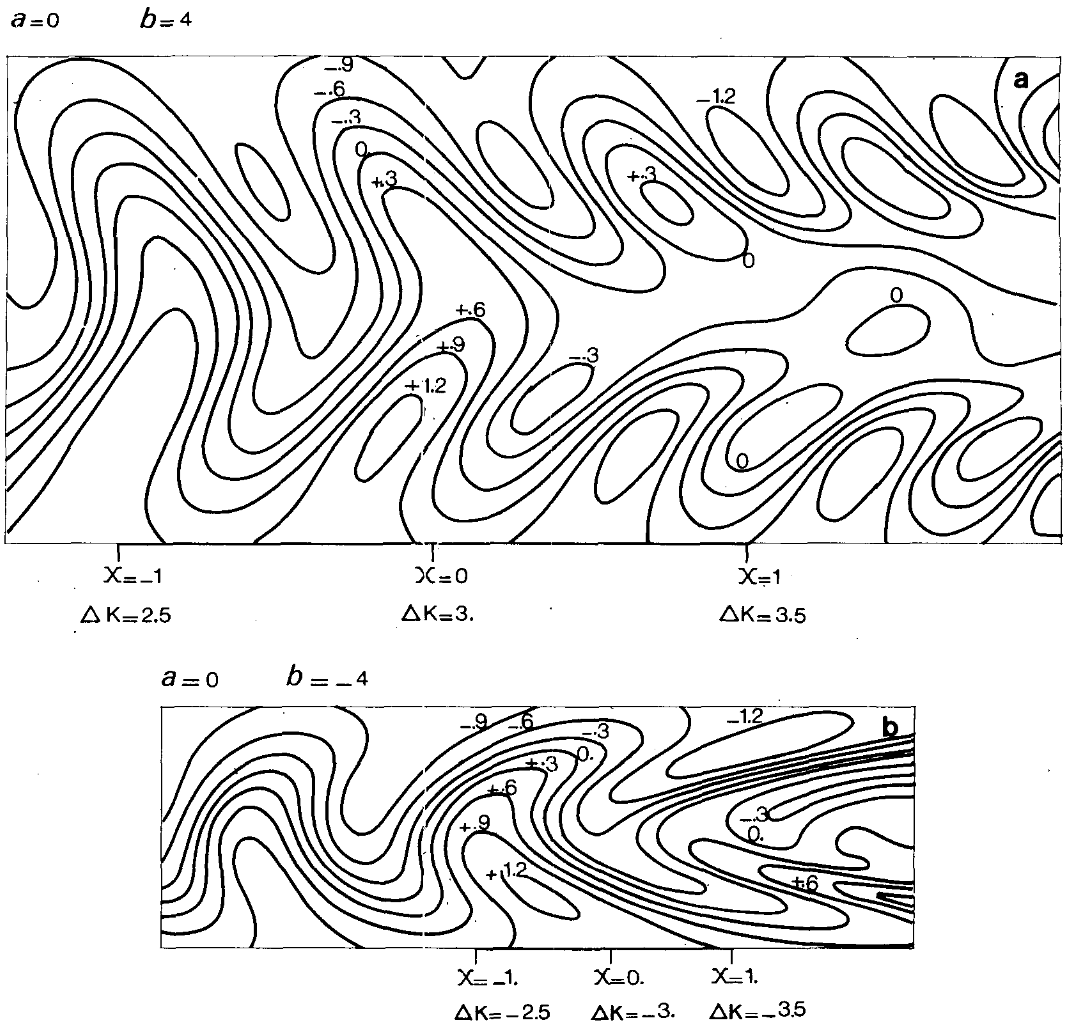

FIG. 5. Streamline patterns in the region of forward jump for different nonlinearities (respectively, $a = 0$, $b = 4$ in Fig. 5a and $a = 0$, $b = -4$ in Fig. 5b). The wavenumber drift velocity is $r = 0.5$. The other parameters are as in Fig. 2.

atmosphere is around $m = 3$. This can be deduced both from theoretical-numerical estimates and from observations. Particularly relevant is the evidence based on the tracking of free Rossby waves (see, e.g., Madden, 1979) which shows how Rossby waves corresponding to $m = 1, 2$ are always regressive, the ones characterized by $m \geqslant 4$, always progressive, while for $m = 3$, phase velocity oscillates around zero. It is therefore natural in the context of our theory to take the topographic components $m = 2, 3, 4$ as a modulated wave packet. The resulting behavior of the local amplitude and wavenumber are shown in Fig. 6. It is interesting to note how both wavenumber and amplitude modulations are relevant. However, *while the most conspicuous wavenumber variation is localized in the European region, most of the amplitude modulation takes place over the Pacific Ocean and the North American continent.*

As we have seen in Section 3, both modulations can cause jumps. It can be seen that the wavenumber modulations are negative with respect to the central wavenumber $m = 3$. Consequently, in order to produce resonance overlapping, we assume the nonlinear bending of the curve to be negative as shown in Fig. 7 for different values of h_m.

The stationary response curves to sinusoidal forcing are strongly dependent on the amplitude of the topography. Since the amplitude is variable (as shown in Fig. 6b), we can expect rather complicated features to appear in the streamfunction field. In order to take into account the latitudinal variation of topography and the sphericity of the earth, we have assumed the height h_m to be constant in the interval $-1 \leq \eta \leq 1/3$ and linearly decreasing to zero in the interval $1/3 < \eta < 1$.

The streamfunction pattern corresponding to the solution of Eqs. (3.3) and (3.4) subjected to the topographic forcing described above is shown in Fig. 8.

A rather impressive dipolar high-low blocking is quite evident in the European region, while the large-amplitude wave solution seems to dominate over the Pacific Ocean. A stationary ridge is correctly placed over the Rocky Mountains.

In conclusion, simultaneous action of forcing harmonics in nonlinear resonance seems to be able

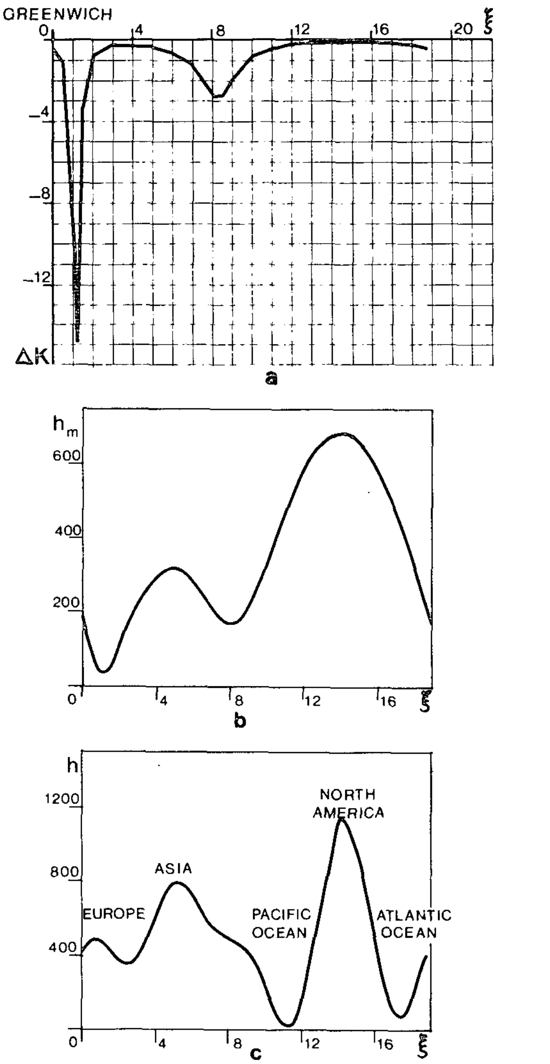

FIG. 6. Longitudinal variation of local wavenumber (a), local amplitude (b) and height (c) of the zonal components 2, 3, 4 of real topography in the Northern Hemisphere. $\gamma = 1/3$.

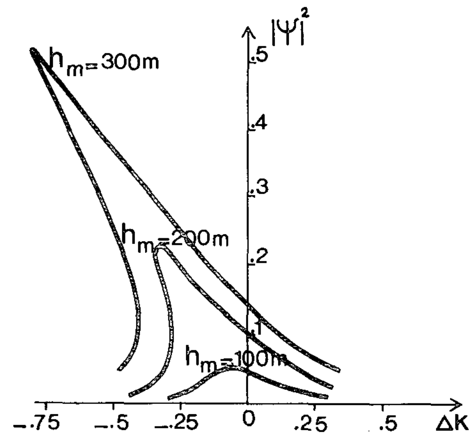

FIG. 7. Negatively bent resonance curve for different values of the maximum height h_m of the zonal topographic components 2, 3, 4. The values of the parameters are $K^* = 0.25$, Ro = 0.166, $\gamma = 1/3$ and $b = -4$. The curves are again evaluated at the central latitude $\eta = 0$.

to produce realistic blocking patterns. In particular, the regional character of observed blocking can be attributed to the jump phenomenon. According to such an interpretation of the regional character of blocking, the atmosphere can enter a blocking state any time the zonal flow assumes a configuration that causes a spectral region of nonlinear resonance to overlap a region of maximum nonsymmetric forcing.

5. Conclusions

Evidence that multiple equilibria are possible under a variety of mathematical approximations to the equations of motion for atmospheric flow is rapidly accumulating, even if nobody has yet been able to identify such different equilibrium states in the real atmosphere or in realistic numerical simulations of its circulations. The interpretative scheme that we suggest in this paper seems to be potentially able to permit such identification, searching in the direction of regional multiple equilibria.

The model can be greatly improved using a numerical approach of the kind used by Egger (1978), who produced blocking-type features with a channel model in which different harmonics of forcing were allowed to interact nonlinearly with a barotropic or baroclinic planetary flow. And, indeed, we think that some of the features identified as blockings by Egger in his study correspond to our local equilibria.

However, the real atmosphere is baroclinic and baroclinicity is known to contribute in an essential way to the maintenance of the stationary waves (Holopainen, 1970; Lau, 1979) and possibly also in the growth of baroclinic waves in the presence

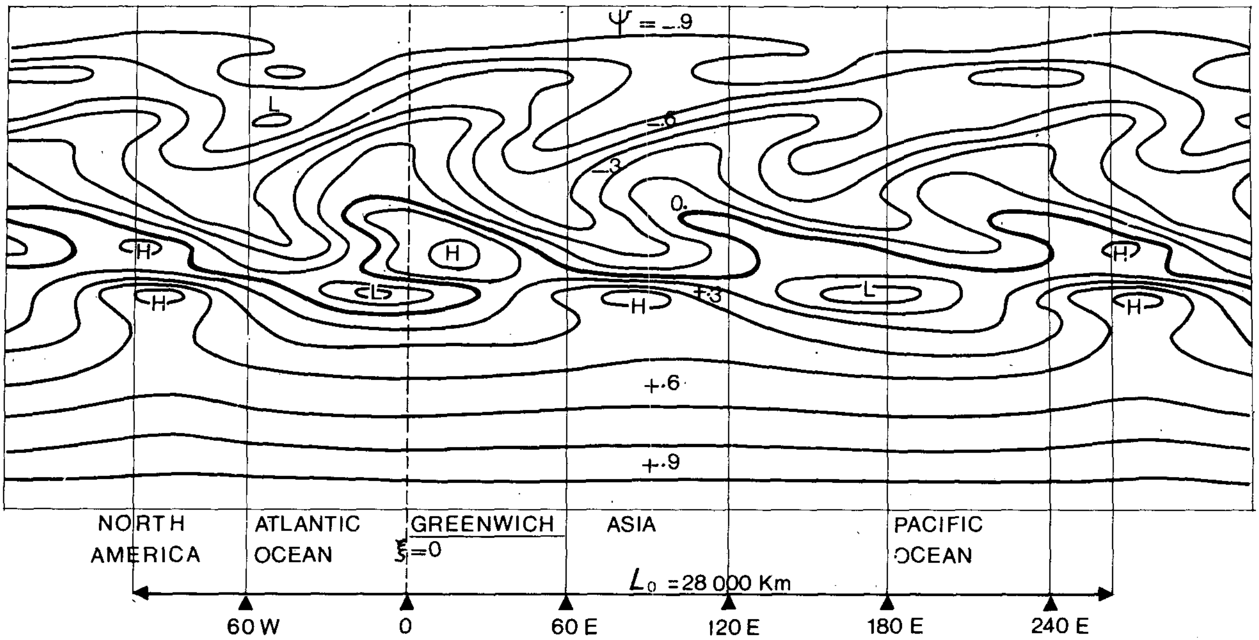

FIG. 8. Streamline patterns corresponding to the solution for the "realistic topography" described in Section 4. $a = 3$ and the rest of the parameters are as in Fig. 7.

of bottom topography (Yao, 1980; Charney and Straus, 1980).

Thus, study of the baroclinic version of multiple equilibria seems an unavoidable step to take in the direction of an application of the theory to real atmospheric flows.

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