

## A Wind Tunnel and Theoretical Study of the Melting Behavior of Atmospheric Ice Particles. I: A Wind Tunnel Study of Frozen Drops of Radius $< 500 \mu\text{m}$

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### ABSTRACT

A study has been made on the melting behavior of frozen drops suspended freely at terminal velocity in the UCLA Cloud Tunnel. The relative humidity of the air ranged between 25 and 95%. The warming rates of the tunnel air stream ranged from 2 to  $5^{\circ}\text{C min}^{-1}$ , which for the studied ice particles of radius between 200 and  $500 \mu\text{m}$ , corresponded to warming rates of 0.8 to  $5.2^{\circ}\text{C}$  per 100 m of fall. The rate of melting of the frozen drops and their fall behavior during melting were continually monitored by motion picture. From these observations the dimensions of the ice core inside the melting drop was determined as a function of time, and from the latter the total melting time was found. The present wind tunnel observations were compared with the theoretical predictions of Mason (1956). Considerable disagreement with Mason's theory was found. This disagreement was attributed to the pronounced asymmetric melting and the internal circulation in the melt water, both of which were disregarded in Mason's theory.

### 1. Introduction

When atmospheric ice particles fall through the  $0^{\circ}\text{C}$  level, they may begin to melt. The exact temperature at which melting begins depends mainly on the humidity of the air. This melting process may extend over considerable distances during the fall of the ice particle, depending on the size of the particle, and the temperature and humidity of the environmental air. If the ice particles are of hailstone size, the amount of heat required for their total melting may considerably exceed the amount of heat which can be supplied to it by the air during the fall of the hailstone from the cloud to the ground. As a result, the hailstone reaches the ground as a hard, and therefore destructive, ice particle. This fact is well substantiated by worldwide observations.

Melting of snowflakes, on the other hand, proceeds over considerably smaller distances leading to what is called the "bright band" in radar meteorology. This bright band is the result of high reflectivity for microwaves from the layer of partially melted snowflakes. Near this level pronounced changes in the cloud microstructure are observed. Thus, Passarelli (1978a,b) and Houze *et al.* (1979a,b) found considerable differences in the cloud particle size distribution above and below the melting level, and Knight (1979) found considerable morphological changes in the cloud particles as they melted.

Superficial considerations would lead one to suspect that ice particles falling through the melting

level would lose some of their melt water. However, observations made by Ohtake (1969, 1970) on snowflakes above and below the melting level show that snowflakes do not break up as they pass the melting level. Also, small frozen drops melt without shedding their melt water. On the other hand, Blanchard's (1957) observations of giant raindrops suspended in a wind tunnel suggest that drops  $> 10 \text{ mm}$  in diameter will shed their melt water. This shedding proceeds from a ring-shaped accumulation of melt water near the ice particle's equator. Similarly, Macklin (1963), Bailey and Macklin (1968), Carras and Macklin (1973), Newman and Gokhale (1977) and List *et al.* (1976) provided experimental evidence that the melting and growth of hailstones is affected by the fact that some of the melted or collected water is shed from the hailstones as a result of the bouncing of colliding drops, the detachment of water sheets or the splashing of impacting drops.

Mason (1956) and Drake and Mason (1966) assumed that ice particles  $< 3 \text{ mm}$  will not shed their water during melting. They also assumed that melting proceeded in a fashion such that the melt water remained concentrically arranged around the melting ice core and that no forced convection due to internal circulation occurred in the melt water.

Obviously, the rate at which an ice particle melts, and thus the total time it requires to completely melt, will crucially depend on such factors as the shedding of the melt water, the geometric arrangement of the melt water around the melting ice core, any internal

circulations inside the melt water due to forced convection, and the bulk density and structure of the original ice particle. The melting rate of ice particles in the atmosphere, in turn, profoundly affects the rate at which the environmental air cools by its consumption of latent heat of melting. Temperature changes in the atmosphere cause changes in the atmosphere's stability which, in turn, affect the vertical motions in the atmosphere. Accurate knowledge of the rate of melting of atmospheric ice particles is therefore of considerable importance for the formation of any cloud dynamics model which allows for their presence.

Unfortunately, very few quantitative data have been provided by previous studies on the rate at which ice particles melt. However, modern experimental and theoretical-numerical techniques now allow us to start to make up for this deficiency. We have undertaken the first series of such studies and report on them below.

In this first study we considered the melting of frozen drops of radius  $< 500 \mu\text{m}$ . In the following sections our experimental procedure of this study will be described. The result of our experimental study will then be compared with the theoretical results of Mason (1956) and with the experimental results of Drake and Mason (1966). The conclusions from this comparison will be used to suggest studies needed to be done in the future.

**2. Experimental set-up**

The present experimental study on the melting of ice particles involved the UCLA Cloud Tunnel. This tunnel allows us to suspend water drops of  $20 \mu\text{m}$  to  $8 \text{ mm}$  in diameter and ice particles from  $50 \mu\text{m}$  to  $2 \text{ mm}$  in diameter. The experimental set-up was essentially that of Pitter and Pruppacher (1973), Pflaum *et al.* (1978) and Pflaum and Pruppacher (1979). During a particular experiment a super-cooled water drop between  $200$  and  $500 \mu\text{m}$  in radius, freely floating in the airstream of the tunnel, was converted to a frozen drop by contact nucleation. Once the ice particles were formed and freely supported in the airstream, the temperature of the tunnel air was raised from about  $-10^\circ\text{C}$  to about  $+10^\circ\text{C}$  at constant warming rates ranging between  $2$  and  $5^\circ\text{C min}^{-1}$ . This warming rate corresponded to a temperature rise of  $0.8$ – $5.2^\circ\text{C}$  per  $100 \text{ m}$  of fall of the frozen drop. The humidity of the air also was closely monitored and ranged from  $25$  to  $95\%$ . The melting of the ice particle was observed and photographically recorded between crossed polarizers by color motion picture. From our photographic record it was possible to deduce the hydrodynamic motions, the rate of melting, and the total melting time of the ice particle, by measuring the size of the ice core of the melting particle as a function of time.

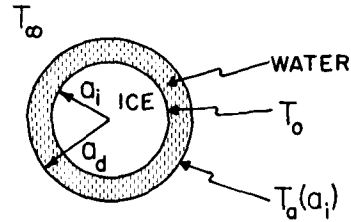


FIG. 1. Auxiliary figure to illustrate the theory of Mason (1956) for idealized melting of an ice sphere.

**3. The theory of Mason (1956)**

Mason (1956) has proposed an equation to describe the rate of melting and total melting time of a spherical ice particle. Since he gave no derivation of this equation the essential steps in this derivation will be briefly outlined.

In its general form, the convective diffusion equation (Fick's generalized second law) can be expressed as

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T, \tag{1}$$

where  $T$  is the temperature,  $t$  time,  $\kappa$  the diffusivity of heat through the medium (water), and  $\mathbf{u}$  the bulk flow in the medium (water). Assuming no internal circulation in the water ( $\mathbf{u} = 0$ ) and steady state ( $\partial T/\partial t = 0$ ), Eq. (1) in spherical coordinates becomes

$$\nabla^2 T = \frac{d^2 T}{dr^2} + \frac{2}{r} \frac{dT}{dr} = 0, \tag{2}$$

where  $r$  is the radial coordinate. Eq. (2) assumes that the problem has spherical symmetry (Fig. 1). Eq. (2) can more simply be expressed in the form

$$\frac{d^2(rT)}{dr^2} = 0. \tag{3}$$

Integration of Eq. (3) leads to

$$T = \frac{K_1}{r} + K_2. \tag{4}$$

The integration constants  $K_1$  and  $K_2$  may readily be obtained from the boundary conditions  $T = T_0$  at  $r = a_i$ , and  $T = T_a$  at  $r = a_d$ , where  $a_i$  is the radius of the ice core,  $a_d$  the radius of the frozen drop, and  $T_0 = 273.15 \text{ K} = 0^\circ\text{C}$ . One then finds

$$K_1 = \frac{(T_0 - T_a)a_i a_d}{(a_d - a_i)}, \tag{5}$$

$$K_2 = \frac{a_i T_0 - a_d T_a}{(a_i - a_d)}, \tag{6}$$

From Eqs. (4)–(6) follows

$$T = \frac{(T_0 - T_a)a_d a_i}{r(a_d - a_i)} + \frac{a_i T_0 - a_d T_a}{(a_i - a_d)}, \tag{7}$$

from which

$$\left(\frac{dT}{dr}\right)_{r=a_i} = \frac{(T_0 - T_a)a_d a_i}{a_i^2(a_d - a_i)} \quad (8)$$

Since the heat flux to the ice core (assumed to be spherical) is given by  $4\pi a_i^2 k_w (dT/dr)_{r=a_i} = L_m (dm_i/dt) = 4\pi \rho_i L_m a_i^2 (da_i/dt)$ , where  $L_m$  is the latent heat of melting,  $m_i$  the mass of the ice core,  $k_w$  the heat conductivity of water, and  $\rho_i$  the bulk density of the ice core, we find together with Eq. (8), that

$$4\pi \rho_i L_m a_i^2 \frac{da_i}{dt} = \frac{4\pi k_w [T_0 - T_a(a_i)] a_d a_i}{(a_d - a_i)} \quad (9)$$

which is the equation given by Mason (1956). Writing Eq. (9) as

$$\frac{da_i}{dt} = \frac{k_w [T_0 - T_a(a_i)] a_d}{\rho_i L_m (a_d - a_i) a_i} \quad (10)$$

which, for a given size  $a_d$  of the melting particle, may be integrated to obtain the total melting time from

$$t_m = \int_0^{a_i^m} dt = - \frac{L_m \rho_i}{k_w a_d} \int_{a_i=0}^{a_i=a_d} \frac{(a_d - a_i) a_i}{[T_0 - T_a(a_i)]} da_i \quad (11)$$

Eq. (11) has been given by Drake and Mason (1966) without derivation.

For steady state, the rate of heat transfer through the water layer is balanced by the rate of heat dissipation by forced convection and evaporation at the surface of the falling, melting ice particle. Thus, from Pruppacher and Klett (1980, Chap. 16)

$$\frac{4\pi k_w [T_0 - T_a(a_i)] a_d a_i}{(a_d - a_i)} = -4\pi a_d k_a [T_\infty - T_a(a_i)] \bar{f}_h - 4\pi a_d L_e D_{va} [\rho_{v\infty} - \rho_{va_d}] \bar{f}_v \quad (12)$$

where  $L_e$  is the latent heat of evaporation,  $\bar{f}_h$  and  $\bar{f}_v$  are the mean ventilation coefficients for heat and vapor flux, respectively,  $D_{va}$  is the diffusivity of water vapor in air, and  $\rho_{v\infty} = \phi_v \rho_{v,\text{sat}}(T_\infty)$  and  $\rho_{va_d} = \rho_{v,\text{sat}}[T_a(a_i)]$  are the water vapor densities far away and at the drop surface, respectively, where  $\phi_v$  is the relative humidity of air, and  $\rho_{v,\text{sat}}[T_a(a_i)]$  is the saturation water vapor density at the drop surface temperature  $T_a(a_i)$ .

For given values of  $T_\infty$ ,  $a_d$  and  $\phi_v$ , the quantities  $D_{va}$ ,  $k_a$ ,  $k_w$ ,  $\bar{f}_h$ ,  $\bar{f}_v$ ,  $\rho_{v,\text{sat}}$  and  $L_e$  can be obtained from Pruppacher and Klett [1980, Eqs. (13.3), (13.6), (16.47), (13.58), (A.4.1) and (4.85a), respectively]. With these quantities given,  $T_a(a_i)$  can be computed from Eq. (12) which, in turn, allows computing  $t_m$  from Eq. (11).

For the derivation of Eqs. (10), (11) and (12), it

was assumed that (i) melting proceeds under steady-state conditions, (ii) the overall radius  $a_d$  of the melting particle remains constant, (iii) the melting ice particle is spherical, and remains so at all times, (iv) the ice core remains spherical and concentric with the outer boundary of the melting particle at all times, and (v) heat transfer inside the melting particle takes place by molecular heat conduction only while forced convection of heat by means of internal circulation in the melt water is not important.

Assumption (i) can be justified on the basis of the thermal relaxation time for a falling ice sphere melting in subsaturated air. This is the time required by the ice sphere to reach  $(1 - 1/e)$  or 63% of its steady-state temperature difference after it has been suddenly transferred into a new environment. Pruppacher and Klett [1980, Eq. (13-68)] derived the thermal adaptation time for a water drop in subsaturated air. This equation has also been used by Mason (1956) and Drake and Mason (1966) to justify steady state. Simple considerations show that this equation is inadequate to describe the thermal adaptation time of a melting ice sphere. Following the procedure outlined by Pruppacher and Klett, one finds for the  $e$ -folding time of an ice sphere melting in subsaturated air

$$\tau = \frac{(a_d^3 - a_i^3) \rho_w c_w}{3a_d \left[ k_a \bar{f}_h + L_e D_{va} \left(\frac{d\rho_v}{dT}\right)_{\text{sat}} \bar{f}_v + \frac{\bar{f}_w a_i k_w}{(a_d - a_i)} \right]} \quad (13)$$

differing essentially by the last term in the denominator from the equation for a liquid drop [Eq. (13.68) in Pruppacher and Klett (1980)]. For  $a_i = 0$ , Eq. (13) is identical to Eq. (13.68) of Pruppacher and Klett for a water drop. In Eq. (13)  $\rho_w$  and  $c_w$  are the bulk density and the specific heat of water, respectively,  $k_w$  is the thermal conductivity of water, and  $f_w$  is the effect of forced convection in the circulating melt water on the transport of heat.

Obviously,  $\tau$  depends on the stage of melting, i.e., on  $a_i$ . For an ice particle of  $a_d = 350 \mu\text{m}$  which has completely melted ( $a_i = 0$ ), one finds from Eq. (13) for  $T_\infty = 0^\circ\text{C}$ ,  $\tau = 1.04 \text{ s}$ , in agreement with Fig. 13.23 (Pruppacher and Klett, 1980, p. 447). For  $a_i = 175 \mu\text{m}$  and  $f_w = 2.5$  (from our observations of the internal circulation inside the melt water), one finds from Eq. (13) that  $\tau = 0.21 \text{ s}$ . For an unmelted ice particle for which  $\rho_w \rightarrow \rho_i$ ,  $c_w \rightarrow c_i$  and  $L_e \rightarrow L_s$ , the relaxation time is even lower because  $\rho_i < \rho_w$ ,  $c_i > c_w$  and  $L_s > L_e$ .

Thus, our computations show that for  $a_d \leq 500 \mu\text{m}$  the thermal relaxation time is of the order of a second. During this time the ice particles considered fall at most a few meters. Over this distance the environmental atmospheric temperature changes by

<0.01°C. No serious errors will therefore be introduced by assuming steady state during the melting of ice particles of  $a_d \leq 500 \mu\text{m}$ .

Assumption (ii) may be verified by consulting Beard and Pruppacher (1971, Tables 1 and 2). For relative humidities in the ambient air of  $RH \geq 50\%$  one finds that for a melting time of 35 s the relative radius change  $\Delta a_d/a_d \leq 3\%$ . Our observations show that such a small radius change due to drop evaporation affects the melting time by <1 s.

Assumption (iii) is difficult to justify for ice particles which have shapes considerably different from that of a sphere. Thus, graupel particles are often irregular or conical in shape, while hailstones often have highly irregular shapes, and ice crystals are hexagonal. However, for comparison with Mason's theory one may consider the melting of frozen drops, which are usually quite spherical in shape.

Assumption (iv) is highly questionable since it is well known that the convective heat transfer into a sphere is most pronounced on the upstream side of the sphere, somewhat lower on the downstream side, and least near the sphere's equator (Woo and Hamielec, 1971). In addition, once the ice core is embedded in the melt water of the melting ice particle, it will tend to "float up," i.e., rise toward the downstream side of the falling and melting ice particle due to the difference between the bulk densities of ice and water. From both of these considerations one would expect that the ice core does not remain concentric with the outer boundary of the melting particle but that melting proceeds asymmetrically.

Assumption (v) must be questioned on the basis of the fact that Pruppacher and Beard (1970) and Le Clair *et al.* (1972) have shown experimentally and theoretically that inside a water drop falling in air, a strong internal circulation exists. Such a circulation is expected to develop also in the melt water as soon as the ice particle begins to melt. Considering these findings one must expect that the transport of heat through the melt water, as well as through the air outside the falling ice particle, is affected by forced convection and is not merely controlled by molecular diffusion. One therefore would expect to observe melting rates which are larger and total melting times which are lower than those theoretically predicted by Eqs. (10)–(12).

**4. Results of theoretical study**

In Fig. 2 a numerical solution to Eqs. (10) and (12) is given in terms of a plot of the variation with time of the radius of the ice core inside a melting ice sphere for a given warming rate and for various sizes of the melting ice sphere. We note that for a given size of the melting ice sphere the ice core initially varies slowly with time, but after some time rapidly decreases in size. Obviously, the larger the melting

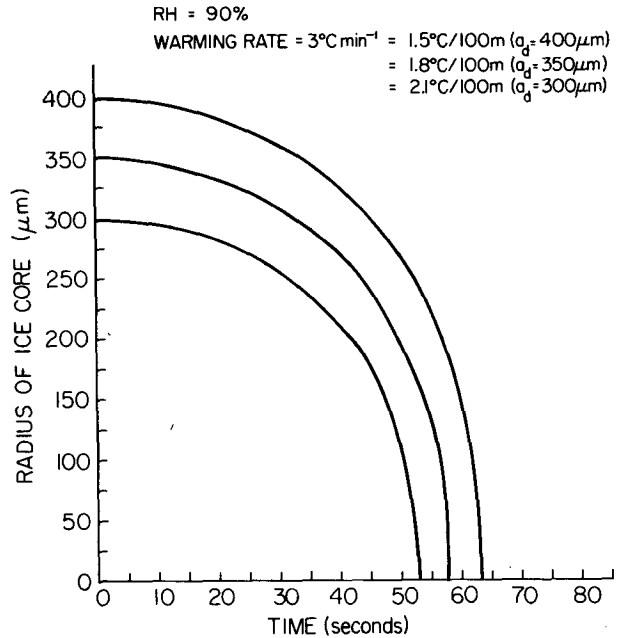


FIG. 2. Radius of the ice core inside melting ice spheres of various sizes, as a function of time for a given warming rate; theory of Mason (1956).

ice particle, the longer the time needed for complete melting.

In Fig. 3 a numerical solution to Eqs. (10) and (12) is given in terms of a plot of the variation with time of the radius of the ice core inside a melting ice sphere for a given size of the melting particle and various warming rates. As expected, the larger the warming rate the larger the rate of melting of the ice core and the shorter the total melting time. Total melting times computed from Eq. (11) for selected conditions and ice particle sizes are listed in Table 1.

Obviously, evaporative cooling of the surface of a melting ice particle falling in subsaturated air counteracts the melting process and therefore increases the time and distance a melting ice particle must fall before it has completely melted. In fact, evaporative cooling may considerably "delay" the onset of melting, causing melting to begin at temperatures considerably warmer than 0°C. Thus, for each environmental relative humidity  $(RH)_\infty$ , there is a critical environmental temperature  $T_{\infty, \text{crit}}$  at which melting begins. This temperature can be found from Eq. (12) by considering that melting begins when  $T_a(\alpha_i) = T_0$ . One then finds for  $\bar{f}_h \approx \bar{f}_v$  that

$$T_{\infty, \text{crit}} = T_0 - (L_s D_{vg}/k_a)[\phi_v \rho_{v, \text{sat}}(T_\infty) - \rho_{v, \text{sat}}(T_a)]. \quad (14)$$

Results from an evaluation of Eq. (14) are given in Fig. 4 where  $T_{\infty, \text{crit}}$  for onset of melting is given as a function of the relative humidity of the air. We

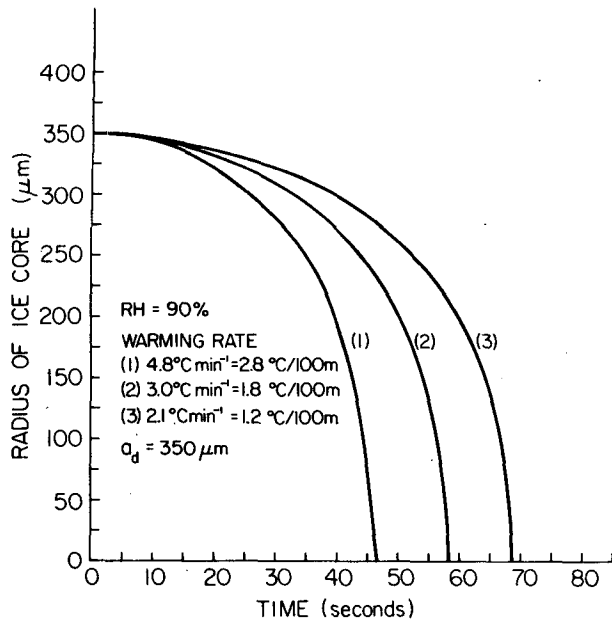


FIG. 3. Radius of the ice core of a melting ice sphere of 350  $\mu\text{m}$  as a function of time, for various warming rates; theory of Mason (1956).

note that  $T_{\infty, \text{crit}}$  is independent of the ice particle size, and increases notably with decreasing relative humidity.

**5. Results of experimental study**

Drake and Mason (1966) made some qualitative studies on the rate at which ice spheres of 200–3000  $\mu\text{m}$  in diameter melt. Unfortunately, the ice spheres were held fixed in the airstream so that they could not adjust hydrodynamically as in free fall. Also, the air temperature was held fixed at some value above 0°C rather than allowing it to warm up gradually, as is typically encountered by an ice particle falling through the melting layer in the atmosphere. In ad-

TABLE 1. Comparison of theoretically predicted [theory of Mason (1956)] with experimentally observed total melting times for frozen drops of various sizes in air of various relative humidities, warming at various rates.

Radius of sphere at time = 0 ( $\mu\text{m}$ )	Warming rate ( $^{\circ}\text{C min}^{-1}$ )	RH of air (%)	Total melting time (s)	
			Present experiment	Theory of Mason (1956)
370	2.02	86.5	65 $\pm$ 4	72
329	2.05	85	51 $\pm$ 4	63
319	1.45	80	47 $\pm$ 4	76
367	2.51	80	48 $\pm$ 5	63
364	1.6	90	52 $\pm$ 4	80
357	2.92	92.5	21 $\pm$ 2	57
388	2.80	91	30 $\pm$ 3	62
373	2.19	91	52 $\pm$ 6	71
355	3.46	84	34 $\pm$ 4	52
360	5.85	87	17 $\pm$ 3	37
371	4.73	90	29 $\pm$ 4	42
364	1.8	86	37 $\pm$ 4	72
370	4.1	88	31 $\pm$ 5	47
355	2.67	87	36 $\pm$ 5	57
370	1.48	93	38 $\pm$ 5	85
319	3.19	84	41 $\pm$ 8	52
370	6.0	87	26 $\pm$ 6	42
345	4.73	84	26 $\pm$ 6	42
397	0.946	83	50 $\pm$ 7	105

dition, melting was studied in air of wind speeds which did not match the terminal velocity of the individual ice particles whose melting rates were determined. Therefore, the experiments of Drake and Mason cannot be considered quantitative. On the other hand, our own experiments carried out in the UCLA Cloud Tunnel, as described in Section 2, did not suffer from the shortcomings of the experiments of Drake and Mason and therefore allowed a considerably more detailed study of the melting behavior of small ice particles. It must be pointed out, however, that both Mason's theory and Drake and Mason's experiment are able to predict qualitatively the correct trend for the rate of melting and total melting time with ice particle size and heating rate.

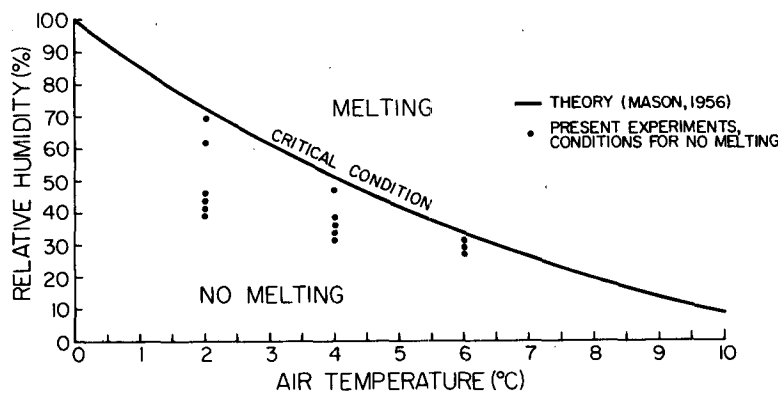


FIG. 4. Conditions (relative humidity of the air and air temperature) at which an ice sphere does/does not melt due to evaporative cooling at the ice-sphere surface. Comparison of experiment with theory of Mason (1956).

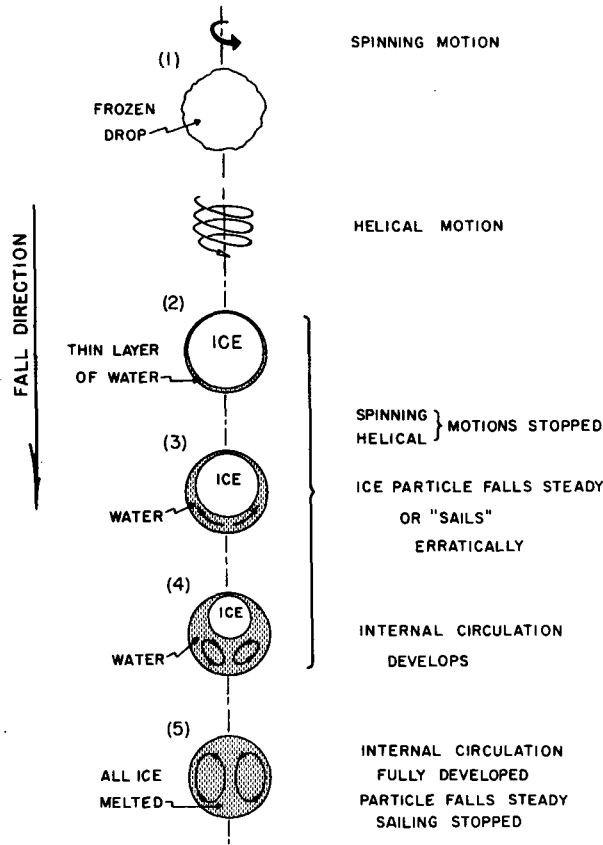


FIG. 5. Schematic of the actual melting behavior of a spherical ice particle, as revealed by motion pictures taken during present experiments.

Our results on the melting behavior of frozen drops of  $\alpha_d \leq 500 \mu\text{m}$  are summarized in Fig. 5. This figure shows that melting has a pronounced effect on the hydrodynamic behavior of the falling ice particle. Thus, while a frozen drop at temperatures below  $0^\circ\text{C}$  exhibits helical and spinning motions of the type described by Pitter and Pruppacher (1973), melting induces an erratic "sailing motion" which stops when the thickness of the melt water is about one-tenth

of the overall particle diameter, at which point the melting ice particle becomes stable. Also, as the particle melts, its terminal velocity increases, due to the fact that its density increases as it changes from ice to water. As Fig. 4 shows, the temperature and relative humidity dependence of the onset of melting predicted by Eq. (14) are verified by our experiments.

Melting was always found to begin on the upstream side of the ice particle and to progress more efficiently on this side. As mentioned above, this is because ventilated heat transfer is progressing most efficiently on the upstream side of a falling particle. Once the ice core is embedded in a sufficient amount of melt water, the ice core was observed to move toward the top of the drop due to the difference in bulk density between ice and water. This results in an eccentric location of the melting ice core with respect to the outer boundary of the drop, the eccentricity increasing as the melting of the ice core progresses, until the ice core eventually disappears on the downstream side of the drop.

As long as the water layer on the ice particle is thin, i.e., its thickness is  $\leq 1/10$  of the diameter of the original frozen drop, the layer behaves as adsorbed water. With increasing thickness, the layer assumes bulk behavior and begins to exhibit an internal circulation which rapidly increases in speed, finally reaching the internal circulation inside an ice-free water drop (Pruppacher and Beard, 1970; Le Clair *et al.*, 1972). Unfortunately, the internal circulation in the melt water which was found by our motion pictures cannot be visually documented within the framework of this article. On the other hand, the eccentric melting is documented here by a succession of motion picture frames (Fig. 6). Notice that the ice core tends toward a somewhat conical shape, because of the internal circulation. This shape also verifies the experimentally observed feature that no tumbling motions of the ice core occurred.

Obviously, our findings for the melting behavior of frozen drops with radii  $< 500 \mu\text{m}$  prove that the fourth and fifth assumptions for Eqs. (10)-(12) of Mason's (1956) theory do not hold. The consequence

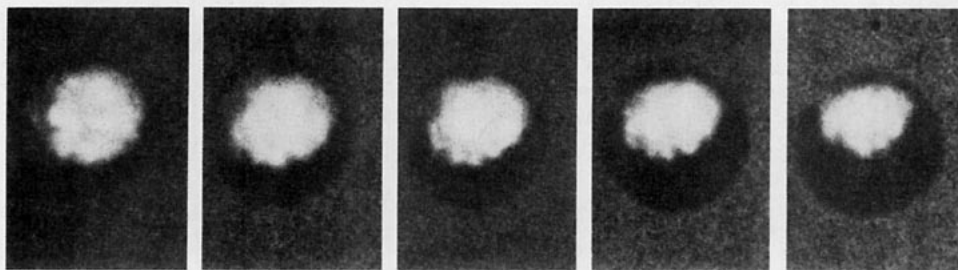


FIG. 6. Selected sequence of frames from a motion picture taken during the melting of a  $350 \mu\text{m}$  radius frozen drop in the present experiment. Ice particle was viewed between crossed polarizers with a tele-lens from a distance of  $\sim 15 \text{ cm}$ ; water appears black, ice appears whitish; note the asymmetric melting behavior.

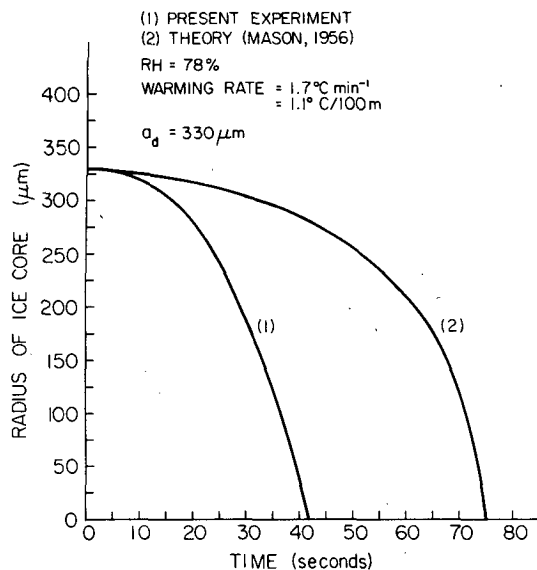


FIG. 7. Radius of the ice core inside a melting frozen drop of  $330\ \mu\text{m}$  radius for a given warming rate and relative humidity of air. Comparison of present experiment with theory of Mason (1956).

of this fact is exhibited in Fig. 7 and Table 1 which show that, as expected, the rate of melting of a frozen drop proceeds much more rapidly and the total melting time is considerably shorter than that expected on the basis of the theoretical predictions of Eqs. (10)–(12). We believe that forced convective heat transport in the water, as compared to purely molecular heat transport as assumed by Mason, is the cause of this difference. A new theoretical approach to the melting problem of small ice spheres must therefore be found which includes the effects of internal circulation in the melt water.

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