Radar Backscattering of Microwaves by Spongy Ice Spheres

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ABSTRACT

The radar backscattering cross section of a spongy ice hailstone—a mixture of ice and liquid water—depends on its size, shape and dielectric function. There are two types of theories of the effective dielectric function of two-component mixtures: Maxwell-Garnet and Bruggeman theories. In the latter, the two components are treated symmetrically, whereas in the former they are not. We have generalized the Maxwell-Garnet expression, originally derived for spherical inclusions in an otherwise homogeneous matrix, to ellipsoidal inclusions. When this expression, with all ellipsoidal shapes equally probable, is used in calculations of radar backscattering by ice spheres coated with spongy ice, the results are in generally good agreement with measured cross sections. Agreement is better if the inclusions are ellipsoidal rather than spherical, but only slightly so. Shape considerations are less important than taking ice to be the inclusions and liquid water to be the matrix.

1. Introduction

The problem of determining the electrical and magnetic properties of a mixture, given those of its components, is as old as electromagnetic theory itself, having been considered by Maxwell (1891, pp. 435–449) in his classic work *A Treatise on Electricity and Magnetism*. Since then, a great many prescriptions for treating heterogeneous mixtures have appeared in the scientific literature. Where radar meteorology intersects this literature on mixtures, is the problem of determining the radar backscattering cross sections of melting snowflakes and spongy-ice hailstones. Because of the laboratory measurements of List (1959) and Macklin (1961), it is believed that hailstones in the free atmosphere may be composed, at least in part, of mixtures of water and ice. The radar cross section of such a hailstone, at any wavelength, depends on its size, shape and complex dielectric function; this last quantity varies with the composition of the hailstone, the wavelength and, to a lesser extent, the temperature. If a scatterer is a relatively smooth sphere and its size and dielectric function are known, its backscattering cross section can be calculated by means of Mie theory.

In the late 1940's, radar meteorologists began using an equation attributed to Debye to calculate the dielectric function of mixtures of water and ice. But it was found by Joss (1964) and Atlas *et al.* (1964) that calculated backscattering cross sections of spongy ice spheres, with dielectric functions given by the Debye formula, did not agree with measured cross sections. Bohren and Battan (1980), however, reported that an expression first derived by Maxwell Garnet (1904) yielded values of the dielectric function for a mixture of small ice spheres in a water matrix that resulted in calculated backscattering cross sections that agreed reasonably well with observations.

In this paper, after a brief general discussion of mixture formulas, we extend the Maxwell-Garnet expression, originally derived for spherical inclusions, to randomly oriented ellipsoidal inclusions in a matrix with a different dielectric function. The expression so obtained is then used in calculations of backscattering by spongy-ice spheres, which are compared with observations. A briefer discussion of these results was presented by Bohren and Battan (1981).

2. Maxwell-Garnet and Bruggeman theories

If we assume that the concept of an average, or effective, dielectric function ϵ_{av}, for a two-component mixture is valid, then it is plausible that it depends on the dielectric functions of its components and their volume fractions

$$\epsilon_{av}(\omega) = F(\epsilon, \epsilon_m, f),$$

where f and $1 - f$ are the volume fractions of *inclusions* and *matrix* and their dielectric functions ϵ and ϵ_m are complex functions of the circular frequency ω. What, then, is the form of F? Unfortunately, the choice is large. Many such functions have been put

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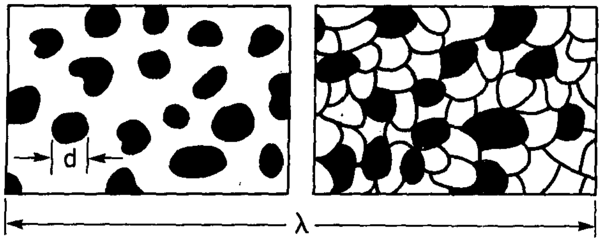

FIG. 1. Representation of spongy ice, when $d \ll \lambda$, as visualized by the Maxwell-Garnet theory (left) and the Bruggeman theory (right).

forward over the years. Some of them, although they appear to be quite different, are in fact identical. Others are not compatible with certain constraints imposed by electromagnetic theory. Still others are totally without theoretical foundation, having been determined by empirical fits to limited sets of data.

In a previous paper (Bohren and Battan, 1980), we critically examined several such functions. In particular, we expanded these functions in powers of the relative difference in the dielectric functions. All of these functions agree to first order:

$$\epsilon_{av} \approx f\epsilon + (1-f)\epsilon_m \quad \text{if} \quad \left|\frac{\epsilon - \epsilon_m}{\epsilon_m}\right| \ll 1.$$

That is, if the dielectric functions of the components are not too different, then the average dielectric function of the mixture is just the volume-weighted sum of the dielectric functions of the components. Indeed, any respectable mixture formula must have this property, because it has been shown by Landau and Lifshitz (1960, pp. 45–47) to follow from very general considerations. It is, therefore, pointless to argue the merits of this or that mixture formula on the basis of agreement with experiments in which the two components are dielectrically similar. A more stringent test is provided by mixtures of components with very different dielectric properties. Spongy ice, at microwave frequencies, is such a mixture.

A formula given by Debye (1929, pp. 44–47), which is a generalization of the Clausius-Mosotti relation, has been used in radar backscattering calculations for spongy-ice particles, and found to be unsatisfactory (Joss, 1964; Atlas *et al.*, 1964). This should come as no surprise, because it is evident from reading Debye that he had in mind homogeneous mixtures, for example, mixtures of two solutions. But spongy ice is not such a mixture, so there is no *a priori* reason to expect the Debye formula to be appropriate.

Therefore, if we exclude from consideration the Debye formula and also eliminate redundant formulas, then the choice narrows down to two types with some theoretical foundation; we shall refer to these as Maxwell-Garnet and Bruggeman theories. Although we used the term "effective medium" for the latter in our previous paper (Bohren and Battan, 1980), this was, perhaps, a poor choice because it has often been used as a general label for all theories of the dielectric properties of mixtures; also, for consistency, if one theory carries a person's name, then so should the other.

The physical distinction between these two types of theories can be illustrated with a simple diagram (Fig. 1); a similar diagram has been given by Niklasson *et al.* (1981). The difference between the Maxwell-Garnet and Bruggeman theories is that, in the latter, the matrix and inclusions are treated symmetrically. That is, it is not possible to identify one component as particles and the other as the medium surrounding them. Both enter into the mixture on an equal footing, although, of course, one may be more abundant than the other. In this instance, the terms *inclusions* and *matrix* are merely convenient labels for the two components. The distinction between the two types of theories is reflected in the symmetry properties of the function F. In the Bruggeman theory, the average dielectric function is invariant with respect to interchange of the rôles of matrix and inclusions, whereas in the Maxwell-Garnet theory it is not:

$$F(\epsilon, \epsilon_m, f) = F(\epsilon_m, \epsilon, 1-f) \quad \text{Bruggeman}$$

$$F(\epsilon, \epsilon_m, f) \neq F(\epsilon_m, \epsilon, 1-f) \quad \text{Maxwell Garnet}$$

Note also that Fig. 1 shows the conditions under which the concept of an average dielectric function for a heterogeneous medium is valid: the individual heterogeneities must be small compared with the wavelength, and there must be many of them to a wavelength.

3. Maxwell-Garnet theory generalized to ellipsoids

The literature on the structure of spongy ice is not very helpful in guiding one's choice of theory. But preliminary calculations of radar backscattering cross sections for spongy ice spheres, and comparison of these calculations with backscattering data indicated to us that, for this particular application, the balance may be tipped slightly in favor of the Maxwell-Garnet theory (Bohren and Battan, 1980). Accordingly, we have generalized the Maxwell-Garnet theory, which was first derived for spherical inclusions (Maxwell Garnet, 1904), to randomly oriented ellipsoidal inclusions, where the probability of a given shape—sphere, prolate spheroid, oblate spheroid, and all shapes in between—is specified by an arbitrary continuous function. There are, undoubtedly, many methods for achieving the result of the derivation in the following paragraphs, some complex and others direct. We have, we believe, chosen the direct method, preferring physical plausibility to mathematical elegance.

Our heterogeneous medium is a mixture composed of inclusions embedded in an otherwise homogeneous matrix, where ϵ and ϵ_m are their respective dielectric functions. The inclusions are identical in composition, but may be different in volume, shape and orientation. We shall restrict ourselves, however, to ellipsoidal inclusions. The average electric field $\langle \mathbf{E} \rangle$ over a volume V surrounding the point $\mathbf{x}$ is defined as

$$\langle \mathbf{E}(\mathbf{x}) \rangle = \frac{1}{V} \int_V \mathbf{E}(\mathbf{x} + \boldsymbol{\xi}) d\boldsymbol{\xi},$$

where V contains many inclusions but is otherwise arbitrary. V is composed of the matrix volume and the volume of all the inclusions; therefore, we may write

$$\langle \mathbf{E}(\mathbf{x}) \rangle = (1 - f) \langle \mathbf{E}_m(\mathbf{x}) \rangle + f \sum_k w_k \langle \mathbf{E}_k(\mathbf{x}) \rangle,$$

$$\langle \mathbf{E}_m(\mathbf{x}) \rangle = \frac{1}{V_m} \int_{V_m} \mathbf{E}(\mathbf{x} + \boldsymbol{\xi}) d\boldsymbol{\xi},$$

$$\langle \mathbf{E}_k(\mathbf{x}) \rangle = \frac{1}{V_k} \int_{V_k} \mathbf{E}(\mathbf{x} + \boldsymbol{\xi}) d\boldsymbol{\xi},$$

where V_m is the matrix volume, V_k is the volume of the kth inclusion, f is the volume fraction of inclusions, w_k is f_k/f and f_k is V_k/V. Similarly, the average polarization is given by

$$\langle \mathbf{P}(\mathbf{x}) \rangle = (1 - f) \langle \mathbf{P}_m(\mathbf{x}) \rangle + f \sum_k w_k \langle \mathbf{P}_k(\mathbf{x}) \rangle.$$

If we assume that the linear, isotropic constitutive relations $\mathbf{P}_m = \epsilon_0 \chi_m \mathbf{E}_m$ and $\mathbf{P}_k = \epsilon_0 \chi \mathbf{E}_k$ are valid for the matrix and inclusions, then it follows that

$$\langle \mathbf{P}_m(\mathbf{x}) \rangle = \epsilon_0 \chi_m \langle \mathbf{E}_m(\mathbf{x}) \rangle, \quad \langle \mathbf{P}_k(\mathbf{x}) \rangle = \epsilon_0 \chi \langle \mathbf{E}_k(\mathbf{x}) \rangle,$$

where ϵ_0 is the permittivity of free space, and $\chi_m = \epsilon_m - 1$ and $\chi = \epsilon - 1$ are the susceptibilities of the matrix and the inclusions, respectively.

The average susceptibility tensor χ_{av} of the mixture is defined by

$$\langle \mathbf{P}(\mathbf{x}) \rangle = \epsilon_0 \chi_{av} \cdot \langle \mathbf{E}(\mathbf{x}) \rangle,$$

where χ_{av} is independent of position if the medium is statistically homogeneous. These equations can be combined to yield

$$(1 - f)(\epsilon_{av} - \epsilon_m \mathbf{1}) \cdot \langle \mathbf{E}_m(\mathbf{x}) \rangle + f \sum_k w_k (\epsilon_{av} - \epsilon \mathbf{1}) \cdot \langle \mathbf{E}_k(\mathbf{x}) \rangle = 0, \quad (1)$$

where $\epsilon_{av} = \mathbf{1} + \chi_{av}$ is the average dielectric tensor and $\mathbf{1}$ is the unit tensor. It is clear that, if ϵ_{av} is to be independent of position, then $\langle \mathbf{E}_m \rangle$ and $\langle \mathbf{E}_k \rangle$ must be linearly related.

Consider an *isolated* ellipsoid in a *uniform* field $\mathbf{E}_m$; the uniform field $\mathbf{E}_k$ in the ellipsoid is given by $\mathbf{E}_k = \boldsymbol{\lambda}_k \cdot \mathbf{E}_m$, where the principal components of the tensor $\boldsymbol{\lambda}_k$ are (see, e.g., Stratton, 1941, pp. 211–212)

$$\lambda_j = \frac{\epsilon_m}{\epsilon_m + L_j(\epsilon - \epsilon_m)}, \quad (j = 1, 2, 3).$$

The geometrical factors L_j are elliptical integrals of the form

$$L = \frac{abc}{2} \int_0^\infty \frac{dq}{(d^2 + q)[(a^2 + q)(b^2 + q)(c^2 + q)]^{1/2}},$$

where d is one of the three lengths $a \geqslant b \geqslant c$ of the semiaxes. In general, there are three distinct geometrical factors: $L_1 \leqslant L_2 \leqslant L_3$; they satisfy the conditions $0 \leqslant L_j \leqslant 1$ and $L_1 + L_2 + L_3 = 1$. All the geometrical factors are ⅓ for a sphere; two of them are equal for a spheroid (prolate or oblate).

With the *assumption*, and this is our major assumption, that the average fields are similarly related, that is, $\langle \mathbf{E}_k \rangle = \boldsymbol{\lambda}_k \cdot \langle \mathbf{E}_m \rangle$, (1) becomes

$$(1 - f)(\epsilon_{av} - \epsilon_m \mathbf{1}) + f(\epsilon_{av} - \epsilon \mathbf{1}) \cdot \sum_k w_k \boldsymbol{\lambda}_k = 0. \quad (2)$$

So our task reduces to that of determining the sum in (2); $\boldsymbol{\lambda}_k$ depends upon the shape and orientation of the kth ellipsoid, and w_k is the ratio of its volume to that of all ellipsoids. It is convenient to approximate the sum in (2) by an integral

$$\sum_k w_k \boldsymbol{\lambda}_k \approx \int_k w(k) \boldsymbol{\lambda}(k) dk,$$

where the continuous variable k in the integral represents all the variables that specify an ellipsoid: shape, volume and orientation. We now make several assumptions: there is no correlation between the volume of an inclusion and its shape or orientation; there is no correlation between shape and orientation; and all orientations are equally probable. It follows from these assumptions that

$$\sum_k w_k \boldsymbol{\lambda}_k \approx \beta \mathbf{1},$$

$$\beta = \iint \mathrm{P}(L_1, L_2) \left(\frac{\lambda_1 + \lambda_2 + \lambda_3}{3} \right) dL_1 dL_2. \quad (3)$$

The shape probability distribution function $\mathrm{P}(L_1, L_2)$ depends on only two variables because of the relation $L_1 + L_2 + L_3 = 1$. The average (scalar) dielectric function is therefore

$$\epsilon_{av} = \frac{(1 - f)\epsilon_m + f\beta\epsilon}{1 - f + f\beta}. \quad (4)$$

When all the inclusions are spheres, then (4) reduces to the expression first obtained by Maxwell-Garnet (1904), which gives us some confidence that it is correct, but subject to the limitations underlying its derivation. It is worth emphasizing that all the inclusions need not be the same size, provided they are small compared with the wavelength. Indeed, if they are to fill all space ($f = 1$), they *must* not all be the same size.

An approach somewhat similar to that above was taken by O'Neill and Ignatiev (1978), who obtained an expression for the average dielectric function of a mixture containing spheroidal inclusions with ratios of semi-minor to semi-major axes given by a probability distribution function.

It is instructive to expand (4) in powers of the relative difference between the dielectric functions of the two components, $\delta = (\epsilon - \epsilon_m)/\epsilon_m$:

$$\epsilon_{av} = \epsilon_m\left[1 + f\delta - \frac{f(1-f)}{3}\delta^2 + \frac{f(1-f)(3\langle L^2\rangle - f)}{9}\delta^3 + \ldots\right],$$

$$\langle L^2\rangle = \iint P(L_1, L_2)(L_1^2 + L_2^2 + L_3^2)dL_1dL_2 .$$

The most important conclusion to be drawn from this expansion is that, to terms of order δ^2, the shape of the inclusions is irrelevant. Therefore, one need not be concerned about the shape of the inclusions if they are embedded in a dielectrically similar matrix. However, spongy ice is not such a mixture.

When one adopts the Maxwell-Garnet, rather than the Bruggeman approach, one has to choose which component is the matrix and which the inclusions. Again, the literature on the structure of spongy ice does not compel a choice. Knight (1968) states that "*. . . in spongy hailstone growth, the ice forms completely from supercooled liquid water . . .*" and that "*. . . the surface zone of the hailstone must consist of projections of ice, with liquid water in between.*" This suggests that it would be best to take ice as the inclusions and water as the matrix. Spongy ice, however, has also been described as liquid water in a skeletal framework of ice (Mason, 1971, p. 341), which suggests opposite rôles for ice and water. It may be too much to expect spongy ice to fit neatly into one category or another; indeed, its structure undoubtedly depends on the stage of its growth. But we have found that ice inclusions embedded in a liquid water matrix seem to be more compatible with the experimental evidence.

In the absence of a reason to assume otherwise, we have assumed that all ellipsoidal shapes are equally probable; that is, $P(L_1, L_2)$ is a constant. With this assumption, (3) can be integrated to yield (Huffman and Bohren, 1979)

$$\beta = \frac{2\epsilon_m}{\epsilon - \epsilon_m}\left[\left(\frac{\epsilon}{\epsilon - \epsilon_m}\right)\text{Log}(\epsilon/\epsilon_m) - 1\right], \quad (5)$$

where $\text{Log}(\epsilon/\epsilon_m)$ denotes the principal value of the complex number (ϵ/ϵ_m).

In treating backscattering by spongy ice hailstones, therefore, we have decided to place our hope in the Maxwell-Garnet theory generalized to a random distribution of ice ellipsoids with all shapes equally probable, that is, Eqs. (4) and (5). But the test of any theory is how well it agrees with measurements; this is what we consider next.

4. Comparison between theory and experiment

Bohren and Battan (1980) compared observations and calculations of the backscattering of 5.05 cm waves by spherical ice cores surrounded by shells of spongy ice. The observed spheres had overall radii ranging from 1.0 to 1.2 cm, while the shell thicknesses ranged from 0.4 to 2 mm. The quantity of water in the spongy ice was expressed in various ways, one of which, the "equivalent water thickness," was used by Atlas *et al.* (1964) as the abscissa in a diagram having relative normalized backscattering cross section (in dB) as the ordinate. Equivalent water thickness is the thickness of a water shell, at the outer limit of the sphere, having the same total volume of water as that in the spongy ice shell.

Calculations of 5.05 cm backscattering by spheres 1.0 cm in overall diameter having shells of spongy ice, were found to agree reasonably well with observations when the dielectric function ϵ_{MG} was obtained from the Maxwell-Garnet equation [Eq. (1) in Bohren and Battan, 1980] for spherical ice inclusions in a water matrix.

A new set of backscattering calculations has been made for ice spheres having shells of ellipsoidal ice inclusions in a water matrix. Eq. (5) was used to obtain the dielectric function of the spongy ice. The new set of curves closely resembles that given in Fig. 5 of Bohren and Battan (1980). The principal differences are that the backscattering cross sections in the new calculations are ~0.5 dB smaller and the positions of the minima are shifted ~0.01 mm toward lower equivalent water thicknesses.

Joss and Aufdermaur (1965) made measurements, at various microwave frequencies, of the backscattering cross sections of individual ice spheres. One set of measurements illustrates the changes with time of the normalized backscattering (δ_n) by a sphere that, at the initial time, consisted of a core of ice surrounded by a shell of spongy ice, and that was exposed to sub-freezing temperature. Over a period of minutes, the water in the outer shell froze, leaving an all-ice sphere. Fig. 2 shows the measurements of the backscattering by one sphere identified as EW 64.39. Initially, it had an outer diameter of 1.36 cm and a shell of spongy ice 0.26 cm thick, 53% water, by mass. The fraction of water in the shell diminished with time until, at the 5.2 min mark, all the water was frozen. In order to compare theory with measurements, we assumed that W_{tot}, the percent mass of water in the shell with respect to the total mass of the sphere, changed linearly with time, from the initially known 40%, to zero at 5.2 min.

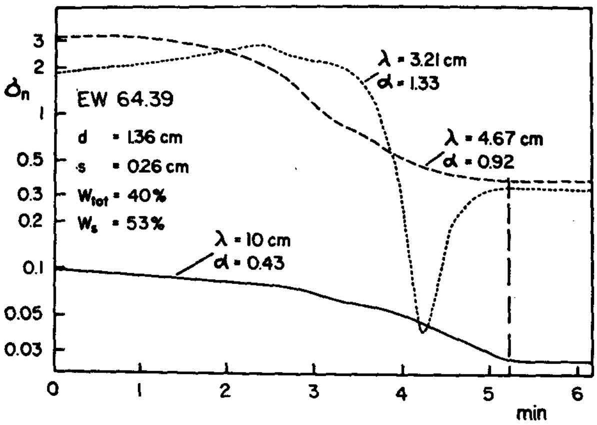

FIG. 2. Normalized cross section δ_n of spongy ice sphere EW 64.39. d is diameter; s, shell thickness; W_s, water content of the shell; W_{tot}, water content of the total stone. It is assumed that all the water in the stone was frozen at slightly after 5 min (dashed vertical line). From Joss and Aufdermaur (1965).

Calculations were made of the dielectric function (ϵ_{MG}) for water–ice mixtures at wavelengths of 3.21, 4.67 and 10 cm for two models of spongy ice. In the first model, the inclusions were taken to be spheres, and Eq. (1) in Bohren and Battan (1980) was used to calculate ϵ_{MG} for water inclusions in an ice matrix and *vice versa*. In the second model of spongy ice, the inclusions were taken to be ellipsoids and values of ϵ_{MG} were obtained from Eq. (5).

The best correspondence between observations and calculations was obtained when the spongy ice consisted of ellipsoidal ice inclusions in a water matrix. The results are shown in Fig. 3. The similarity to Fig. 2 is obvious: the magnitudes are about right, as are the relative positions of the three curves; the two measured intersection points (top two curves) are reproduced by the calculations.

Results obtained by assuming ice *spheres* in a water matrix are somewhat similar to those in Fig. 3, but they do not correspond as well with the measurements as do the ones in Fig. 3. The calculations of δ_n of spongy ice having water spheres or ellipsoids in ice matrices differ appreciably from the observations.

Figs. 4 and 5 allow a comparison of measurements and calculations of the 3.21 cm backscattering cross sections of ice spheres coated with spongy ice. The spongy ice was assumed to be made up of ice ellipsoids in a water matrix. Except for the shift of the minima towards the right, the calculations are in fair agreement with the observations. Again, as noted above, if other ice structures are assumed, the agreement between observations and measurements is not as good as in the case of ice inclusions in a water matrix. When the mixture involves water inclusions in ice, the calculated minima are closer along the W_{tot} scale than in Figs. 4 and 5, but the calculated values of δ_n at the minima are approximately an order of magnitude smaller than the observations.

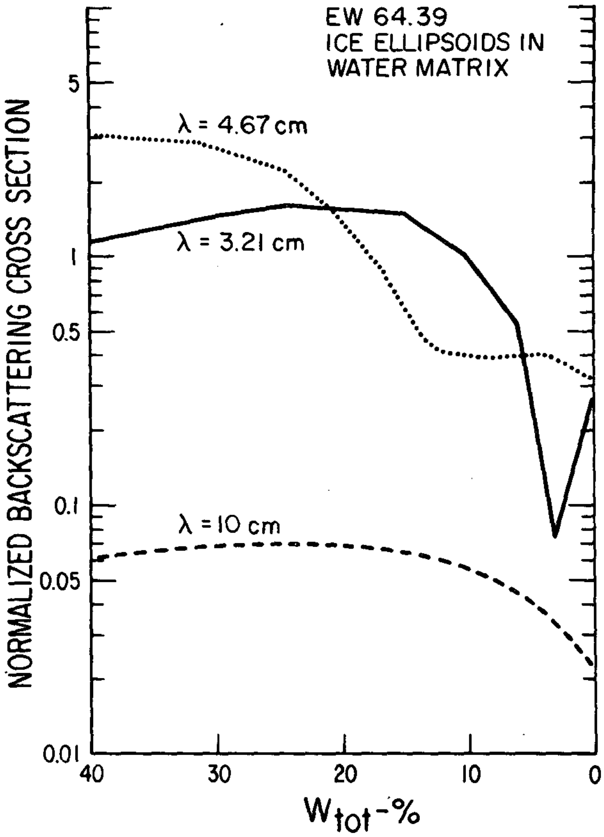

FIG. 3. Calculated values of normalized backscattering cross sections for spongy ice sphere EW 64.39.

5. Summary

The average, or effective, dielectric function of a heterogeneous medium, composed of inclusions embedded in a homogeneous matrix, depends on the dielectric functions of the two components, as well

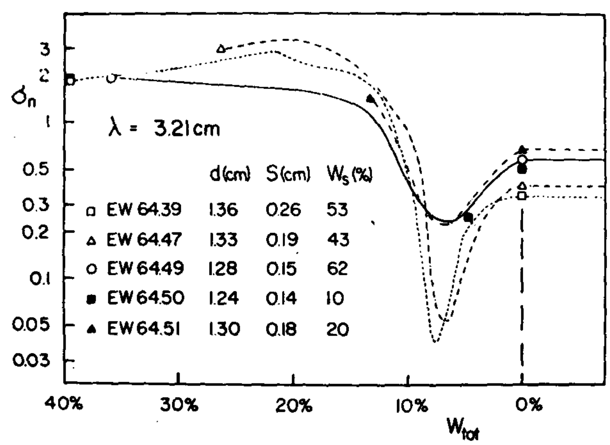

FIG. 4. Normalized cross section δ_n versus liquid water content W_{tot} of five different particles prepared in the hail tunnel and measured during freezing. (EW 64.39 is also presented in Fig. 2.) From Joss and Aufdermaur (1965).

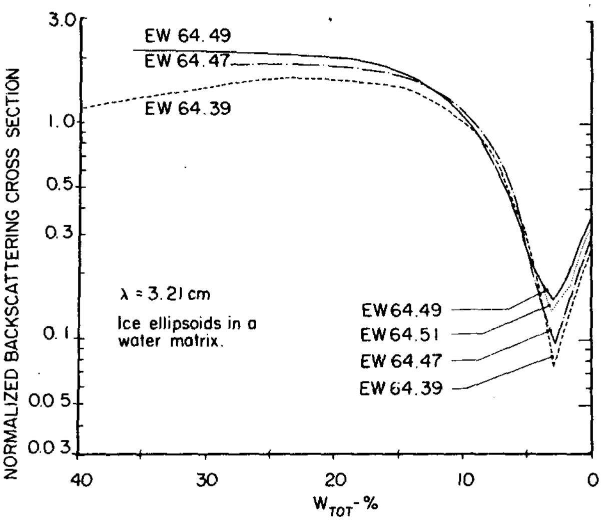

FIG. 5. Calculated values of normalized backscattering cross sections for all but one of the spheres described in Fig. 4. The curve for EW 64.50 was omitted for the sake of clarity.

as the volume fraction and shape of the inclusions. A great number of analytical expressions, superficially different, exist for the effective dielectric function of such heterogeneous media. If, however, the components are dielectrically similar, then these expressions give similar results (Bohren and Battan, 1980). Moreover, the shape of the inclusions is a third-order effect. But the dielectric functions of liquid water and ice are vastly different at microwave frequencies. In this instance, therefore, more care must be taken in constructing effective dielectric functions. We have generalized the Maxwell-Garnet theory, originally derived for small spherical inclusions, to small ellipsoids with shapes specified by a uniform probability distribution; that is, all ellipsoids—spheres, oblate and prolate spheroids, needles and discs—are equally probable in the mixture. Calculations of radar backscattering cross sections of spongy-ice spheres, using this dielectric function, agree well with measurements, provided that ice is taken to be the inclusions and the matrix is liquid water. In the cases examined, the shapes of the inclusions had only small effects on ϵ_{MG} and on the backscattering cross sections.

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