# NOTES AND CORRESPONDENCE

## A Note on QBO–SO Interaction, the Quasi-Triennial Oscillation and the Sunspot Cycle

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ABSTRACT

When quasi-triennial periodicities have been found in climatic data, their presence has often been attributed to QBO–SO interaction. Although a mixture of idealized QBO and SO signals can give rise to a quasi-triennial carrier wave modulated at the sunspot-cycle period, neither of the latter periods appears in spectral analysis of the resultant time series. It is therefore concluded that quasi-triennial spectral peaks are more likely to be of physical origin than a result of QBO–SO interaction.

### 1. Introduction

The Quasi-biennial and Southern Oscillations (QBO and SO) are the only recognized cycles longer than a year exhibited regularly by meteorological data (Holton and Tan, 1979; Horel and Wallace, 1981). The former is best shown by the alternating east- and west-wind regimes in the equatorial stratosphere of periods 24–27 months (Holton, 1979) and the latter by an index computed from variations in MSL barometric pressure at a number of stations stretching from Cape Town to Honolulu (Wright, 1977). A recent spectral analysis of this index for the period 1860–1975 (see Fig. 1) indicates peaks at 46, 67½, 34 and 107 months, in order of decreasing strength, and an autocorrelation analysis of the same series exhibits peaks at 43½ months and 11½ years.

Evidence, however, has appeared from time to time of the existence of an approximate 3-year cycle, henceforth referred to as the Quasi-triennial Oscillation (QTO). It is mentioned briefly by Lamb (1972) with reference to the extent of Baltic Ice and the pressure differences between Madeira and Iceland. Barry and Perry (1973) quote analyses of the annual frequencies of meridional and zonal hemispheric Grosswetter circulation types for the period 1900–65 which show a mean period of 3.1 years for the zonal cases and 2.5 years for the meridional ones. More importantly, such a cycle is mentioned by Joseph (1975) in relation to the 100 mb wind field in the equatorial regions of Asia and Africa. In the peak westerly phase of the cycle, the subtropical westerly wind belt of the Northern Hemisphere has been found to extend southward to the equator, and even to cross it in the Arabian Sea sector. Furthermore, the Indian monsoon has been found to respond to the oscillation, with monsoon failures occurring during the westerly phase of the oscillation in the upper tropospheric wind field. Bhalme and Mooley (1980), *inter alia*, refer to a triennial cycle of monsoon circulation events in India. Coughlan (1978) reports a 3.1-year quasi-periodicity in Australian rainfall. Finally, Gordon (1982) has found a highly significant three-year period in the zonally-averaged geopotential heights of the 50 mb pressure surface at 10°N. The power spectrum for this time series is shown in Fig. 2.

Holton (private communication to AHG) suggests that it is unlikely that the QTO is an independent phenomenon and more likely that it can be interpreted as a mixture of the QBO and SO. Acting upon this suggestion, the authors have investigated the consequences of mixing a noiseless time series corresponding to the SO with one corresponding to the QBO. The manner in which this has been done and the rather surprising results obtained therefrom are described below.

### 2. Linear mixture of idealized SO and QBO signals

Let $T$ and $T/\alpha$ denote the periods of the idealized (i.e., noiseless) SO and QBO signals, respectively, and suppose that the amplitudes of these respective signals are $S$ and $B$. If the mixing of one signal with the other is represented by the linear superposition of their corresponding time series and if $\phi$ denotes the phase difference between the two time series at some epoch $t = 0$, then the series representing the mixture may be written in the form

$$x(t) = S\cos\left(\frac{2\pi}{T}t\right) + B\cos\left(\frac{2\pi\alpha}{T}t + \phi\right). \quad (1)$$

Upon setting

$$E(t) = \frac{2\pi}{T}\left(\frac{\alpha - 1}{2}\right)t + \frac{\phi}{2}, \quad (2)$$

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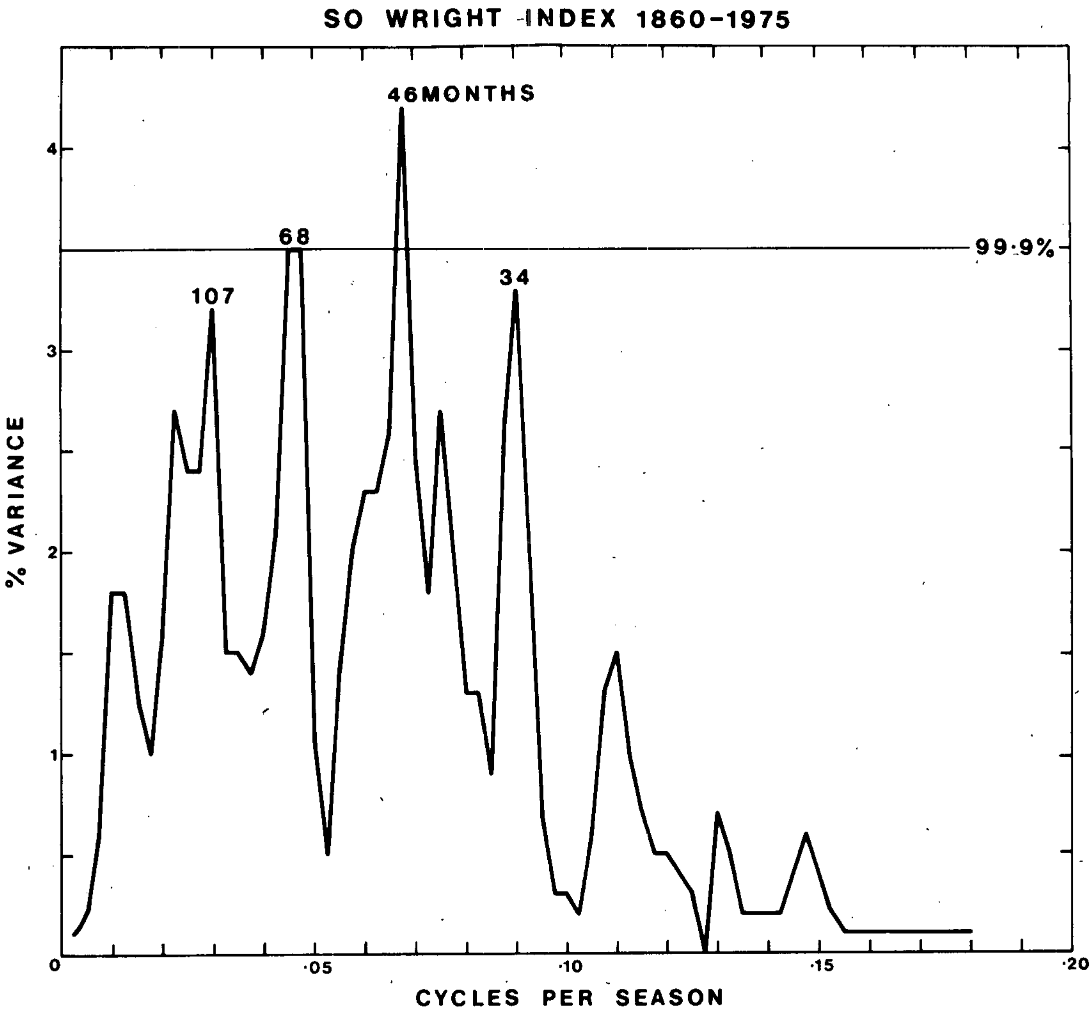


FIG. 1. Power spectrum of the Wright (1977) SO Index for the period 1860–1975.

however, it is easily shown that

$$x(t) = R(t)\cos\left[\frac{2\pi}{T}\left(\frac{\alpha+1}{2}\right)t + \theta(t)\,\mathrm{sgn}(B-S) + \frac{\phi}{2}\right], \quad (3)$$

where $R(t) = [(S+B)^2\cos^2E(t) + (S-B)^2 \times \sin^2E(t)]^{1/2}$ and $\theta(t)$ satisfies

$$[\sin\theta(t), \cos\theta(t)] = \frac{1}{R(t)}[|B-S|\sin E(t), (B+S)\cos E(t)].$$

Clearly, $R(t)$ and $\theta(t)$ are both periodic functions of period $2T/(\alpha-1)$.

The behaviour of $R(t)$ and $\theta(t)$ during one cycle of $E(t)$ is displayed in the harmonic dial of Fig. 3. The latter contains an ellipse with semi-major and semi-minor axes of length $(B+S)$ and $|B-S|$, respectively, about which has been circumscribed an auxiliary circle of radius $(B+S)$; both conics are centered at 0, the origin of the dial. Suppose that P′ on the auxiliary circle is the point whose polar coordinates are $(B+S, E)$ and that the perpendicular from P′ to Q on the major axis intersects the ellipse at P. Then it turns out that the polar coordinates of P are $(R, \theta)$ as defined above.

From (2), it is clear that P′ must describe the auxiliary circle at a uniform angular speed. The variations of $R$ and $\theta$ with respect to time are then obviously given by the motion of the corresponding point P around the ellipse. Note that this interpretation extends to the case $B = S$, since the line A0A′0A can be regarded as a degenerate ellipse of unit eccentricity for which P coincides with Q. We also mention in passing that (1) is mathematically equivalent to the superposition of an epicycle upon the deferent of a planet under the Ptolemaic system,

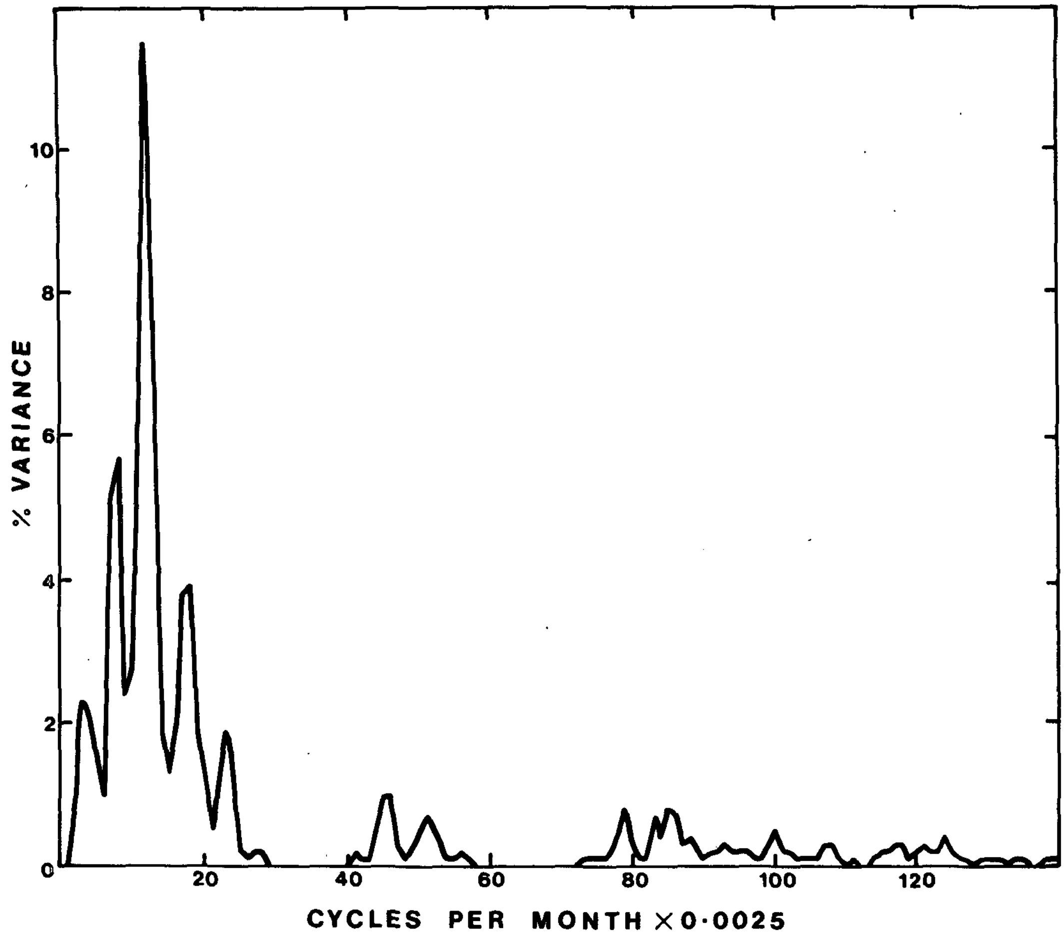


FIG. 2. Power spectrum of the zonally-averaged geopotential heights at 50 mb and 10°N.

whereas the angle $E(t)$ is analogous to the eccentric anomaly of a planet in the Keplerian system.

In the case $2T/(\alpha + 1) \ll 2T/(\alpha - 1)$, we could certainly interpret (3) as representing a carrier wave of period $2T/(\alpha + 1)$ whose amplitude $R(t)$ and phase $\theta(t)$ are modulated with a period of $2T/(\alpha - 1)$. As will be seen below, our interest lies more in situations where $2T/(\alpha + 1) < 2T/(\alpha - 1)$, but the separation between these periods is barely sufficient to justify the use of the "modulated carrier wave" concept. For want of a better terminology, however, we will still use the latter.

Bearing in mind the variability of the QBO and SO periods which have been determined from data analysis, we have investigated how the periods of the carrier wave and its modulations depend upon those of the QBO and SO. The results are shown in Fig. 4, which is a graph of carrier-wave period (in months) against QBO and SO periods (also in months), and Fig. 5, which displays the modulation period as a function of QBO and SO periods.

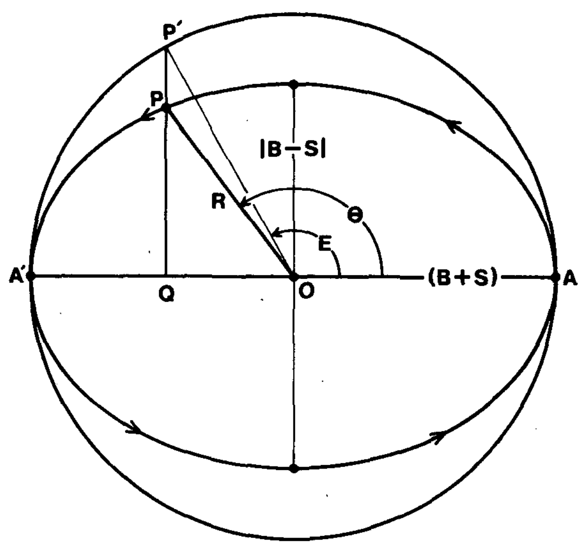


FIG. 3. Harmonic dial for the variations of $R$ and $\theta$ with time. As P′ describes the auxiliary circle at a uniform angular speed, the point P with coordinates ($R$, $\theta$) describes the ellipse.

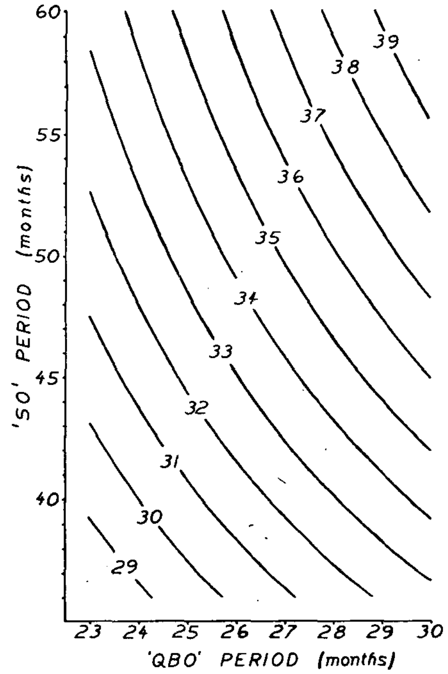


FIG. 4. Carrier-wave period (months) as a function of QBO and SO periods.

## 3. Discussion and conclusions

Referring to Fig. 4, it is clear that QBO's of period 24–27 months could combine with SO's of period about 46 months (corresponding to the strongest spectral peak mentioned in Section 1) to produce carrier waves of period 31.5–34.1 months and, referring to Fig. 5, it is evident that these same QBO and SO periods could produce a modulation of period 100–131 months, which remarkably is a bit shorter than the Sunspot Cycle (SSC) period of about 11 years. It therefore appears to be possible for the QBO and SO to combine in such a way as to produce a QTO modulated at the SSC period!

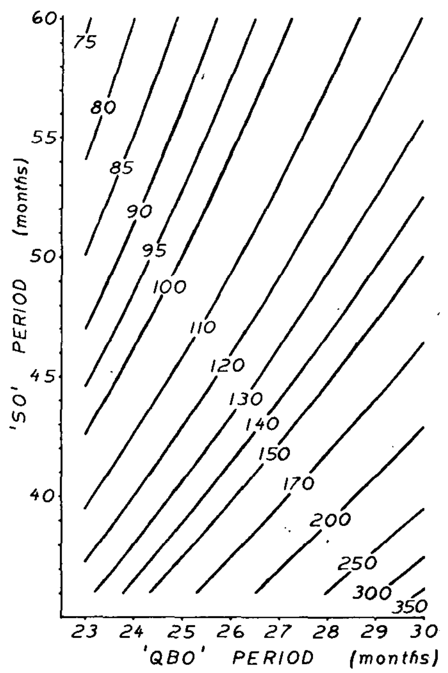


FIG. 5. Modulation period (months) as a function of QBO and SO periods.

At first sight, the above analysis appears to support Holton's hypothesis that spectral peaks at the QTO (and SSC) periods are more likely to be due to QBO–SO mixing than to independent physical effects. We should point out, however, that a perfect spectral analysis of the time series represented by (3) would yield only the components of (1), even though an experienced visual scanner of meteorological time series might see a modulated carrier wave in the data. As examples, we synthesized time series of lengths 252 and 996 months, by adding a QBO of 26½-month period to an SO of 43½-month period and similar amplitude, and then analyzed them by both the Fourier and autocorrelation techniques. In every case, peaks appeared at approximately 26½ and 43½ months, as expected, and not at the carrier and modulation periods of 33 and 135 months, respectively. Furthermore, three experienced visual scanners failed to detect with confidence any periodicity except the QBO.

While it is possible to envisage other (nonlinear) forms of QBO–SO interaction which might yield spectral peaks at or near the QTO and SSC periods, the conclusion to be drawn from the present analysis is that a simple linear mixing of the QBO and SO cannot masquerade as a QTO or SSC, even in the rather short data series that are the norm in climatic studies. In other words, the above discussion suggests that if a careful spectral analysis of data extending over several sunspot cycles yields a highly significant peak at the QTO period, then the latter cannot be lightly dismissed as a by-product of QBO–SO linear mixing.

The only conclusion to be drawn is that for certain climatic data series, a QTO is a better representative of the variability than either the QBO or the SO. In reality, however, the QBO, SO and QTO may all be aspects of the same physical phenomenon, but a discussion of this question is beyond the scope of our note.

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