MEASUREMENT OF APPROXIMATE RAINDROP SIZE BY MICROWAVE ATTENUATION

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ABSTRACT

It is not possible to deduce the size of raindrops from the average intensity of the radar signal produced by the precipitation region. However, if it is assumed that all the drops are of the same size, a relation between the diameter of the drops and the number per unit volume may be obtained. When measurable relative attenuation between radar waves of different lengths occurs, a different relation between drop size and concentration may be derived therefrom. The combination of these two types of measurement yields a single solution for the size and concentration of the drops. The solution can be considered as an approximation to the size of the largest drops present in the precipitation region, since the smaller drops contribute only slightly to the radar echo. Measurable attenuation of 3-cm waves may be expected to occur under two conditions: (a) heavy rain with large drops, and (b) moderately heavy rain with fairly uniform characteristics over a large area. Measurable attenuation of 1-cm waves can be expected even with light rain.

Frequent observations of radar echoes from precipitation areas have led to investigations of the possible usefulness of radar not only as a means of detecting storm areas but also as a source of meteorological information concerning the nature of the storm. Radar photographs in the horizontal and vertical planes give interesting descriptive information concerning the structure of various types of storms. The general scope and background of radar meteorology have been discussed by Bemis (1947) and by Wexler and Swingle (1947). There is little doubt that the radar echoes are caused by electromagnetic energy scattered by the water drops or ice crystals in the precipitation region; thus the radar echoes may be expected to yield information concerning the nature, size, and number of these scatterers. However, it does not seem possible to determine from observations on a single radar whether a signal of given intensity is caused by a relatively small number of large raindrops or by a very large number of smaller drops. This paper presents a discussion of the possibility of determining the approximate size of drops by comparing measurements of echo signal strength and of attenuation on two different wave lengths.

1. Average signal intensity of precipitation echo

The intensity of the radar echo from a precipitation region fluctuates rapidly with time because of interference between waves scattered by different drops. It can be shown that the average signal intensity is given by

$$P_r = P_t A_p^2 (9\pi\lambda^2 R^4)^{-1} V \eta K, \qquad (1)$$

where P_r is the average received power, P_t the transmitted power, A_p the area of the antenna dish for a paraboloid antenna, and λ the wave length. The length R is the distance of the target, in this case the part of the precipitation region which is under observation. The quantity V is the volume of the space from which scattered radiation can reach the receiver at a single instant. If h is the length of a wave train corresponding to a single pulse on the radar, and if θ is the beam width in degrees,

$$V = \pi[(\pi/180)\tfrac{1}{2}\theta R]^2 \tfrac{1}{2} h. \qquad (2)$$

Thus, for a given radar, V is proportional to R^2. The quantity η in (1) is the reflectivity per unit volume of the precipitation region and is equal to the sum of the radar scattering cross sections of all the drops in a unit volume. If $\sigma(D, \lambda)$ is the scattering cross section for a drop of diameter D and a radar wave length λ, and if $N(D)$ is the number of drops of diameter D in a unit volume,

$$\eta = \sum_D N(D)\, \sigma(D, \lambda). \qquad (3)$$

The last factor in (1), K, is an attenuation factor.

$$\log_{10} K = -0.2 \int_0^R k(R, \lambda)\, dR, \qquad (4)$$

where $k(R, \lambda)$ is the attenuation of the radar beam in decibels per unit length of path at a distance R from the transmitter in the direction of the beam.

All quantities in (1) can be measured directly on the radar except η and K. If the attenuation is neglected by assuming that K is equal to unity, (1) can be solved for η. If further it is assumed that all the drops are of the same size, the value of η serves to establish a relationship between N, the number of drops per unit volume, and D, the drop diameter, for (3) then becomes

$$\eta = N\, \sigma(D, \lambda). \qquad (5)$$

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The value of the scattering cross section $\sigma(D, \lambda)$ can be calculated for any values of wave length and drop size as shown by Stratton (1941) and Ryde.[2] For very small values of $\pi D/\lambda$ the scattering cross section for a spherical water drop is approximated by

$$\sigma(D, \lambda) \sim \frac{\pi^5 D^6}{\lambda^4}\left|\frac{n^2 - 1}{n^2 + 2}\right|^2, \tag{6}$$

where n is the complex index of refraction for water at the given wave length. This is the well known scattering formula derived by Rayleigh in order to explain the blue color of the sky.

An example of the quantitative information obtainable from a single radar measurement may be considered: Let it be assumed that the average signal intensity of the echo from a given storm is measured on a radar using a 10-cm wave length. The transmitted power, antenna area, and beam width are assumed to be known for the particular radar set, and the range can be read directly. Thus, if the attenuation is neglected by equating K to unity, (1) can be solved for the reflectivity per unit volume. Assume further that this solution yields the value 10^{-9} cm^{-1}. A value of η of this order of magnitude would be obtained, using a wave length of 10 cm, for moderately heavy rain. From (5) and (6) it follows that

$$\eta = \frac{N\pi^5 D^6}{\lambda^4}\left|\frac{n^2 - 1}{n^2 + 2}\right|^2. \tag{7}$$

For λ equal to 10 cm, $|(n^2 - 1)/(n^2 + 2)|^2$ is approximately 0.9. With these values, (7) expresses the following relation between N and D,

$$ND^6 = 3.5 \times 10^{-6}\ \mathrm{cm}^3. \tag{8}$$

The full curve in fig. 1 illustrates the relation between drop size and concentration expressed in (8). Each point on this curve gives values of N and D which would produce a reflectivity per unit volume of 10^{-9} cm^{-1} at a wave length of 10 cm, but to determine which point on the curve is most representative of the actual conditions in the precipitation region is impossible.

The relationship (8) between drop size and concentration for a given measured average signal intensity on the radar depends on several assumptions. The restrictions imposed by these assumptions will be considered briefly. In the first place, the attenuation was neglected. Theoretical calculations made by Ryde and Ryde[3] and also experimental evidence indicate that, for wave lengths longer than about 10 cm, the attenuation of the radar beam is always negligible except in extremely heavy cloudbursts. For wave lengths near 3 cm the attenuation is usually negligible for light precipitation but may be appreciable for moderate or heavy rain. For shorter wave lengths the attenuation can rarely be neglected. The effect of the attenuation might be minimized by taking the measurement near the front edge of the precipitation region. It was also assumed that all the drops in the precipitation region are of the same size. This is probably erroneous in practically all cases. However, most of the scattered energy comes from the largest drops present, so that neglecting the smaller ones introduces a relatively small error in computing the intensity of the radar echo. The use of the Rayleigh scattering formula for the scattering cross section is a valid approximation for wave lengths longer than 5–6 cm for all drops up to 0.6 cm in diameter. For shorter wave lengths the exact expression for $\sigma(D, \lambda)$ should be used unless there is reason to believe that the drops are small.

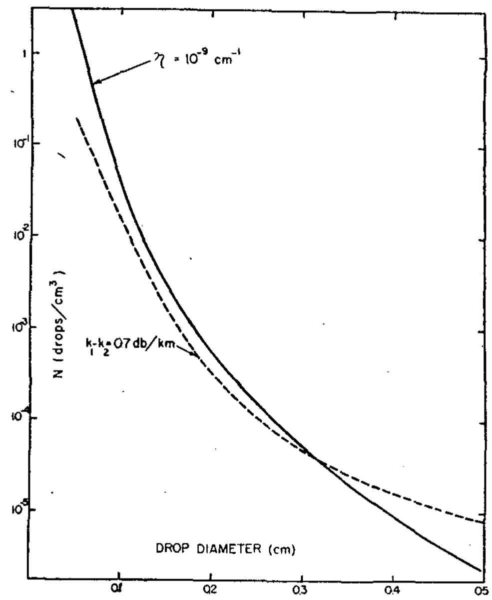

FIG. 1. Relation between raindrop diameter and number of drops per unit volume for (full curve) reflectivity per unit volume equal to 10^{-9} cm^{-1} for a 10-cm wave length and for (dashed curve) relative attenuation between 10-cm and 3-cm waves equal to 0.7 db/km.

Thus the conclusion is reached that the measurement of the average intensity of the radar echo from a precipitation region gives a relation between the size of the raindrops and their concentration, provided that the attenuation of the radar beam may be neglected and that the drops are assumed to be uniform in size. In order to obtain a more precise solution, further measurements are necessary.

2. Comparison of average intensity at two wave lengths

It has been shown in the preceding section that the measurement of the average power in the radar echo from a precipitation region can be used to obtain the

[2] J. W. Ryde, "Echo intensities and attenuation due to clouds, rain, hail, sand and duststorms at centimetre wavelengths," Research Laboratories of the General Electric Company, Ltd., England, Report No. 7831 (unpublished), 1941.

[3] J. W. Ryde and D. Ryde, "Attenuation of centimetre and millimetre waves by rain, hail, fogs and clouds," Research Laboratories of the General Electric Company, Ltd., England, Report No. 8670 (unpublished), 1945.

value of η, the reflectivity per unit volume of the rain area. The value of η establishes a relationship between D, the diameter of the raindrops, and N, the number of drops per unit volume. Since η depends only on drop size, number of drops, and radar wave length, measurements of average signal intensity on two radars would not yield any additional information unless the radars are operating on different wave lengths.

If the measurements of η are made on two wave lengths and if a curve similar to the full curve in fig. 1 is plotted for each wave length, any point at which the curves coincide represents a possible solution for the drop size and concentration which would produce the measured value of η at each wave length. From (7) it can be seen that, for drops small relative to the wave length and for a given value of η, N is proportional to $1/D^6$. If the precipitation is actually composed of very small drops, the curves for the two measured values of η must have a point of coincidence at a small value of D. But, since N is proportional to $1/D^6$ for both values of η, the curves must be identical in this region. For larger values of D the scattering of the shorter wave length begins to depart from the Rayleigh approximation and the curves for the two values of η separate. In this case a single solution for drop size and concentration in the precipitation region is not obtained, but an upper limit to the value of D is provided by the point where the two curves separate. If large drops are present in the rainstorm, the curves for the two measured values of η must have an intersection for a fairly large value of the diameter. Then for small values of D, where N is proportional to $1/D^6$, the curves should be parallel to each other. In this case a single solution for drop size and concentration may be obtained, but an appreciable error might be introduced in calculating the value of η from (1) because the attenuation of the radar beam is probably not negligible.

It is apparent that the comparison, on two wave lengths, of average signal intensity of radar echoes from a precipitation region would not, in most cases, yield a satisfactory solution for the size of the raindrops. In some cases, however, an upper limit for the drop size might be obtained.

3. Comparison of attenuation at two wave lengths

The discussion thus far has been restricted to those cases where the attenuation of the radar beam may be neglected. The cases with appreciable attenuation are considered in this section. The attenuation between the ranges R and $R + r$ can be obtained by measuring the average signal intensity at the two ranges. Since V is proportional to R^2 and since η is assumed to be the same at both ranges, (1) yields

$$\frac{(P_r)_R}{(P_r)_{R+r}} = \left(\frac{R+r}{R}\right)^2 \frac{\eta_R}{\eta_{R+r}} 10^{0.2kr}. \qquad (9)$$

The difference in signal intensity in decibels is

$$\Delta \equiv 20 \log_{10} \frac{R+r}{R} + 10 \log_{10} \frac{\eta_R}{\eta_{R+r}} + 2kr. \qquad (10)$$

In (9) and (10) k is the average attenuation in the range r. If this difference is measured on two different wave lengths at the same ranges and if all the drops are assumed to be of the same size, the first two terms in (11) are the same for both wave lengths and hence

$$\Delta_1 - \Delta_2 = 2r(k_1 - k_2) = 2Nr(k_1' - k_2'), \qquad (11)$$

where k' is the value of the attenuation coefficient for a concentration of one drop per unit volume. Values of k' for various drop sizes and wave lengths have been calculated and tabulated by Ryde and Ryde.[3] If Δ_1 and Δ_2 are measured, the value of N for any drop size can be obtained from Ryde and Ryde's tables and (11), and thus it is possible to plot a curve showing N as a function of D such that any point on the curve corresponds to a drop size and concentration which would produce the measured attenuation. As an example let it be assumed that the two wave lengths are 10 cm and 3 cm and that the measured attenuation, $k_1 - k_2$, is 0.7 db/km. An attenuation coefficient of this magnitude for a wave length of 3 cm would be produced by a moderately heavy rain, while the attenuation caused by the same rain would be negligible for λ equal to 10 cm. The dashed curve in fig. 1 shows the combinations of N and D which would cause this amount of attenuation. It can be seen from fig. 1 that a combination of the average signal intensity on one wave length and the relative attenuation at two wave lengths can give a single solution for N and D, namely, the intersection of the two curves.

Each side of fig. 2 shows the value of the relative attenuation, $k_1 - k_2$, as a function of the reflectivity per unit volume, η_2, for a specified set of conditions. The quantity η_2 was calculated from Rayleigh's formula, the attenuation being neglected. These graphs are constructed by eliminating N from (7) and (11) and thus obtaining, for λ_2 equal to 10 cm,

$$k_1 - k_2 = (3.5 \times 10^3/D^6)(k_1' - k_2')\eta_2, \qquad (12)$$

where D is in centimeters, η_2 in cm^{-1}, k in db/km, and k' in db/km for a concentration of one drop per cubic centimeter. For a given value of D, k_1' and k_2' can be obtained from the tables of Ryde and Ryde so that the curve for $k_1 - k_2$ as a function of η_2 may be plotted directly.

Since the limit of accuracy for quantitative radar measurements is in the vicinity of 3 db, the measured attenuation should be about 5 db or more. Table 1 gives the approximate distance required to produce 5-db attenuation. The average attenuation for different precipitation rates has been calculated by Ryde and Ryde[3] using average drop-size distributions for

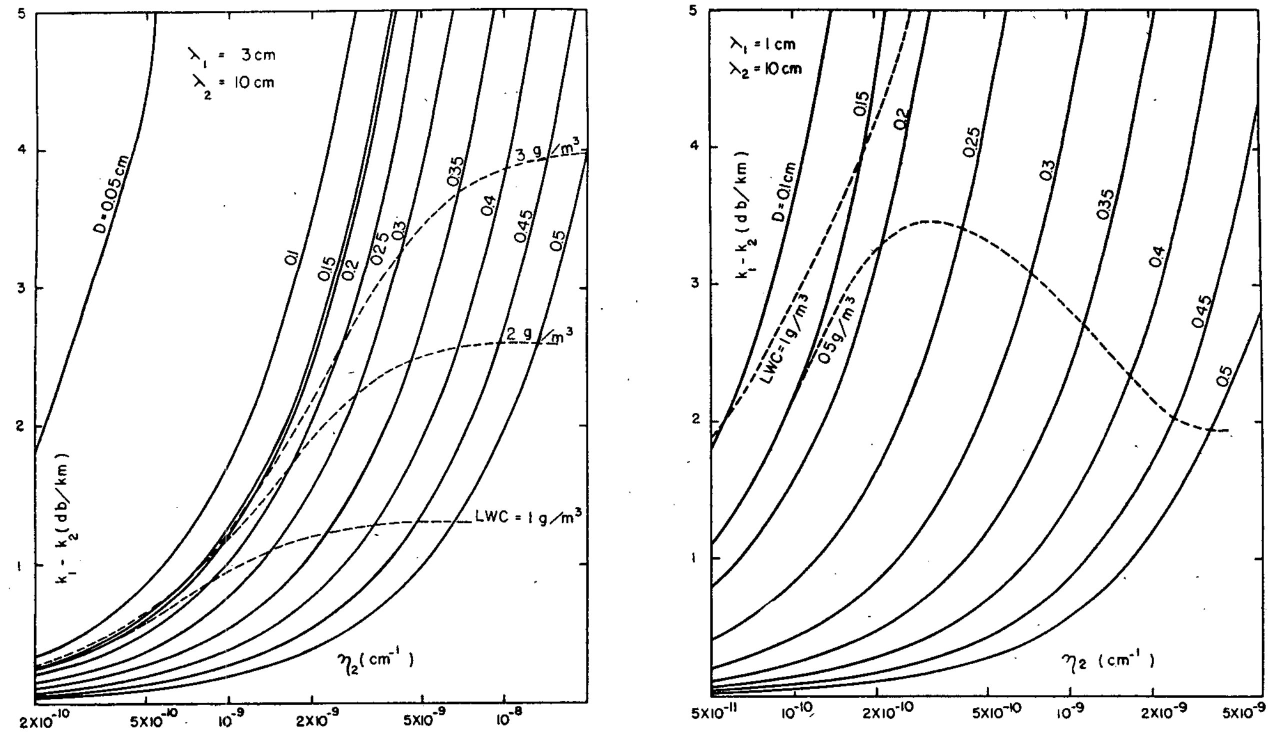

FIG. 2. Relation between η_2, the reflectivity per unit volume for a given wave length (λ_1 = 3 cm, 1 cm), and $k_1 - k_2$, the relative attenuation between the given wave length and 10 cm, for various values of raindrop diameter. The dashed curves indicate values of the liquid water content.

each precipitation rate. Absorption due to oxygen and water vapor in a saturated atmosphere at 18C as calculated by Van Vleck (1947a; b) is included in the table.

TABLE 1. Approximate distance in kilometers required to produce 5 decibels attenuation in a radar beam traveling through various intensities of precipitation (two-way transmission).

Precipitation rate mm/hr	λ = 10 cm	λ = 3 cm	λ = 1 cm
0	420	230	31
2.5	370	50	4.4
7.5	310	16	1.5
12.5	260	8.4	0.9
25	190	3.7	0.5
100	70	0.8	0.1

It can be seen from the table that measurable difference in attenuation between radars using 10-cm and 3-cm waves can be expected under two conditions: (a) very heavy rain, and (b) moderate to heavy rain with reasonably uniform characteristics over a large area. With a wave length of 1 cm the attenuation should be measurable even with light rain.

Calculation of drop sizes from measurements of relative attenuation in combination with average signal intensity is subject to the following restrictions:

1. Because of the assumption that all the drops are of the same size, the result obtained may be considered as an approximation to the size of the largest drops present in the precipitation region. The value of drop concentration obtained has little significance, because a large number of small drops may be present in the precipitation and have little effect on the radar echo.

2. The result must also be considered as an average for the meteorological conditions existing over the range *r*, the distance required to obtain measurable attenuation.

Recent measurements of attenuation of 1.25-cm waves by Anderson *et al.* (1947) show experimental values which definitely exceed the theoretical values calculated by Ryde. Also the correlation between drop size and attenuation was poor. Earlier measurements by Robertson and King (1946) agreed reasonably well with theoretical predictions. It is desirable that such discrepancies between theoretical and experimental results be understood before measurements of attenuation are used in calculating drop sizes.

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