The Evolution of a Rossby-Wave Packet in a Three-Dimensional Baroclinic Atmosphere

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ABSTRACT

The development of an individual quasi-geostrophic disturbance in a three-dimensional baroclinic atmosphere is investigated by using a wave-packet representation and the WKB method. The results obtained indicate that the development of a Rossby-wave packet in the upper level of the atmosphere depends on the packet's structure and location with respect to the zonal flow, whether the zonal flow is stable or not. The wave packet develops (decays) if the three-dimensional rays are directed up-gradient (down-gradient) in the zonal flow. All characteristics of the wave packet are changing with time. The spatial scale or the three-dimensional wavelength of the developing (decaying) wave packet increases (decreases). The tilt of barotropic decaying (developing) trough line away from the meridian increases (decreases), while the vertical tilt of the baroclinic decaying (developing) trough line increases (decreases). The maximum amplitude of the developing (decaying) Rossby-wave packet moves toward (out from) the jet region, if the zonal flow is stable. Unlike a single normal mode, most wave packets cause considerable divergence of momentum and heat flux; hence there exists strong interaction between a Rossby-wave packet and the zonal flow.

1. Introduction

There is a great deal of literature devoted to the instability problems. The two-dimensional cases such as pure barotropic instability or pure baroclinic instability have been thoughtfully investigated. However, due to the complexity of the general three-dimensional problem with horizontal and vertical shear of the basic flow, until recently, we have only certain criteria of instability (Blumen, 1968, 1978; Charney and Stern, 1962; Pedlosky, 1964, 1979) and a few numerical results about the three-dimensional structure of normal modes (Brown, 1969; Simmons and Hoskins, 1976, 1980). Undoubtedly, it needs more investigation.

Although the investigation of instability problems as well as the normal mode method are a good approach to the understanding of the evolution of weather systems, they do not include all kinds of evolutional processes. For example, many rapidly growing or decaying disturbances can exist under both stable and unstable conditions of the basic current, as can be seen from daily weather maps. Therefore, it is desirable to study the more general problem, namely, the problem of evolution or development, and to solve the general initial-value problem. One way is to draw qualitative conclusions from the integral properties of the system. As has been shown by Blumen (1968), Pedlosky (1979) and Zeng (1982a,b), the results obtained from such properties and those obtained by using the normal mode method are in agreement and supplement each other. It is worth extending these results.

Moreover, an individual weather system such as a Rossby wave, trough and ridge can be more or less considered as a wave packet (see Appendix). Hence the problem can be simplified, and more detailed conclusions can also be obtained. Dickinson (1968), Grose and Hoskins (1979), Hoskins and Karoly (1981) and Held (1981) have investigated the propagation of forced stationary wave-trains. The results obtained give an excellent illustration of the nature of the steady response of atmospheric motions to orographic and thermal forcing.[2] On the other hand, the wave-packet representation of disturbances which emphasizes the energetic aspect, such as the development of individual disturbances in a two-dimensional pure barotropic or pure baroclinic quasi-geostrophic model, has been investigated by Lu and Zeng (1981) and Zeng (1982a,b); and some simple but very clear relationships between the development of disturbances and their structures as well as their spatial scale changes have been obtained. We now consider the more general problem of development, applying the WKB method to three-dimensional disturbance superimposed on an arbitrary zonal flow with both

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[2] After the manuscript was submitted to the JOURNAL, there appeared a very interesting paper (Karoly and Hoskins, 1982), in which the propagation of Rossby-wave packets in a three-dimensional shear zonal flow is also investigated.

horizontal and vertical shears. A systematic qualitative analysis of all characteristics of development such as the changes in total energy of the disturbance, the slopes of trough or ridge line, the horizontal and vertical scale of the disturbance, and so on, will be made. In addition, the transport properties as well as the interaction between zonal flow and disturbances will also be investigated.

2. The evolution of a Rossby-wave packet

The nondimensional inviscid and adiabatic quasi-geostrophic model is written as

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)q=0, \quad (2.1)$$

$$q=\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial}{\partial \zeta}\left(\frac{\zeta^2}{C^2}\frac{\partial}{\partial \zeta}\right)\right]\psi+\beta y, \quad (2.2)$$

where q is the nondimensional departure of quasi-geostrophic potential vorticity from some standard value of planetary vorticity, $\zeta = p/p_s$, p_s is the pressure on the ground (usually ζ is written as σ), $\beta = (df/dy)_0 L^{*2}/u^*$, $(df/dy)_0$ is the dimensional mean meridional gradiant of the Coriolis parameter, L^* and u^* are the length and velocity scale, respectively; and C is the non-dimensional characteristic velocity of propagation of gravity waves in the baroclinic atmosphere (see Zeng, 1982b).

Denoting the zonal mean of a function $F(x, y, \zeta, t)$ by $\bar{F}(y, \zeta, t)$ and its departure by $F'(x, y, \zeta, t)$ and linearizing (2.1) for ψ' yields

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)q'=-Bv', \quad (2.3)$$

$$q'\equiv\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial}{\partial \zeta}\left(\frac{\zeta^2}{C^2}\frac{\partial}{\partial \zeta}\right)\right]\psi', \quad v'=\frac{\partial\psi'}{\partial x}, \quad (2.4)$$

where $B \equiv \partial\bar{q}/\partial y$ is the meridional gradient of potential vorticity of the zonal current.

Note that during the linearization we have neglected only the terms $-J(\psi', q') + \overline{J(\psi', q')}$ in (2.3), so that it is approximately valid not only for small perturbations, but also for finite-amplitude disturbances, provided the contours of ψ' and q' are essentially coincident, i.e., that $J(\psi', q')$ is one order of magnitude smaller than ψ' and q'. This is the case, for example, when ψ' is represented by a wave packet.

We restrict ourselves to the investigation of disturbances located at upper levels far away from the bottom boundary. It is convenient to express an individual disturbance as a wave packet and to use the WKB method to investigate its evolution.

As in our previous paper (Zeng, 1982), by introducing a new vertical coordinate ξ and a new variable $\hat{\psi}$, i.e.,

$$\xi=\int_{\zeta}^{1}\sqrt{\gamma(\zeta')}d\zeta', \quad (2.4)$$

$$\psi'=W(\xi)\hat{\psi}(x, y, \xi, t), \quad (2.5)$$

Eq. (2.3) is transformed into

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\hat{q}+B\frac{\partial\hat{\psi}}{\partial x}=0, \quad (2.6)$$

where $\gamma \equiv C^2/\zeta^2$, $W \equiv (\gamma/\gamma_s)^{1/4}$, $H(\xi) \equiv Wd^2W^{-1}/d\xi^2$, and

$$\hat{q}\equiv\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial \xi^2}-H(\xi)\right]\hat{\psi}. \quad (2.7)$$

We assume $H > 0$, and that $\bar{u}$, B and H are all slowly varying functions of y, ξ and t. We now let

$$X=\epsilon x, \quad Y=\epsilon y, \quad Z=\epsilon\xi, \quad T=\epsilon t, \quad (2.8)$$

and let the disturbance $\hat{\psi}$ be a wave packet, i.e.,

$$\hat{\psi}=\Psi(X, Y, Z, T)e^{i\theta(X,Y,Z,T)/\epsilon}, \quad (2.9)$$

$$\Psi(X, Y, Z, T)=\Psi_0(X, Y, Z, T)+\epsilon\Psi_1(X, Y, Z, T)+\cdots, \quad (2.10)$$

where $\epsilon > 0$ is a small parameter, and θ, Ψ_j and their derivatives appearing in the formulas of expansion are of O(1).

The local and instantaneous frequency σ and wavenumbers (m, n, k) are determined as follows:

$$\frac{\partial\theta}{\partial T}=-\sigma, \quad \frac{\partial\theta}{\partial X}=m, \quad \frac{\partial\theta}{\partial Y}=n, \quad \frac{\partial\theta}{\partial Z}=k. \quad (2.11)$$

Substituting (2.8)–(2.11) into (2.6) and (2.7), performing almost the same calculations as in the two-dimensional pure baroclinic case, we obtain essentially the same dispersive relation and the same equation for the evolution of the amplitude Ψ_0, i.e.,

$$(\sigma-m\bar{u})\nu^2+Bm=0, \quad (2.12)$$

$$\left(\frac{\partial}{\partial T}+\bar{u}\frac{\partial}{\partial X}\right)\nu^2\Psi_0-(\sigma-m\bar{u})\left[2\left(m\frac{\partial}{\partial X}+n\frac{\partial}{\partial Y}+k\frac{\partial}{\partial Z}\right)\Psi_0+\Psi_0\left(\frac{\partial m}{\partial X}+\frac{\partial n}{\partial Y}+\frac{\partial k}{\partial Z}\right)\right]-B\frac{\partial\Psi_0}{\partial X}=0, \quad (2.13)$$

where

$$\nu^2\equiv m^2+n^2+k^2+H. \quad (2.14)$$

Now, the equations governing the change of frequency and wavenumbers are

$$D_g\sigma/DT=-\left(m\frac{\partial\bar{u}}{\partial T}-\frac{m}{\nu^2}\frac{\partial B}{\partial T}\right), \quad (2.15)$$

$$D_gm/DT=0, \quad (2.16)$$

$$D_gn/DT=-\left(m\frac{\partial\bar{u}}{\partial Y}-\frac{m}{\nu^2}\frac{\partial B}{\partial Y}\right), \quad (2.17)$$

$$D_gk/DT=-\left(m\frac{\partial\bar{u}}{\partial Z}-\frac{m}{\nu^2}\frac{\partial B}{\partial Z}+\frac{mB}{\nu^4}\frac{\partial H}{\partial Z}\right), \quad (2.18)$$

where

$$D_g/DT \equiv \frac{\partial}{\partial T} + C_{gX}\frac{\partial}{\partial X} + C_{gY}\frac{\partial}{\partial Y} + C_{gZ}\frac{\partial}{\partial Z}, \quad (2.19)$$

and the group velocity $\mathbf{C}_g = \mathbf{i}C_{gX} + \mathbf{j}C_{gY} + \mathbf{k}C_{gZ}$ is given by

$$\left.\begin{aligned} C_{gX} &= \frac{\partial\sigma}{\partial m} = \frac{\sigma}{m} + \frac{2m^2B}{\nu^4} \\ C_{gY} &= \frac{\partial\sigma}{\partial n} = \frac{2mnB}{\nu^4} \\ C_{gZ} &= \frac{\partial\sigma}{\partial k} = \frac{2mkB}{\nu^4} \end{aligned}\right\}. \quad (2.20)$$

We now let

$$\Psi_0 = |\Psi_0|e^{i\alpha(X,Y,Z,T)},$$

and from (2.13) we obtain the equations governing the change of α and $|\Psi_0|$, i.e.,

$$D_g\alpha/DT = 0, \quad (2.21)$$

$$D_g\nu^2|\Psi_0|/DT + \tfrac{1}{2}\nu^2|\Psi_0|\left(\frac{\partial C_{gX}}{\partial X} + \frac{\partial C_{gY}}{\partial Y} + \frac{\partial C_{gZ}}{\partial Z}\right) - \frac{\nu^2|\Psi_0|}{2B}\left(\frac{D_gB}{DT} - \frac{\partial B}{\partial T}\right) = 0. \quad (2.22)$$

Eqs. (2.16)–(2.18) and (2.21)–(2.22) together with (2.12) [or (2.15)] form a closed set to describe the evolution of all the properties of the disturbance. However, we need further transformations to get simpler and more convenient formulas for making qualitative analyses.

The equations governing the changes of the three-dimensional wavenumber along the ray $m^2 + n^2 + k^2$, and of the meridional and vertical slopes of the trough or ridge line $(-n/m, -k/m)$ can easily be obtained from (2.16)–(2.18). We have

$$D_g(m^2 + n^2 + k^2)^{1/2}/DT = -m\left(\frac{\partial\bar{u}}{\partial l} - \nu^{-2}\frac{\partial B}{\partial l} + B\nu^{-4}\frac{\partial H}{\partial l}\right), \quad (2.23)$$

$$D_g\left(-\frac{n}{m}\right)\Big/DT = \left(\frac{\partial\bar{u}}{\partial Y} - \nu^{-2}\frac{\partial B}{\partial Y}\right), \quad (2.24)$$

$$D_g\left(-\frac{k}{m}\right)\Big/DT = \left(\frac{\partial\bar{u}}{\partial Z} - \nu^{-2}\frac{\partial B}{\partial Z} + B\nu^{-4}\frac{\partial H}{\partial Z}\right), \quad (2.25)$$

where $\partial\bar{u}/\partial l = \mathbf{l}^0\cdot\text{grad}\bar{u}$ is the derivative of $\bar{u}$ along the ray, and so on, and $\mathbf{l}^0$ is the unit vector along the ray, i.e.,

$$\mathbf{l}^0 = \frac{1}{(m^2 + n^2 + k^2)^{1/2}}(\mathbf{i}m + \mathbf{j}n + \mathbf{k}k). \quad (2.26)$$

From (2.22) and (2.23) we obtain the equation governing the change of energy density and some integrals:

$$D_g\left(\frac{\nu^2}{2}|\Psi_0|^2\right)\Big/DT + \frac{\nu^2}{2}|\Psi_0|^2\nabla\cdot\mathbf{C}_g = m(m^2 + n^2 + k^2)^{1/2}|\Psi_0|^2\frac{\partial\bar{u}}{\partial l}, \quad (2.27)$$

$$\frac{\partial}{\partial T}\iiint_{(W)}\frac{\nu^2}{2}|\Psi_0|^2dXdYdZ = \iiint_{(W)}\left[mn\frac{\partial\bar{u}}{\partial Y} + mk\frac{\partial\bar{u}}{\partial Z}\right]|\Psi_0|^2dXdYdZ = \iiint_{(W)}m(m^2 + n^2 + k^2)^{1/2}|\Psi_0|^2\frac{\partial\bar{u}}{\partial l}dXdYdZ, \quad (2.28)$$

$$\iiint_{(W)}B^{-1}\frac{\partial\nu^4|\Psi_0|^2}{\partial T}dXdYdZ = 0, \quad (2.29)$$

$$\iiint_{(W)}\left[\frac{\partial}{\partial T}\left(\frac{1}{2}\nu^2|\Psi_0|^2\right) + \frac{\bar{u}_r - \bar{u}}{B}\frac{\partial}{\partial T}(\nu^4|\Psi_0|^2)\right] \times dXdYdZ = 0, \quad (2.30)$$

where W denotes the whole region, occupied by the wave packet; $\bar{u}_r$ is an arbitrary constant; and $\nu^2|\Psi_0|^2$ and $\nu^4|\Psi_0|^2$ are the analogues of the density of eddy energy and enstrophy, respectively. The total energy of the wave packet is given as

$$E' = \iiint_{(W)}\tfrac{1}{2}\nu^2|\Psi_0|^2dXdYdZ, \quad (2.31)$$

so that (2.28) describes the change of total energy with time. While (2.29) is the conservation of total wave action (see Zeng, 1982b) and (2.30) is the conservation of energy-modified enstrophy, provided $\bar{u}$ and $\partial\bar{q}/\partial y$ are independent with time and $B \neq 0$. In the case of time-dependent zonal flow with $B \neq 0$ we usually have $O(\partial\bar{u}/\partial T) = O(1)$ and $O(\partial B/\partial T) \leqslant O(\epsilon)$ [see (4.11) and (4.12)]; hence, the total wave action still is conserved approximately, but the energy-modified enstrophy is not. Moreover, these two quantities might even not exist if $B = 0$ at some lines, but (2.29) and (2.30) are still valid.

Eqs. (2.23)–(2.30) are the direct extension of our previous results (Zeng, 1982a,b), and (2.29)–(2.30) are the extension of the above-mentioned conservation laws.

It must be pointed out that by using the normal mode method one can obtain either neutral mode with invariant amplitude or modes exponentially growing or decaying. Hence the total energy and enstrophy of a single mode grow or decay simultaneously for the non-neutral modes, and they both are invariant for the neutral modes. However, this relationship between the changes in total energy and enstrophy is not a commonly valid one, because the

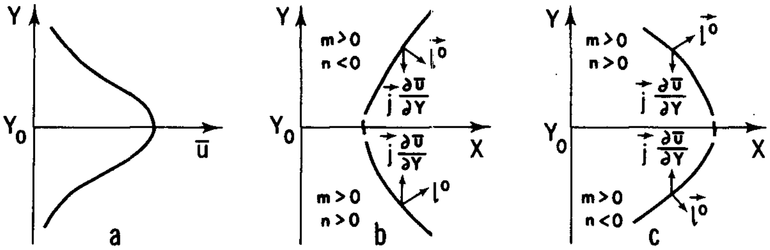

FIG. 1. (a) An ideal barotropic jet profile $\bar{u}(Y)$. (b) A typical trough or ridge line (solid line) of barotropic developing disturbance in the (X, Y) plane; the direction $\mathbf{l}^0$ of the ray and the gradient of $\bar{u}$ $(\mathbf{j}\partial\bar{u}/\partial Y)$ are shown by small arrows; the local wavenumbers (m, n) are also indicated. (c) As in (b), but for decaying disturbance.

normal modes are not orthogonal and not complete unless the continuous spectrum is added. It is clear from Eqs. (2.28) and (2.29) that in general the total energy of a disturbance changes with time, although its weighted enstrophy, i.e., the total wave action, is conserved approximately.

3. The structures of developing and decaying disturbances

We call a disturbance as developing if its total energy E' increases with time and as decaying if E' decreases with time. Since m can always be taken positive, the only factor which determines the direction of development is $\partial\bar{u}/\partial l$ in accordance with (2.28). The disturbance develops as $\partial\bar{u}/\partial l > 0$, i.e., the ray is directed up-gradient with respect to the zonal flow, and decays as $\partial\bar{u}/\partial l < 0$, i.e., the ray is directed down-gradient. In the following section we will show that under some conditions generally satisfied in practice the energy density $\nu^2|\Psi_0|^2/2$ as well as the amplitude $|\Psi_0|$ increase simultaneously, or, exactly speaking, $D_g \frac{1}{2}\nu^2|\Psi_0|^2/DT > 0$ and $D_g|\Psi_0|^2/DT > 0$, as $\partial\bar{u}/\partial l > 0$; and the opposite law is true as $\partial\bar{u}/\partial l < 0$. Therefore, the total and local energy considerations as well as the concept commonly used are consistent with each other.

In the pure barotropic case, $\partial\bar{u}/\partial Z = 0$ and $\partial\bar{u}/\partial l = n(m^2 + n^2 + k^2)^{-1/2}\partial\bar{u}/\partial Y$. Suppose there is a pure barotropic jet-like westerly flow with $\partial\bar{u}/\partial Y = 0$ at $Y = Y_0$ (Fig. 1a). A trough or ridge line develops as it is tilted westward with an increase of meridional distance $|Y - Y_0|$; thus the ray is directed up-gradient with respect to $\bar{u}$ (Fig. 1b); and it decays as it is tilted eastward with increase of $|Y - Y_0|$, and the ray is directed down-gradient (Fig. 1c). Of course, a trough line, located on one side of the jet, for example, $Y - Y_0 > 0$, and having $\partial\bar{u}/\partial l > 0$, is a developing one. Similar conclusions are true for the other cases and for the baroclinic and general three-dimensional cases.

In the pure baroclinic case, the results are similar. Suppose there is a pure baroclinic jet with $\partial\bar{u}/\partial Z = 0$ at $Z = Z_0$ (Fig. 2a). The typical developing and decaying troughs are given in Figs. 2b and 2c, respectively.

In the three-dimensional case we have trough or ridge surfaces in the (X, Y, Z) space. Suppose there is a barotropic-baroclinic westerly jet with $\partial\bar{u}/\partial Y = 0$ at $Y = Y_0(Z)$ and $\partial\bar{u}/\partial Z = 0$ at $Z = Z_0(Y)$ (Fig. 3). For simplicity, $Y = Y_0(Z)$ and $Z = Z_0(Y)$ are taken as two straight lines. In the general case, these two curves, as well as the structure of developing and decaying disturbances, must be somehow modified, but the main characteristics are essentially the same. Fig. 4a shows a typical three-dimensional trough or ridge surface for a barotropic and baroclinic developing disturbance. A typical trough or ridge surface for a decaying disturbance is given in Fig. 4b, and the barotropic decaying but baroclinic developing one is given

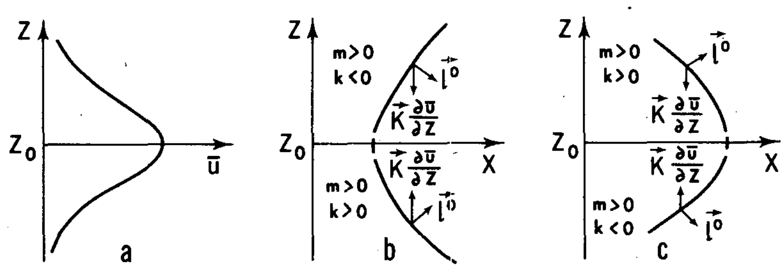

FIG. 2. (a) An ideal baroclinic jet profile $\bar{u}(Z)$. (b) A typical trough or ridge line (solid line) of a baroclinic developing disturbance in the (X, Z) plane; the direction $\mathbf{l}^0$ of the ray and the gradient of $\bar{u}$ $(\mathbf{k}\partial\bar{u}/\partial Z)$ are shown by small arrows; the local wavenumbers (m, k) are also indicated. (c) As in (b), but for decaying disturbance.

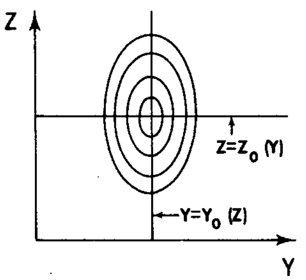

FIG. 3. An ideal westerly jet. Its center is at the point (Y_0, Z_0) and isotaches are given by solid lines.

in Fig. 4c. Since the barotropic developing but baroclinic decaying disturbance seldom appears in the atmosphere, it will not be given here.

It is difficult to determine whether the disturbance given in Fig. 4c is developing or decaying, unless numerical integration or further analysis is made. For this purpose it is convenient to use the following energetic diagram.

Fixing X, we have a trough or ridge line, then project it onto the (Y, Z) plane. Hence, we have a family of such trough or ridge lines depending on X parametrically in (Y, Z) plane. The projection of $\mathbf{l}^0$ on the

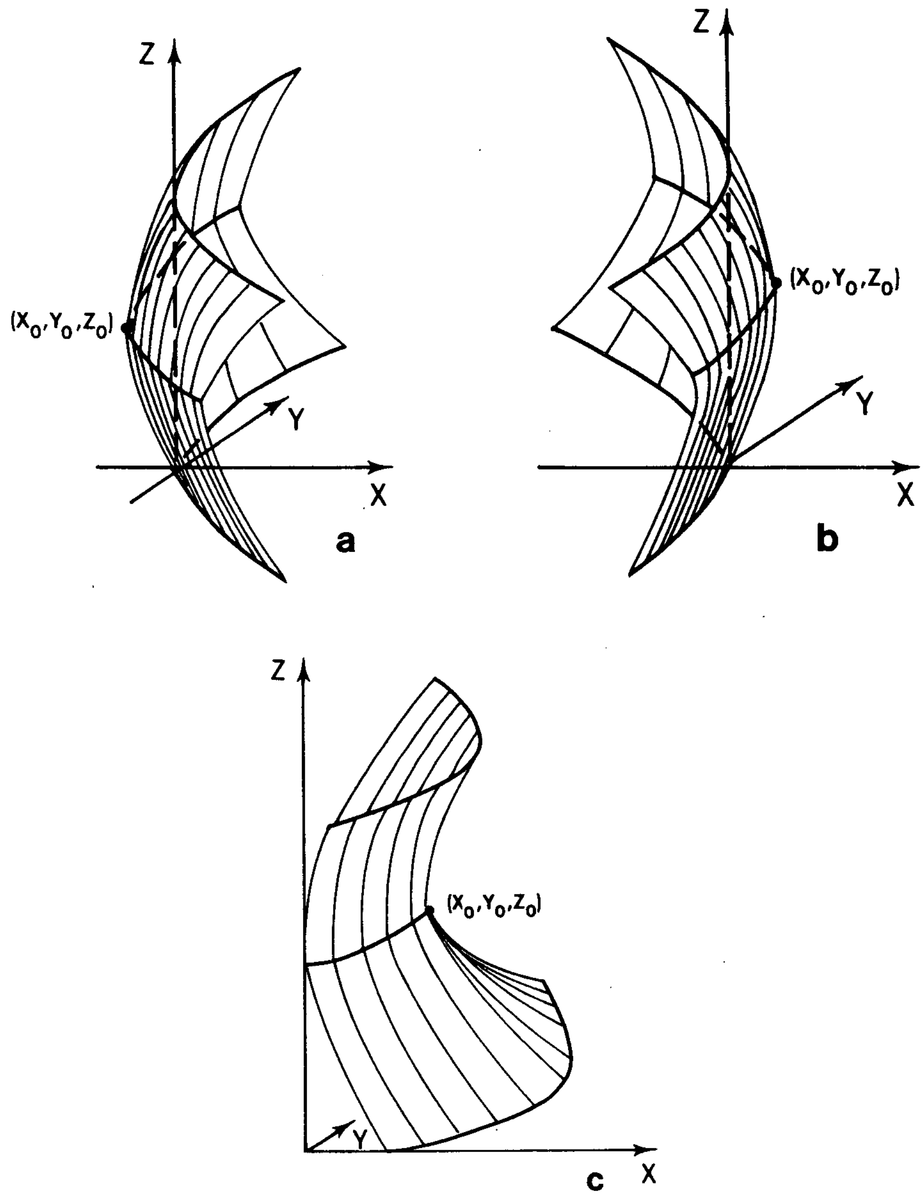

FIG. 4. Some typical three-dimensional trough or ridge surfaces. (a) For a barotropic and baroclinic developing disturbance, (b) for barotropic and baroclinic decaying disturbance, (c) for a barotropic decaying but baroclinic developing disturbance.

(Y, Z) plane, $\mathbf{l}^{0\prime}$, is perpendicular to the projection of the trough or ridge line. Overlapping these lines and vectors with isotachs of $\bar{u}(Y, Z)$, we get an energetic diagram. We implicitly take symmetric jet and symmetric trough surfaces. In such a case, the isotachs form a family of ellipses with center at (Y_0, Z_0), and the trough lines are also ellipses with center at (Y_0, Z_0), corresponding to Figs. 4a and 4b.

In the barotropic and baroclinic developing case (Fig. 4a), all $\mathbf{l}^{0\prime}$ are directed toward the center (Y_0, Z_0), i.e., $\partial\bar{u}/\partial l > 0$ (Fig. 5a). In the barotropic and baroclinic decaying case (Fig. 4b), the $\mathbf{l}^{0\prime}$ are directed from (Y_0, Z_0), i.e., $\partial\bar{u}/\partial l < 0$ (Fig. 5b), while the trough lines corresponding to Fig. 4c contain straight line segments (Figs. 5c and 5d). It is not difficult to imagine that the baroclinic effect exceeds the barotropic effect near $Y = Y_0$, since there $|\partial\bar{u}/\partial Y|$ is small; and the reverse is true near $Z = Z_0$, since there $|\partial\bar{u}/\partial Z|$ is small. Thus, in the baroclinic developing but barotropic decaying case, $\partial\bar{u}/\partial l$ cannot keep the same sign in the whole plane. The eddy energy is produced near $Y = Y_0$, but it is absorbed by the zonal flow near $Z = Z_0$. Fig. 5c shows the case with relative large vertical tilt, i.e., $|-k/m|$ is larger than $|-n/m|$; here the area with $\partial\bar{u}/\partial l > 0$ is much larger than that with $\partial\bar{u}/\partial l < 0$ (shaded), hence the production of eddy energy is larger than the absorption, and the disturbance develops. Fig. 5d shows a decaying case, where the area with $\partial\bar{u}/\partial l > 0$ is shaded. Note that the net effect is determined by the difference of these two areas alone if the jet and trough surface are symmetric. In the general case, more detailed calculation is needed.

4. The change of the structure

According to (2.30), the necessary condition for local instability is that there is no constant $\bar{u}_r$ such

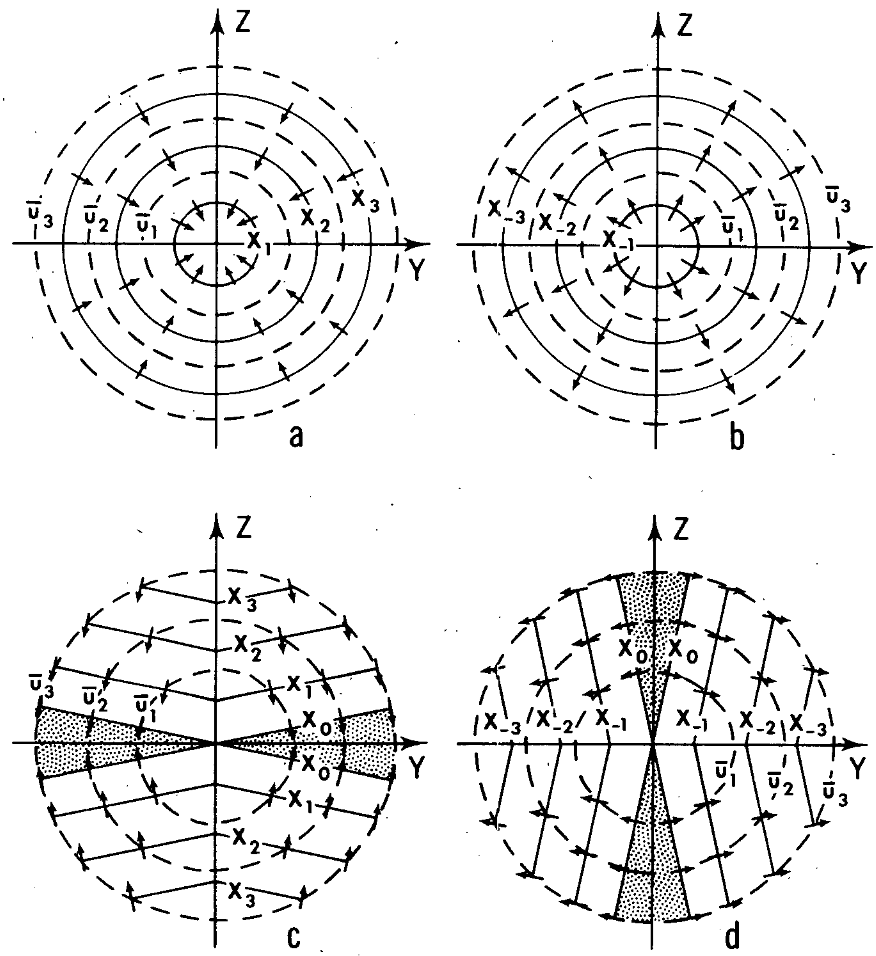

FIG. 5. Energetic diagram. Solid lines show the trough or ridge lines in (Y, Z) plane for various X, $\cdots < X_{-2} < X_{-1} < X_0 < X_1 < X_2 < \cdots$; and dashed lines show isotaches of $\bar{u}$, $\bar{u}_1 > \bar{u}_2 > \cdots$. Direction of the rays are given by small arrows. (a) For a barotropic and baroclinic developing disturbance, (b) for a barotropic and baroclinic decaying disturbance, (c) for a barotropic decaying but baroclinic developing disturbance, and the baroclinic effect is dominant, and (d), as in (c), but the barotropic effect is dominant. See text for detail.

that $(\bar{u}_r - \bar{u})/B > 0$ everywhere in the area occupied by the disturbance. In the actual atmosphere, the region with $B < 0$ is limited to a narrow meridional belt and vertical layer, which is too narrow to involve a disturbance completely, so that we will simply identify the local stability with $B > 0$ in the further discussion.

It can be seen from (2.28) that $\partial E'/\partial T$ depends only on the structure of the disturbance and its location related to the basic current. Hence the conclusions mentioned in the above paragraph are valid, no matter whether the basic current satisfies the instability condition. The differences between a disturbance superimposed on a stable basic current and one on an unstable basic current are included in the propagation and deformation of the shape as can be seen below.

Taking $\Psi \approx \Psi_0$, the real part of (2.9) represents a wave packet with modulation of amplitude and phase angle, i.e.,

$$\hat{\psi} = |\Psi_0| \cos\left(\frac{\theta}{\epsilon} + \alpha\right). \tag{4.1}$$

According to (2.21) and (2.22), both $|\Psi_0|$ and α propagate at a velocity $\mathbf{C}_g$, given by (2.20). In addition, $|\Psi_0|$ undergoes some change (development), while θ propagates along the ray on a phase velocity $\mathbf{C}$ given by

$$\mathbf{C} = \frac{\sigma}{(m^2 + n^2 + k^2)^{1/2}} \mathbf{l}^0. \tag{4.2}$$

It is also convenient to observe the movement of trough or ridge lines along the X axis under fixed Y and Z. We call $\mathbf{C}'_X = \mathbf{i}\sigma/m$ a partial X-phase velocity of propagation of θ. Note that $C_{gX} > C'_X > C_X$ if $B > 0$. The projections of $\mathbf{C}_g$, $\mathbf{C}$ and $\mathbf{C}'_X$ on the plane (X, Y) are schematically shown in Fig. 6. Generally, the trough or ridge line propagates on a velocity between $\mathbf{C}$ and $\mathbf{C}_g$, and its partial X-phase velocity is between C_{gX} and σ/m due to the spatial variation of α and $|\Psi_0|$.

We first investigate the propagation of the maximum of $|\Psi_0|$. As mentioned above, $|\Psi_0|$ propagates nearly along C_g. Denoting the projection of $\mathbf{C}_g$ on the (Y, Z) plane as $\mathbf{C}'_g$, we obtain from (2.20)

$$\mathbf{C}'_g = 2Bm\nu^{-4}(m^2 + n^2 + k^2)^{1/2}\mathbf{l}^{0\prime}. \tag{4.3}$$

In the stable zonal flow, i.e., $B > 0$ everywhere, $\mathbf{C}'_g$ and $\mathbf{l}^{0\prime}$ are pointed in the same direction, hence the maximum $|\Psi_0|$ or the main trough or ridge line of barotropic and baroclinic developing (decaying) disturbance moves toward (out from) the jet. For the barotropic decaying but baroclinic developing disturbance, $\mathbf{C}'_g$ is directed to the jet near $Y = Y_0$ from below and above, but goes out from the jet near $Z = Z_0$.

In the region with $B < 0$, the direction of $\mathbf{C}'_g$ is opposite to that of $\mathbf{l}^{0\prime}$, and the change of $|\Psi_0|$ is complicated. Again we assume that the jet and disturbance are symmetric, and that $B < 0$ within a ring, $r_1 < r < r_2$, where $r = [(Y - Y_0)^2 + (Z - Z_0)^2]^{1/2}$. Suppose that the disturbance is barotropic and baroclinic developing, and that $\mathbf{C}'_g$ is directed from the center (Y_0, Z_0) within $r_1 < r < r_2$, but toward the center outside this ring (Fig. 7). Thus, the slope of

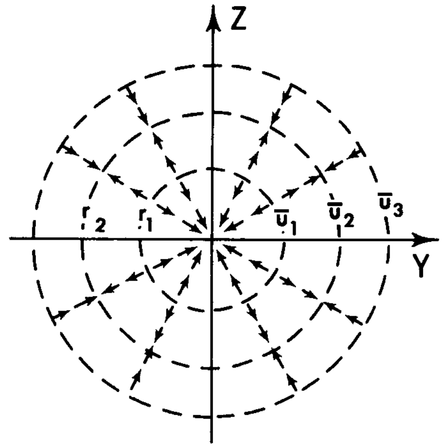

FIG. 7. A sketch showing the unstable jet with $B < 0$ within a ring $r_1 < r < r_2$. The projection of the group velocity on the (Y, Z) plane, $\mathbf{C}'_g$, is indicated by small arrows.

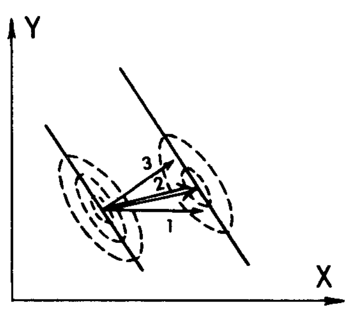

FIG. 6. A sketch showing the trough or ridge line (solid line), isopleths of $\hat{\psi}$ (dashed), and the vectors $\mathbf{C}'_X$ (1), $\mathbf{C}_g$ (2) and $\mathbf{C}$ (3). See text for detail.

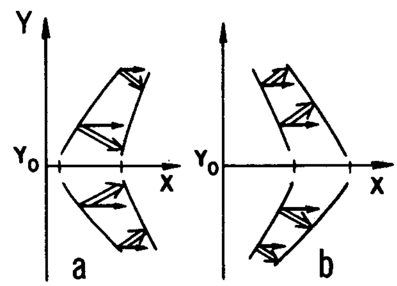

FIG. 8. The propagation and evolution of the tilt of a trough or ridge line in the (X, Y) plane. The partial X-phase velocity, $\mathbf{C}'_X$, and the group velocity, $\mathbf{i}C_{gX} + \mathbf{j}C_{gY}$, are indicated by single and double arrows, respectively. (a) For a barotropic developing disturbance, (b) for a decaying one.

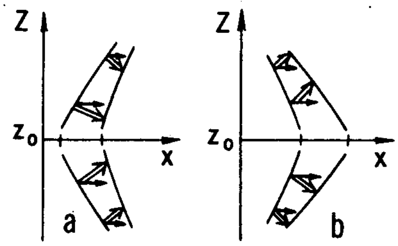

FIG. 9. As in Fig. 8, but in the (X, Z) plane and for group velocity $\mathbf{i}C_{gX} + \mathbf{k}C_{gZ}$. (a) For a baroclinic developing disturbance, (b) For a decaying one.

$|\Psi_0|$ becomes larger at r_2 and smoother at r_1. This means that the shape of $|\Psi_0|$ gradually becomes more complicated, and the assumption about the slowly varying nature of $|\Psi_0|$ might even break down after a sufficient long time.

Next, we discuss a change of the tilt of the trough or ridge line. We restrict ourselves to two cases, that either

$$O\left(\frac{\partial B}{\partial Y}, \frac{\partial B}{\partial Z}\right) \ll O(\epsilon) \qquad (4.4)$$

or $\nu^{-2} \ll 1$ is satisfied. In both cases, we have the partial X-phase velocity $\sigma/m \approx \bar{u}$, and also $C_{gX} \approx \bar{u}$. Hence, the trough or ridge line will be more and more (less and less) tilted in the (X, Y) plane for the barotropic decaying (developing) disturbance (Fig. 8). The change of tilt in the (X, Z) plane is similar (Fig. 9). Combining these results, one can easily reach the conclusions about the change of tilt in the three-dimensional case. It is also convenient and, probably more reasonable, to seek the change of slopes of the main trough line based on Eqs. (2.24) and (2.25); in either event we come to the same conclusions.

Now we discuss the change of the spatial scale of the disturbance. When either (4.4) or $\nu^{-2} \ll 1$ is satisfied, from (2.23) we have

$$D_g(m^2 + n^2 + k^2)^{1/2}/DT \approx -m\frac{\partial \bar{u}}{\partial l}. \qquad (4.5)$$

This indicates that the mean three-dimensional wavenumber decreases (increases) as the disturbance develops (decays). Note that the scale of a disturbance can be identified with the zero contour of $\hat{\psi}$. More detailed conclusions can be obtained by analyzing (2.16)–(2.18). We find that 1) both meridional and vertical scales of the disturbance increase (decrease) as it is barotropic and baroclinic developing (decaying) (Figs. 10a and 10b), and 2) that the barotropic decaying but baroclinic developing disturbance enlarges its vertical scale but shortens its meridional scale (Figs. 10c and 10d).

Finally, we turn to the discussion of changes in the energy density and amplitude. In practice, one might approximately represent the synoptic disturbance by a wave packet with constant m, n and k, independent of spatial coordinates; in this case $\nabla\cdot\mathbf{C}_g$ is directly proportional to $n\partial B/\partial Y + k\partial B/\partial Z$, and its magnitude is of $O(\epsilon)$, provided (4.4) is satisfied. Therefore, from (2.27) we obtain

$$D_g{}^1\!/\!_2\nu^2|\Psi_0|^2/DT = (m^2 + n^2 + k^2)^{1/2}|\Psi_0|^2\frac{\partial \bar{u}}{\partial l} + O(\epsilon), \qquad (4.6)$$

$$\nu^2 D_g{}^1\!/\!_2|\Psi_0|^2/DT = 2m(m^2 + n^2 + k^2)^{1/2}|\Psi_0|^2\frac{\partial \bar{u}}{\partial l} + O(\epsilon), \qquad (4.7)$$

where in (4.7) we have used (4.5) and have taken into account the fact that $\partial H/\partial l$ is small. Eqs. (4.6) and (4.7) indicate that the energy density as well as the amplitude propagate along the rays and undergo some definitive evolution, namely, they both are intensified as the rays are directed up-gradient and damped as they are down-gradient.

When neither (4.4) nor $\nu^{-2} \ll 1$ is satisfied, all these conclusions might no longer be valid. Instead, we can use (2.30) to get a general conclusion about the change of weighted scale, l_w, if $\bar{u}$ and B do not depend on t, and $B > 0$, where

$$l_w \equiv l_\beta\left(\frac{E'}{Q'_w}\right)^{1/2}, \qquad (4.8)$$

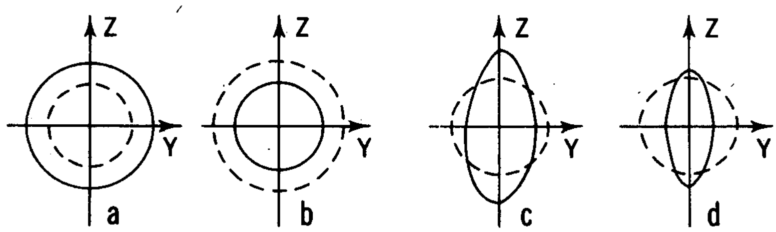

FIG. 10. The change of sizes of contour $\hat{\psi} = 0$. Dashed line, initial; solid line, for $t > 0$. (a) For a barotropic and baroclinic developing disturbance, (b) for a barotropic and baroclinic decaying one, (c) and (d) for a barotropic decaying but baroclinic developing one, where the disturbance in (c) is developing, and in (d) is decaying.

$$Q'_w \equiv \iiint_{(W)} \frac{\bar{u}_r - \bar{u}}{2B} \nu^4 |\Psi_0|^2 dXdYdZ. \quad (4.9)$$

Here the arbitrary constant $\bar{u}_r$ can be taken as the maximum of $\bar{u}$, hence $Q'_w > 0$, and

$$l_\beta \equiv \left[\iiint_{(W)} \frac{\bar{u}_r - \bar{u}}{B} dXdYdZ \Big/ \iiint_{(W)} dXdYdZ\right]^{1/2} \quad (4.10)$$

is a modified nondimensional wavelength of a stationary Rossby wave in the general three-dimensional case. Thus l_w increases (decreases) as the disturbance develops (decays). No general conclusion can be made if B changes its sign.

It must be pointed out that (4.4) usually takes place, if we assume that the meridional scale of the disturbance is small compared with that of the zonal flow. In fact, we have

$$B = \beta - \epsilon^2\left(\frac{\partial^2 \bar{u}}{\partial Y^2} + \frac{\partial^2 \bar{u}}{\partial Z^2} - \frac{\partial \ln\sqrt{\gamma}}{\partial Z}\frac{\partial \bar{u}}{\partial Z}\right). \quad (4.11)$$

Since β is taken to be a constant, we have, for example,

$$\frac{\partial B}{\partial Y} = -\epsilon^2\left(\frac{\partial^3 \bar{u}}{\partial Y^3} + \frac{\partial^3 \bar{u}}{\partial Y \partial Z^2} - \frac{\partial \ln\sqrt{\gamma}}{\partial Z}\frac{\partial^2 \bar{u}}{\partial Y \partial Z}\right). \quad (4.12)$$

Hence we have $O(B) = O(1)$ and (4.4) is satisfied if all third-order derivatives of $\bar{u}$ with respect to (Y, Z, T) are of $O(1)$. Only for an extremely narrow jet can its third-order derivatives be of $O(\epsilon^{-2})$, and (4.4) is not satisfied.

In the vicinity of the jet axis and the tropopause the slowly varying assumption for $\bar{u}$, B and H might break down. However, even in these cases, our results all are still in good qualitative agreement with the observational study. This is probably due to the fact that the trough lines as well as trough surfaces usually have turning points there; hence there the local n and k of the disturbance are very small, and it propagates approximately zonally.

Note that in the very narrow jet case we must take a rather small characteristic scale L^* for the disturbance in order to satisfy the slowly varying assumption for zonal flow; in other words, the meridional scale of the disturbance is rather small but the zonal scale is not necessarily small. Thus, β is small, and B can change its sign. In addition, in the rotating annulus experiments and in the nonrotating two-dimensional flow we have $\beta = 0$, but can also keep the term with B in our equations. Thus B can also change its sign. These results indicate that a gross qualitative analysis can still be made by application of the WKB method even to the unstable case. Of course, a more detailed study of these problems is desirable and will be a subject of further investigations.

5. The transport properties

Meridional fluxes of momentum and sensible heat can be directly calculated by using (4.6), (4.7), (6.1) and $u' = -\partial\psi'/\partial y$, $v' = \partial\psi'/\partial x$ and $T' = -\zeta\partial\psi'/\partial\zeta$. We have

$$\overline{u'v'} = W^2 M + O(\epsilon), \quad M = -\tfrac{1}{2}\overline{mn|\Psi_0|^2}, \quad (5.1)$$

$$\overline{v'T'/C} = W^2 S + O(\epsilon), \quad S = \tfrac{1}{2}\overline{mk|\Psi_0|^2}. \quad (5.2)$$

These formulas indicate that the eddy meridional transports depend only on the structure of the disturbance, no matter what the structure of zonal flow is; and that the divergence of these fluxes is determined by the spatial change of tilt of the trough line (n and k), as well as by the spatial change of amplitude $|\Psi_0|$, provided m is constant.

If we presume that the disturbance is symmetric, and that $|mn|\Psi_0|^2|$ has a maximum at $Y = Y_s$ and Y_n, $Y_s < Y_n$, the barotropic decaying (developing) disturbance causes up-gradient (down-gradient) momentum flux, which is convergent (divergent) within (Y_s, Y_n), and divergent (convergent) or negligibly small outside (Y_s, Y_n) (Fig. 11).

The baroclinic decaying disturbance causes up-gradient meridional heat flux, i.e., it is northward above the jet and southward below the jet. The reverse law is true as the disturbance is baroclinic developing. Fig. 12 shows the vertical profile of $\overline{v'T'/C}$ for the symmetric disturbances.

It is interesting to compare our results with those obtained by again using the normal mode method. The transport properties of normal modes have been thoroughly investigated by Kuo (1951), Green (1970), Stone (1972, 1974), Held (1975) and many others. The results indicate that any normal mode is neutral and does not transport any momentum and heat if the basic zonal flow is stable; while, in the case of unstable zonal flow, the growing disturbance always causes momentum flux from the region with $\partial\bar{q}/\partial y > 0$ to that with $\partial\bar{q}/\partial y < 0$. The reverse is true for

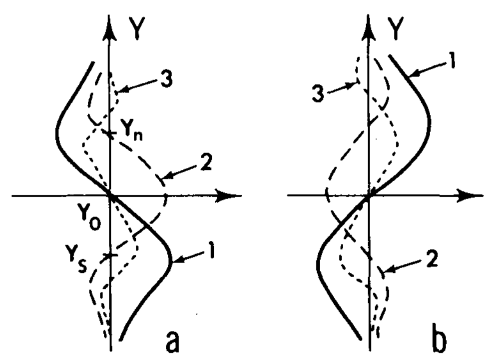

FIG. 11. The meridional distribution of $\overline{u'v'}$ (1), $-\partial\overline{u'v'}/\partial Y$ (2), and $-\partial^2\overline{u'v'}/\partial Y^2$ (3). (a) For a barotropic decaying disturbance, (b) for a barotropic developing one.

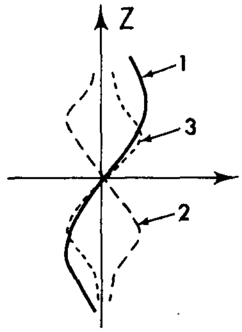

FIG. 12. The vertical distribution of $\partial \bar{T}/\partial Y$ (1), and $\overline{v'T'}/C$ (2) and (3). Curve (2) is for a baroclinic developing disturbance, (3) for a baroclinic decaying one.

decaying disturbance. However, according to our results, an individual disturbance can cause momentum flux even if the zonal flow is stable; and the direction and divergence of momentum depend only on the structure of the disturbances but not the sign of $\partial\bar{q}/\partial y$. These differences are also due to the non-orthogonality and the non-completeness of the normal modes. From the non-orthogonality a linear combination of modes can result in non-zero transports in the case of neutral modes and in a different distribution of these transports in the case of non-neutral modes. On the other hand, due to the non-completeness an arbitrary disturbance generally cannot be represented by the linear combination of normal modes with sufficient accuracy unless the continuous spectrum is added. Usually the project of a local disturbance on the continuous spectrum is large, so that its behavior and transport properties are quite different from those of a single normal mode.

6. The evolution of zonal flow

The reaction of disturbances and the evolution of zonal flow in the quasi-geostrophic model is simply described by

$$\frac{\partial \bar{q}}{\partial t} = -\frac{\partial \overline{v'q'}}{\partial y}. \tag{6.1}$$

Note that

$$\overline{v'q'} = W^2\left(-\frac{\partial M}{\partial y} + \frac{\partial S}{\partial \xi}\right). \tag{6.2}$$

Now, introducing

$$\bar{\psi} = W\hat{\bar{\psi}}, \tag{6.3}$$

and substituting (6.2) and (6.3) into (6.1), we have

$$\left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial \xi^2} - H(\xi)\right]\frac{\partial \hat{\bar{\psi}}}{\partial t} = W\left(\frac{\partial^2 M}{\partial y^2} - \frac{\partial^2 S}{\partial y \partial \xi}\right). \tag{6.4}$$

As the disturbance is limited in a local region, it is more convenient to form a localized solution to (6.4). Hence we have

$$\frac{\partial \bar{u}(y, \xi, t)}{\partial t} = -\iint_{-\infty}^{+\infty}\left[W(\xi)W(\xi')\frac{\partial G(y, \xi; y', \xi')}{\partial y'}\right] \times \frac{\partial^2 M(y', \xi', t)}{\partial y'^2}dy'd\xi' + \iint_{-\infty}^{+\infty}\frac{\partial}{\partial \xi'}[W(\xi)W(\xi') \times G(y, \xi; y', \xi')]\cdot\frac{\partial^2 S(y', \xi', t)}{\partial y'^2}dy'd\xi', \tag{6.5}$$

where $\bar{u} = -\partial\bar{\psi}/\partial y$, and G is the Green's function given by

$$\left.\begin{aligned} G(y, \xi; y', \xi') &= \frac{1}{2\pi}J_0(ir\sqrt{H(\xi)}) \\ &\quad + \text{regular terms} \\ r &\equiv [(y - y')^2 + (\xi - \xi')^2]^{1/2} \end{aligned}\right\}, \tag{6.6}$$

and $J_0(ix)$ is the zero-order cylindrical function of pure imaginary argument. Note that the residual regular terms in (6.6) are zero if H is a constant.

The order of magnitude in the right-hand side of (6.5) is the same as that of $\partial^2 M/\partial y^2$ and $\partial^2 S/\partial y^2$. Hence, $\partial\bar{u}/\partial T$, $\partial^2\bar{u}/\partial Y\partial T$ and $\partial^2\bar{u}/\partial Z\partial T$ are all of $O(\epsilon)$, providing M and S as well as their first and second order derivatives are slowly varying functions of (y, ξ), i.e., $O(\partial^2 M/\partial y^2) = O(\epsilon^2)$, $O(\partial^2 M/\partial Y^2) = \epsilon^2 O(\partial^2 M/\partial y^2) = O(1)$, and so on. These mean that the assumption of a slowly varying $\bar{u}$ and conclusions are consistent.

Supposing that the disturbance is centered at (Y_0, Z_0), the corresponding $\partial^2 M/\partial Y^2$ is given in Fig. 11 schematically, and $\partial^2 S/\partial Y^2$ is given schematically in Fig. 13. Denoting $W(\xi)W(\xi')G$ as G_r, we have

$\partial G_r/\partial y' > 0$ as $y' < y$, and $\partial G_r/\partial y' \to \infty$ as $\xi' = \xi$ and $y' \to y - 0$,

$\partial G_r/\partial y' < 0$ as $y' > y$, and $\partial G_r/\partial y' \to -\infty$ as $\xi' = \xi$ and $y' \to y + 0$,

$\partial G_r/\partial \xi' > 0$ as $\xi' < \xi$, and $\partial G_r/\partial \xi' \to \infty$ as $y' = y$ and $\xi' \to \xi - 0$,

$\partial G_r/\partial \xi' < 0$ as $\xi' > \xi$, and $\partial G_r/\partial \xi' \to -\infty$ as $y' = y$ and $\xi' \to \xi + 0$.

Hence, we have $\partial\bar{u}/\partial t > 0$ (<0) around the jet, and $|\partial\bar{u}/\partial t|$ has a maximum at (Y_0, Z_0) but is small far away from the jet, if the disturbance is barotropic and baroclinic decaying (developing). This means that the barotropic and baroclinic decaying (developing) disturbance strengthens (weakens) the westerly jet as well as its meridional and vertical shear. As to the baro-

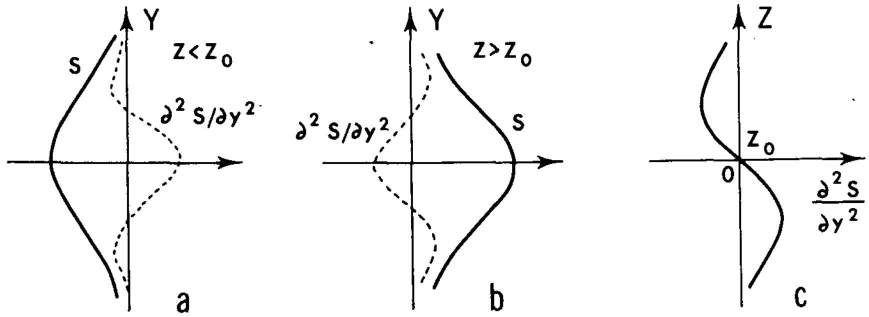

FIG. 13. The meridional and vertical distribution of $S = \overline{v'T'}/C\sqrt{\gamma}$ and $\partial^2 S/\partial y^2$ for a baroclinic developing disturbance. (a) Below the jet, $Z < Z_0$; (b) above the jet, $Z > Z_0$; (c) the vertical profile of $\partial^2 S/\partial y^2$ for fixed y. The reverse is true for a baroclinic decaying disturbance.

tropic decaying but baroclinic developing disturbance, the effect of M and S cancel each other, the meridional shear is strengthened, but the jet becomes broad vertically.

7. Conclusive remarks

Summarizing the results, we have:

1) The development of individual disturbances depends on their structure and location related to the zonal flow, no matter whether the zonal flow is stable or not. The disturbance develops (decays) as the rays are directed up-gradient (down-gradient) with respect to the zonal flow. The development characteristics are more clearly shown in an energetic diagram.

2) The spatial scale or the three-dimensional wavelength of developing (decaying) disturbances increases (decreases).

3) Meridional tilt of barotropic decaying (developing) trough lines increases (decreases), while the vertical tilt of baroclinic decaying (developing) trough lines increases (decreases).

4) The center of developing (decaying) disturbances moves toward (out from) the jet if the zonal flow is stable, but the situation is complicated if the zonal flow is unstable.

5) Meridional fluxes of momentum and sensible heat are determined by the spatial distribution of the amplitude of the disturbance and the slope of the trough line. Barotropic decaying (developing) disturbances cause up-gradient (down-gradient) meridional momentum flux, while baroclinic decaying (developing) disturbances cause up-gradient (down-gradient) heat flux.

6) The westerly jet as well as its meridional and vertical shear are strengthened (weakened) as the disturbance superimposed on it is barotropic and baroclinic decaying (developing).

We now presume that the initial disturbance is barotropic and baroclinic developing. According to the conclusions mentioned above, its amplitude as well as its spatial scale will gradually become larger, but the meridional and vertical tilt of trough or ridge line will gradually become smaller. At the same time, the westerly jet as well as its meridional and vertical shear are weakened. Then, the growth ratio of the disturbance becomes smaller, and its eastward propagation slows down. Then, most probably, one or both of the meridional and vertical tilts become inversed, the disturbance decays, and all aspects of evolutional process become reversed. The disturbance will decay continuously if the basic flow remains stable. In such a case, the disturbance is absorbed by the zonal flow, which we call as rotational adaptation (Zeng, 1979). However, the further evolution of disturbance might be complicated if the basic flow becomes unstable.

It must be pointed out that the influence of bottom boundary has not been taken into account in our investigation. However, the bottom boundary definitely affects the evolution of low-level disturbances, which is very important for the heat transport (Held, 1978). In order to investigate general behaviors of low-level disturbances, the WKB method must be more or less modified. The success of Lindzen and Rosenthal's (1981) work indicates that an appropriate extension of the WKB method is possible.

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APPENDIX

Representation of an Arbitrary Perturbation in the Form of a Wave Packet

For simplicity of mathematic interpretation we take a one-dimensional problem as an example. We represent a perturbation $\psi'(x)$ in the Fourier integral

$$\psi'(x) = \int_0^\infty \Psi_m e^{imx} dm. \quad \text{(A1)}$$

Supposing that $|\Psi_m|$ as a function of m has a pronounced maximum at $m = m_0$, we can easily rewrite ψ' in the form of a wave packet

$$\psi'(x) = \Psi(x) e^{im_0 x}, \quad \text{(A2)}$$

where the amplitude

$$\Psi(x) \equiv \int_0^\infty \Psi_m e^{i(m-m_0)x} dm \quad \text{(A3)}$$

is a slowly varying function of x. If $|\Psi_m|$ does not have a pronounced maximum, we approximate the upper bound of integration by a sufficiently large number, dividing the whole spectrum into several narrow zones, we can represent ψ' by a superposition of several wave packets

$$\psi'(x) \approx \sum_j \Psi_j(x) e^{im_j x}, \quad \text{(A4)}$$

where every Ψ_j, defined as

$$\Psi_j(x) = \int_{m_j-\Delta m/2}^{m_j+\Delta m/2} \Psi_m e^{i(m-m_j)x} dm, \quad \text{(A5)}$$

is a slowly varying function of x, provided $\Delta m/m_j \ll 1$ or $\Delta m \ll 1$. Of course, in the general case, we must also consider the interference of these wave packets along with the behavior of every individual one.

REFERENCES

Blumen, W., 1968: On the stability of quasi-geostrophic flow. *J. Atmos. Sci.*, **25,** 929–931.

——, 1978: A note on horizontal boundary conditions and stability of quasi-geostrophic flow. *J. Atmos. Sci.*, **35,** 1314–1318.

Brown, J. A., 1969: A numerical investigation of hydrodynamic instability and energy conversions in the quasigeostrophic atmosphere: Parts I and II. *J. Atmos. Sci.*, **26,** 352–365.

Charney, J. G., 1947: The dynamics of long waves in a baroclinic westerly current. *J. Meteor.*, **4,** 135–162.

——, and M. E. Stern, 1962: On the stability of internal baroclinic jets in a rotating atmosphere. *J. Atmos. Sci.*, **19,** 159–173.

Dickinson, R. E., 1968: Planetary Rossby waves propagating vertically through weak westerly wind waveguides. *J. Atmos. Sci.*, **25,** 984–1002.

Green, J. S. A., 1970: Transfer properties of large-scale eddies and the general circulation of the atmosphere. *Quart. J. Roy. Meteor. Soc.*, **96,** 157–185.

Grose, W. L., and B. J. Hoskins, 1979: On the influence of orography on large-scale atmospheric flow. *J. Atmos. Sci.*, **36,** 223–234.

Hayashi, Y., 1981: Vertical-zonal propagation of a stationary planetary wave packet. *J. Atmos. Sci.*, **38,** 1197–1205.

Held, I. M., 1975: Momentum transport by quasi-geostrophic eddies. *J. Atmos. Sci.*, **32,** 1494–1497.

——, 1978: The vertical scale of an unstable baroclinic wave and its importance for eddy heat flux parameterizations. *J. Atmos. Sci.*, **35,** 572–576.

——, 1981: Stationary and quasi-stationary eddies in the extratropical troposphere: Theory. (To be published).

Hoskins, B. J., and D. J. Karoly, 1981: The steady linear response of a spherical atmosphere to thermal and orographic forcing. *J. Atmos. Sci.*, **38,** 1179–1196.

Karoly, D. J., and B. J. Hoskins, 1982: Three-dimensional propagation of planetary waves. *J. Meteor. Soc. Japan*, Ser. II, **60,** 109–123.

Kuo, H. L., 1949: Dynamic instability of two-dimensional flow in a barotropic atmosphere. *J. Meteor.*, **6,** 105–122.

——, 1951: The general circulation and the stability of zonal flow. *Tellus*, **3,** 268–284.

——, 1952: Three-dimensional disturbances in a baroclinic zonal current. *J. Meteor.*, **9,** 260–278.

Lindzen, R. S., and A. J. Rosenthal, 1981: A WKB asymptotic analysis of baroclinic instability. *J. Atmos. Sci.*, **38,** 619–629.

Lu Pei-sheng, and Qing-cun Zeng, 1981: On the evolution process of disturbances in the barotropic atmosphere. *Sci. Atmos. Sinica*, **5,** 1–8.

Pedlosky, J., 1964: The stability of currents in the atmosphere and the ocean: Part I. *J. Atmos. Sci.*, **21,** 201–219.

——, 1979: *Geophysical Fluid Dynamics*. Springer-Verlag, 624 pp.

Simmons, A. J., and B. J. Hoskins, 1976: Baroclinic instability on the sphere: Normal modes of the primitive and quasi-geostrophic equations. *J. Atmos. Sci.*, **33,** 1454–1477.

——, and ——, 1980: Barotropic influence on the growth and decay of nonlinear baroclinic waves. *J. Atmos. Sci.*, **37,** 1679–1684.

Stone, P. H., 1972: On non-geostrophic baroclinic instability, Part III: The momentum and heat transports. *J. Atmos. Sci.*, **29,** 419–426.

——, 1974: The meridional variation of the eddy heat fluxes by baroclinic waves and their parametrization. *J. Atmos. Sci.*, **31,** 444–456.

Zeng Qing-cun, 1979: On nonlinear interaction of motion and process of rotational adaptation in rotating atmosphere. *Sci. Sinica*, **22,** 945–957.

——, 1982a: On the evolution and interaction of disturbances and zonal flow in rotating barotropic atmosphere. *J. Meteor. Soc. Japan*, Ser. II, **60,** 24–31.

——, 1982b: The development characteristics of quasi-geostrophic baroclinic disturbances. *Tellus*, **35** (in press).