A Relationship Between Hailstone Concentration and Size

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ABSTRACT

Hailstone size distributions have been determined from 41 time-resolved hailstone samples collected at the ground from seven storms that occurred in Alberta in the summer of 1980. Most size distributions were found to quite closely fit an exponential function of the form $n(D) = n_0e^{-\lambda D}$. In studying variations in n_0 and λ, it was found that a relationship exists between the two. In particular, correlation coefficients of ~ -0.9 were found when least-squares linear regressions were fitted to the values of $\log n_0$ versus $\log\lambda$. For Alberta storms, therefore, n_0 can be expressed in terms of λ as $n_0 = 115\lambda^{3.63}$, and hail size distributions can be expressed in terms of the single parameter λ as $n(D) = 115\lambda^{3.63}e^{-\lambda D}$. From an examination of hail size distributions from one storm that occurred in Switzerland, it appears likely that similar relationships can be determined for hailstorms from other regions.

1. Introduction

The size distributions of precipitation particles are of considerable interest in the study of cloud physical processes, in numerical cloud simulations, and in the remote measurement of precipitation by radar. Measurements of particle size distributions have been reported in the literature since the 1940's. At ground level, Marshall and Palmer (1948) and Mason and Andrews (1960) studied the size spectra of rain; Gunn and Marshall (1958) examined the size distribution of snowflakes; Douglas (1964) and Federer and Waldvogel (1975, 1978) observed the size spectra of hailstones. At higher levels, measurements of particle size distributions of raindrops were made by Merceret (1974), of ice particles by Simpson and Wiggert (1971) and Houze *et al.* (1979), and of hailstones by Spahn and Smith (1976). These measurements show that, on average, the size distributions are generally exponential, i.e., of the form

$$n(D) = n_0 \exp(-\lambda D), \tag{1}$$

where D is the particle diameter and n_0 and λ are the intercept and slope parameters, respectively, which describe the characteristics of the distribution. Thus $n(D)dD$ gives the number of particles in the size interval D to $D + dD$ observed at a particular instance and location.

Although n_0 is commonly assumed to be constant for rain (e.g., Kessler, 1969) and ice (Smith *et al.*, 1975) in microphysical parameterization schemes used in numerical cloud models, and λ has been suggested to be constant for hail (Douglas, 1964), the measurements (e.g., Waldvogel, 1974; Houze *et al.*, 1979) generally show that neither n_0 nor λ can be considered to be constant, especially in convective situations. Moreover, empirical and theoretical studies suggest that, for raindrops and ice crystals, there may be a relationship between n_0 and λ. From data obtained with a vertically pointing Doppler radar, Sekhon and Srivastava (1971) deduced that n_0 and λ are both dependent upon the rainfall intensity. Srivastava (1978) considered the processes of condensation, coalescence and drop breakup, and concluded that n_0 and λ are interrelated for raindrops. Passarelli (1978) constructed an approximate analytical model of deposition and aggregation growth of snow and showed that an equilibrium relation should exist between n_0 and λ of the form $n_0 = C\lambda^3$, where C depends upon storm dynamics, temperature and crystal type. Houze *et al.* (1979, 1980) flew inside frontal clouds and found that, for ice particles, $n_0 \propto \lambda^{1.6}$. It seems reasonable, then, that a relationship between n_0 and λ might also exist for hailstone size spectra.

A time-resolved hailstone sampling system has been in operation as part of the Alberta Hail Project since the summer of 1979. Exponential distributions have been fitted to the size distributions obtained from the data collected in 1980 and variations among the distributions in n_0 and λ have been examined. It appears that there is, indeed, a relation between n_0 and λ. Thus the size distributions are essentially single parameter functions.

The sampling procedures are described in Section 2. Section 3 gives a discussion on fitting exponential distributions to the observed data and the hail size distribution data are presented in Section 4. A summary of the results appears in Section 5.

TABLE 1. Hailfall intensity (R_H), hail water content (W_H), hailstone concentration (N_H), mean volume diameter (D_m) and mean diameter ($\bar{D}$) integrated from the hailstone samples collected in 1980. Mean, maximum and minimum values and standard deviations σ of the integrated parameters are also listed.

Date	Time (MDT)	R_H (mm h^{-1})	W_H (g m^{-3})	N_H (m^{-3})	D_m (mm)	$\bar{D}$ (mm)
16 July	134200–134550	0.12	0.003	0.02	8.0	6.6
23 July	184348–184448	1.38	0.037	0.28	6.8	6.3
26 July	174600–174700	12.47	0.296	1.35	8.5	7.4
	174700–174800	34.94	0.839	4.18	8.3	7.2
	174800–174900	4.63	0.121	0.97	6.9	6.2
27 July	182700–183100	2.03	0.038	0.08	9.9	8.8
	183100–183200	19.01	0.335	0.38	12.3	10.8
	183200–183300	22.27	0.407	0.43	12.7	11.8
	183300–183400	43.88	0.772	1.07	11.5	9.9
	183400–183700	13.65	0.261	0.70	9.3	8.0
	183700–183900	11.38	0.248	0.80	8.7	8.1
	191500–191630	3.61	0.082	0.32	8.9	8.5
	191630–191800	3.62	0.080	0.24	8.9	8.5
	191800–191930	7.90	0.172	0.61	8.4	7.7
	191930–192100	7.86	0.174	0.54	8.8	8.3
	192100–192230	9.46	0.204	0.64	8.8	8.1
	194300–194500	17.51	0.367	1.04	9.1	8.2
	194500–194800	1.44	0.033	0.14	7.9	7.5
	201030–201230	1.00	0.024	0.11	7.8	7.5
2 August	172400–172500	1.84	0.041	0.19	7.8	7.0
	173345–173645	36.88	0.592	0.54	13.2	10.6
	173645–173745	22.61	0.436	0.88	10.2	9.1
	173745–173849	23.25	0.510	1.78	8.5	7.8
	173849–174017	33.59	0.702	1.88	9.3	8.5
	174000–174100	5.89	0.141	0.66	7.7	7.4
	174100–174200	10.48	0.248	1.11	7.8	7.4
	174200–174300	0.73	0.019	0.13	6.8	6.6
	181630–181730	20.13	0.475	2.25	7.6	7.3
	181730–181830	2.73	0.075	0.69	6.1	6.0
	181830–181930	0.83	0.022	0.17	6.5	6.3
	182035–182239	8.09	0.168	0.33	10.3	10.0
	182241–182412	5.38	0.130	0.64	7.6	7.3
14 August	164700–164800	0.49	0.012	0.09	7.8	6.1
15 August	180000–180130	6.10	0.149	0.77	8.0	7.2
	180130–180230	0.37	0.005	0.04	6.4	6.1
	192100–192230	4.65	0.128	1.21	6.3	6.0
	192230–192400	0.50	0.014	0.15	6.0	5.8
	192430–192630	0.76	0.022	0.24	5.9	5.7
	202150–202322	0.23	0.006	0.04	7.1	6.6
	210800–211000	3.51	0.090	0.61	7.2	6.6
	211000–211200	0.42	0.012	0.11	7.4	6.0
Minimum		0.12	0.003	0.02	5.9	5.7
Maximum		43.88	0.839	4.18	13.2	11.8
Mean		9.94	0.21	0.69	8.3	7.6
σ		11.34	0.22	0.76	1.8	1.4

2. Sampling and measurement procedures

In 1980, two instrumented trucks were directed underneath the high-reflectivity zones of hailstorms by the use of an S- or C-band radar and a radio communication system. The instrumentation on the trucks consisted of a hail catcher and eight bottles of heptane cooled in dry ice. The hail catcher consisted of conical mosquito netting inside a fiberglass drum that had an aperture of 0.21 m^2. The netting, which allowed the separation of the hail from the rain, led to a 10 cm opening with a flexible hose which could be used to direct the hail into any of the heptane bottles. This arrangement permitted the rapid changing of sample containers at controlled time intervals (which varied from 1 to 4 min, but were generally 1 to 1.5 min). Each bottle contained a removable nylon net lining to facilitate removal of the stones from the bottles. The collected samples were put into sealed plastic bags and stored in dry ice.

The sequential hail samples were laid out and photographed in a cold room at a later time. The film negatives were projected onto a screen and the maximum and minimum linear dimensions of the projected images were measured. The scale of magnification was adjusted so that the smallest particles of interest (~4 mm) could be measured with accuracy. To calculate the equivalent volume diameter of each stone, the stones were assumed to be oblate spheroids in shape. The hailstones were then classified according to their equivalent volume diameters in size intervals of 1 mm and the median volume diameter of each size category was used for the size distribution calculations. In 1980, 41 samples, containing 9 to 640 hailstones with a mean of 149 stones per sample, were collected from seven storms (see Table 1).

3. Fitting exponential size distributions to the observations

The integrated characteristics of the hailstone samples collected in 1980 are presented in Table 1. The terminal velocity V_T used to calculate the hail concentration was estimated by equating the drag force and the weight of the hailstone, as in English (1973), i.e.,

$$V_T(D) = \left(\frac{4}{3}\frac{\rho_H}{\rho_a}\frac{g}{C_D}D\right)^{1/2}, \qquad (2)$$

where the density ρ_H of the hailstone is assumed to be 0.9 g cm^{-3}, since most stones were observed to be hard ice by those who collected the samples; the acceleration of gravity g is taken to be 980 cm s^{-2}; the drag coefficient C_D of the hailstone is assumed to be 0.55; and the density ρ_a of air is taken to be 1.05×10^{-3} g cm^{-3}. The terminal velocities calculated using Eq. (2) for hailstone sizes observed in this study differ from those obtained from the expression suggested by Waldvogel *et al.* (1978), i.e., $V_T(D) = 4.41D^{1/2}$ m s^{-1} (D in mm), by only ~2%.

The parameters n_0 and λ of the exponential size distribution (1) have usually been calculated in terms of the general properties of exponential distributions as was done by Sekhon and Srivastava (1970, 1971), who related n_0 and λ to the median volume diameter D_0 and the total concentration N, and by Waldvogel (1974) who expressed n_0 and λ in terms of the water content W and the radar reflectivity factor Z. To derive n_0 and λ in terms of integrated hail parameters would be desirable, if the particle size spectrum extended from $D = 0$ to $D = \infty$. However, the observed size distributions are often truncated at a certain maximum size D_{max}. In addition to this upper limit, there is a clearly defined lower limit D_{min} (5 mm by definition) for hail spectra. If integrated parameters without corrections to account for the truncations of the exponential hailstone size distribution (at 5 mm and the observed maximum diameter) are used to calculate n_0 and λ, considerable error can result.

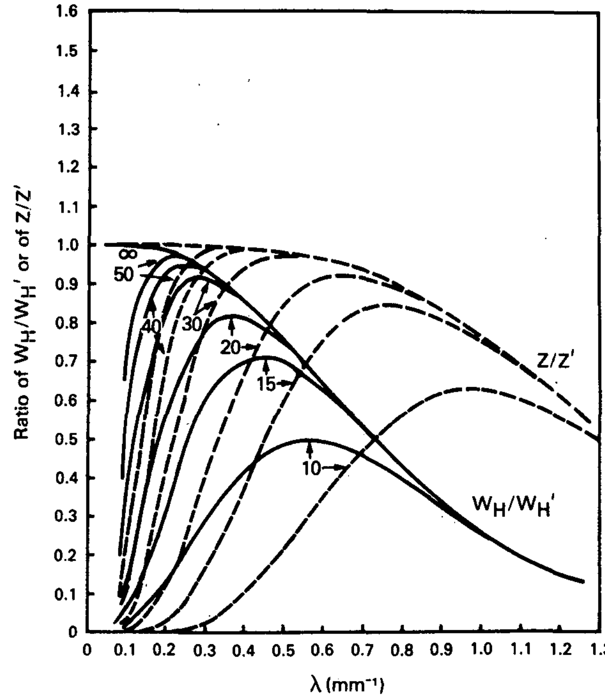

FIG. 1. W_H/W'_H (solid lines) and Z/Z' (dashed lines) as functions of λ for various D_{max} (mm).

If we consider the hail water content, which is the mass of hail per unit volume of air, and the radar reflectivity factor, assuming Rayleigh scattering, of an exponential size distribution of hailstones integrated from 0 to ∞ (W'_H and Z') and from D_{min} (5 mm) to D_{max} (W_H and Z), then

$$W'_H = \int_0^\infty \tfrac{1}{6}\pi\rho_H n_0 D^3 e^{-\lambda D} dD = \tfrac{1}{6}\pi\rho_H n_0 \lambda^{-4}\Gamma(4), \qquad (3)$$

$$W_H = \int_{D_{min}}^{D_{max}} \tfrac{1}{6}\pi\rho_H n_0 D^3 e^{-\lambda D} dD = \tfrac{1}{6}\pi\rho_H n_0 \lambda^{-4}[\gamma(4, \lambda D_{max}) - \gamma(4, \lambda D_{min})], \qquad (4)$$

$$Z' = \int_0^\infty n_0 D^6 e^{-\lambda D} dD = n_0\lambda^{-7}\Gamma(7), \qquad (5)$$

$$Z = \int_{D_{min}}^{D_{max}} n_0 D^6 e^{-\lambda D} dD = n_0\lambda^{-7}\{\gamma(7, \lambda D_{max}) - \gamma(7, \lambda D_{min})\}, \qquad (6)$$

in terms of the gamma function Γ and the incomplete gamma function γ. The ratios of W_H/W'_H and Z/Z' can be readily evaluated for known λ and D_{max} since D_{min} is taken to be 5 mm by definition. Fig. 1 shows the ratios of W_H/W'_H and Z/Z' as functions of λ and D_{max} as solid and dashed lines, respectively. For small

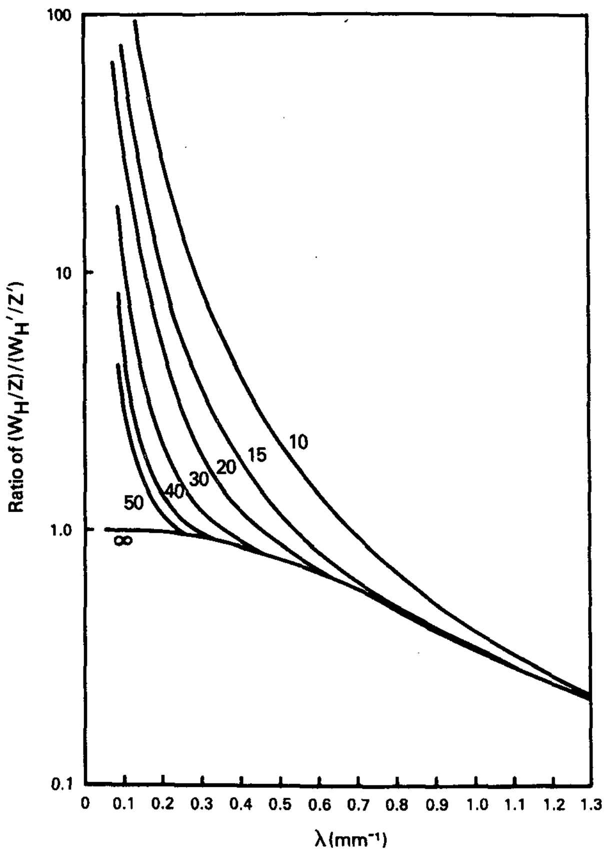

FIG. 2. $(W_H/Z)/(W'_H/Z')$ as a function of λ for various $D_{\max}$ (mm).

$D_{\max}$ the deviations of W_H from W'_H are very large for all λ. For a given $D_{\max}$, the ratio W_H/W'_H varies only with λ and a peak in the value of the ratio is obtained at some point over the range of λ. The peak value of the ratio increases with $D_{\max}$ and approaches unity as $D_{\max}$ approaches ∞. The value of λ corresponding to the maximum W_H/W'_H decreases as $D_{\max}$ increases. Even for $D_{\max} = \infty$, the discrepancies between hail water contents evaluated by Eqs. (3) and (4) are very significant for $\lambda > 0.3$ mm^{-1} because $D_{\min}$ is not zero. The ratio of Z/Z' shows similar variations as W_H/W'_H, but to a lesser extent. However, the differences between Z and Z' are still very large, especially if $D_{\max}$ is small.

Certain adjustments should be applied to methods which calculate n_0 and λ from integrated hail parameters assuming size distributions with limits from 0 to ∞. However, the adjustments depend upon the observed maximum size and the slope of the size distribution. For reference, the ratio W_H/W'_H over Z/Z' is shown in Fig. 2 as a function of $D_{\max}$ and λ. This ratio can be rewritten as W_H/Z over W'_H/Z', which corresponds to the ratio of W_H/Z, used by Federer and Waldvogel (1975) to determine n_0 and λ. It can be seen from Fig. 2 that the value of W_H/Z which results from integration over the range of the observed data can be larger or smaller than the value of W'_H/Z' which assumes the integration limits of 0 and ∞. The magnitude and sign of the difference depend upon the real value of the slope of the distribution.

Two Alberta hailstone size distributions observed in 1980 are plotted in Figs. 3a and 3b. The approximated exponential distributions determined from the expressions suggested by Federer and Waldvogel (1975), i.e.,

$$n_0 = \frac{1}{\pi}\left(\frac{6!}{\pi}\right)^{4/3}\left(\frac{W_H}{\rho_H Z}\right)^{4/3}\frac{W_H}{\rho_H}, \tag{7}$$

$$\lambda = \left(\frac{6!}{\pi}\right)^{1/3}\left(\frac{W_H}{Z\rho_H}\right)^{1/3}, \tag{8}$$

and from a least-squares regression of the logarithm of the concentration and the size are shown in the figures as the dashed and solid lines, respectively. The error bars were calculated assuming a Poisson distribution for concentration (Joss and Waldvogel, 1969). The two approximated exponential distributions evaluated from the regression method described the observations very well, whereas the ones using the integrated parameters as suggested by Federer and Waldvogel did not. In Fig. 3a, the regressed distribution gives an n_0 of 1.05 m^{-3} mm^{-1} and a λ of 0.32 mm^{-1}, while the distribution computed from the integrated W_H and Z gives an n_0 of 2.80 m^{-3} mm^{-1} and a λ of 0.44 mm^{-1}. Assuming a real λ of 0.32 mm^{-1} (the regressed distribution) and using the observed $D_{\max}$ of 17.5 mm and Fig. 2, it is apparent that the observed W_H/Z should be larger than that of the distribution with limits of 0 to ∞ by a factor of 2.2. Thus, the λ calculated from Eq. (8) should be overestimated by a factor of 1.3, a value which is approximately equal to what was obtained. In Fig. 3b, the regressed distribution gives an n_0 of 38.2 m^{-3} mm^{-1} and a λ of 0.79 mm^{-1}, whereas Eq. (8) gives an n_0 of 9.2 m^{-3} mm^{-1} and a λ of 0.68 mm^{-1}. Assuming that 0.79 mm^{-1} represents the more realistic slope for the distribution, the method using the integrated hail quantities to calculate the parameters of the exponential distribution underestimated λ by a factor of 0.86. Fig. 2 suggests that W_H/Z integrated from 5 to 10.5 mm (the observed $D_{\max}$) should be smaller than the one integrated from 0 to ∞ by a factor of ~0.66. Thus, the λ calculated from (8) will be underestimated by a factor of 0.87, which is about equal to what was observed.

It has been demonstrated that correction factors should be applied to the observed integrated quantities when size distributions with limits of 0 and ∞ are assumed to calculate the exponential parameters of the spectrum. Similar discussions have been given by Sekhon and Srivastava (1971), who suggested certain modification factors. These modification factors, however, should be tailored to individual

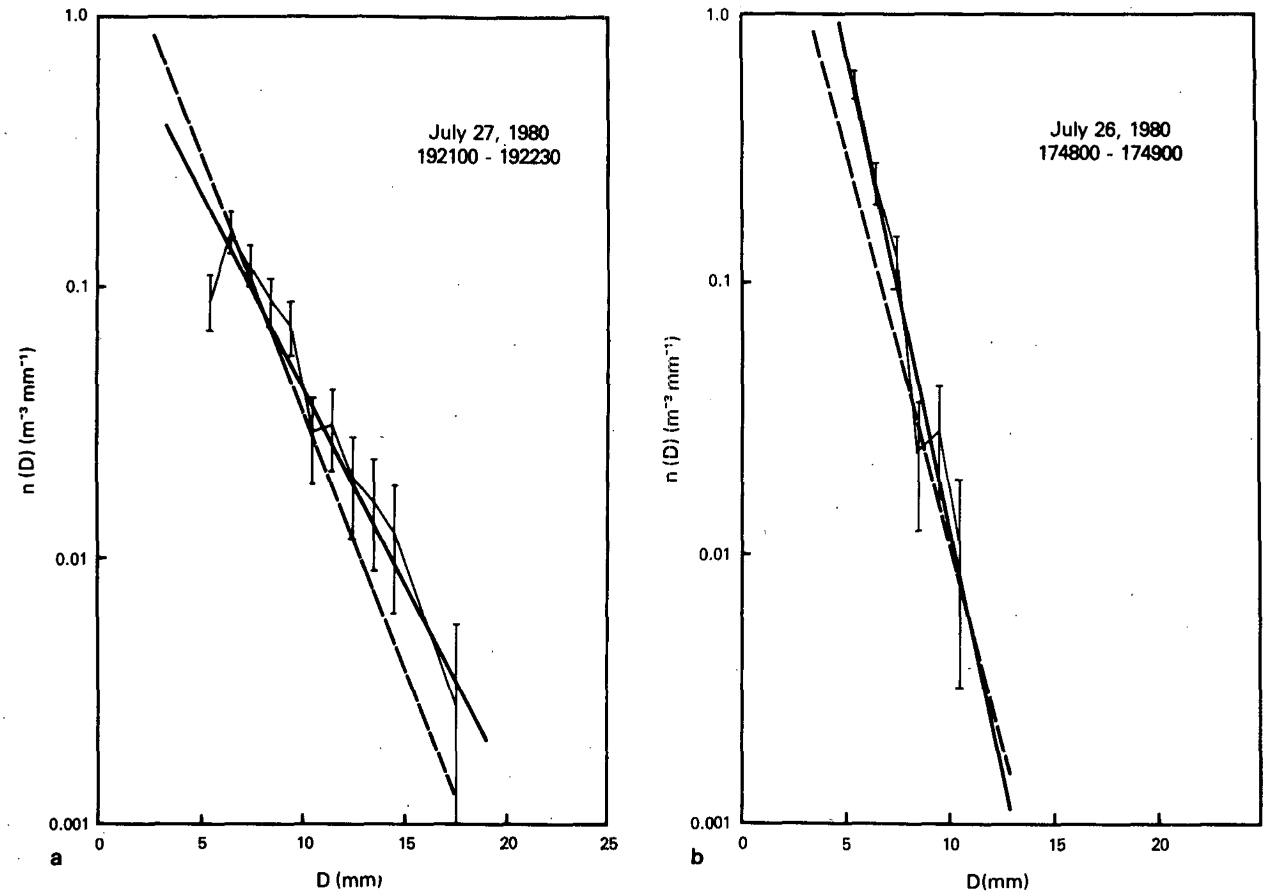

FIG. 3. Examples of measured hailstone size spectra and the approximated exponential distributions obtained by least-squares regression (solid line) and the method suggested by Federer and Waldvogel (dashed line) for (a) 27 July, (b) 26 July.

spectra, unless a common maximum size is imposed. Imposing a common maximum size would sacrifice valuable information on larger sizes which may exist in some spectra. Thus for these reasons, n_0 and λ of each size distribution were obtained in this study by least-squares fitting of the observed $\log n(D)$ and D.

4. Hailstone size distributions

a. Approximated exponential hail distributions

The hail samples collected in 1980 were fitted to exponential functions and a correlation coefficient was calculated for each regression. Fig. 4 shows a histogram of the frequency of occurrence of the correlation coefficients. Note that 39% (16 of 41) of the distributions have correlation coefficients better than −0.95. Approximately 60% (25 of 41) of the distributions have correlation coefficients of −0.90 or better. A correlation coefficient of −0.85 or better was obtained for ~80% of the distributions (33 of 41). These correlation coefficients tend to support the results of earlier studies which have generally shown hail size distributions to be exponential. Federer and Waldvogel (1978) noted exceptions to the exponential character of hail size spectra at the beginning and end of hailfalls. However, in the storms of this study, the non-exponential distributions were found to have occurred at any point in the hailfall and not primarily at the beginning and end of the hailfall. This may be due to differences in the time resolution. As presented in Table 1, the sampling time intervals for this study were longer than the 30 s used in sampling Swiss storms. Generally, the longer sampling duration may tend to smooth out any non-exponential distributions. The hailfall intensities of the Alberta storms in this study were very low. A short sampling period was not possible with the sampling apparatus used; otherwise, the sample size would have been too small for meaningful analysis. It has been recognized that a larger sampling area should be used in future studies.

To test the general reliability of the data and the regression method, all the 1980 samples were combined to form a composite size distribution. The n_0 and λ for this distribution are 0.81 m^{-3} mm^{-1} and 0.33 mm^{-1}, respectively, with a correlation coefficient of 0.99. This mean λ agrees well with the value of 0.31 mm^{-1} given by Douglas (1964) for Alberta storms.

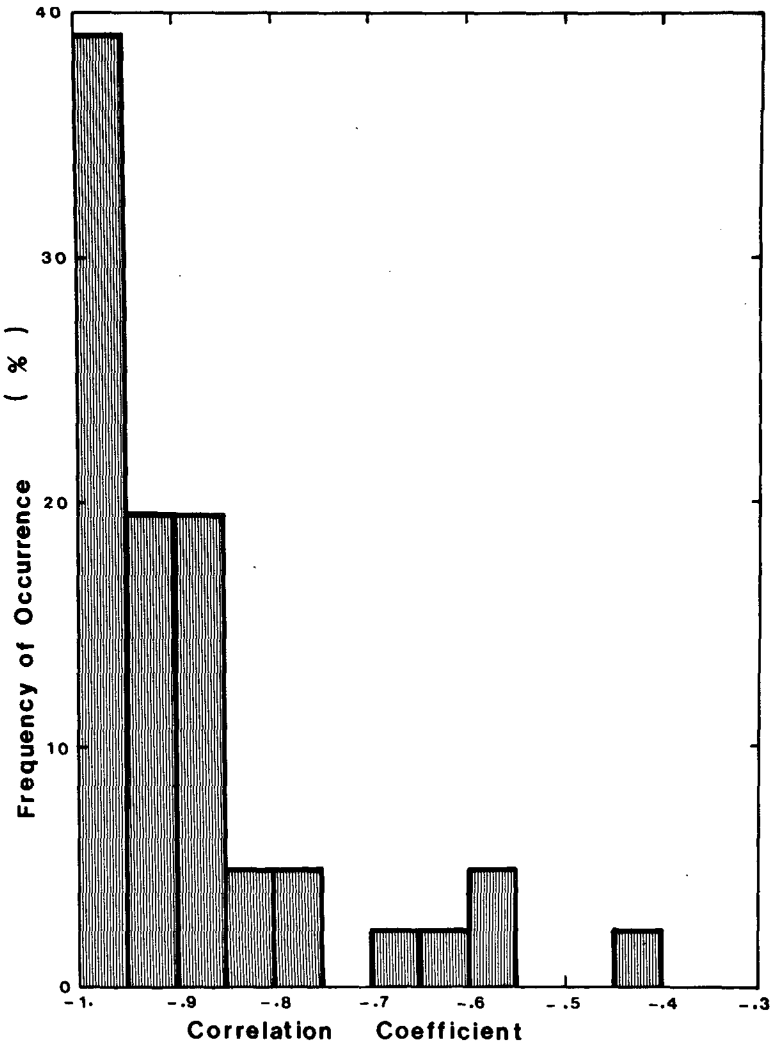

FIG. 4. Frequency distribution of correlation coefficients from least-squares regressions of log$n(D)$ and D of the 1980 time-resolved hailstone samples.

b. Relation between n_0 *and* λ

The n_0 and λ of each of the 41 size distributions are plotted in Fig. 5. A different symbol is used for each of the seven storms. Note that the variations in n_0 and λ do not support the constant slope approximation of Douglas (1964), nor the constant intercept assumption of Smith *et al.* (1975). Moreover, a power-law relationship exists between n_0 and λ for the seven Alberta storms.

Least-squares regression analysis of the data points in Fig. 5 gives the following expression for the line of best fit:

$$\log n_0 = (1.96 \pm 0.16) + (4.18 \pm 0.32) \log\lambda. \quad (9)$$

The standard deviations of the intercept and slope of the regression line are given inside the respective parentheses. The correlation coefficient for the 41 data points to this line is 0.87. This means that the hailstone size distributions can be expressed as

$$n(D) = 91\lambda^{4.18}e^{-\lambda D}. \quad (10)$$

Some of the data points in Fig. 5 represent hailstone samples of limited concentrations and/or with only small sizes. Distribution parameters calculated from such limited observations are probably not representative. If hailstone samples with maximum sizes smaller than 1 cm and total number of hailstones less than 100 are eliminated from the data sample, the number of samples reduces to 21 and Fig. 5 becomes Fig. 6. The minimum, maximum, mean and standard deviation of the integrated hailfall intensity, hail water content, hail concentration, mean volume diameter and mean diameter of the 21 discriminated samples of 1980 are summarized in Table 2. Note that large variations in the hailfalls still exist among the 21 discriminated samples.

The n_0 and λ for the 21 discriminated samples are plotted in Fig. 6. Least-squares regression gives the following expression for the line of best fit to the data points:

$$\log n_0 = (2.06 \pm 0.13) + (3.63 \pm 0.27) \log\lambda. \quad (11)$$

Thus, n_0 can be expressed in terms of λ as $n_0 = 115\lambda^{3.63}$. The correlation coefficient of the data points in Fig. 6 to the line of best fit is 0.95 and the hailstone size distributions can be expressed as

$$n(D) = 115\lambda^{3.63}e^{-\lambda D}. \quad (12)$$

c. Hail water content

One may argue that the regression parameters n_0 and λ fit the shape of the spectrum measured as closely as possible, but may not give good approximations for parameters like W_H, R_H and Z. To examine the representativeness of the regressed n_0 and λ of the 21 discriminated samples, the hail water contents W_H were computed from these samples using

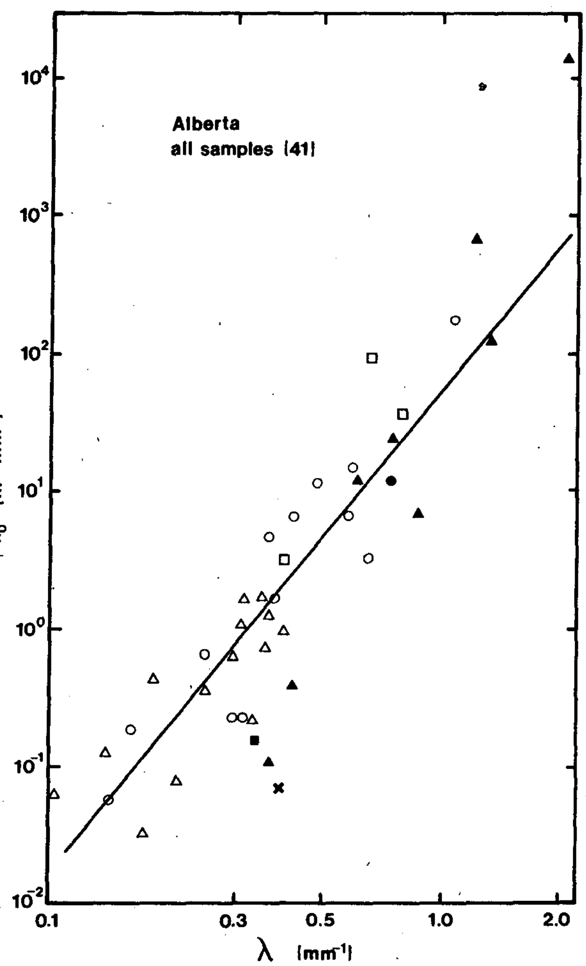

FIG. 5. Variation of the intercept parameter (n_0) as a function of the slope parameter (λ) of exponential size distributions for the 1980 time-resolved hailstone samples. Symbols represent samples collected on various days: crosses, 16 July; solid circles, 23 July; squares, 26 July; triangles, 27 July; circles, 2 August; solid squares, 14 August and solid triangles, 15 August.

TABLE 2. Statistical summary of the integrated characteristics of hailfall for the discriminated samples of 1980. (Symbols as in Table 1.)

	R_H (mm h^{-1})	W_H (g m^{-3})	N_H (m^{-3})	D_m (mm)	$\bar{D}$ (mm)
Minimum	3.51	0.09	0.33	6.9	6.2
Maximum	43.88	0.84	4.18	13.2	10.6
Mean	16.17	0.34	1.11	8.8	8.0
σ	11.75	0.22	0.84	1.4	1.1

$$W_{Hm} = \sum_{i=1}^{k} \tfrac{1}{6}\pi\rho_H D_i^3 n_i \quad (13)$$

and

$$W_{Hp} = \int_{D_{\min}}^{D_{\max}} \tfrac{1}{6}\pi\rho_H D^3 n_0 e^{-\lambda D} dD, \quad (14)$$

where n_i is the number of hailstones in size category D_i, k is the number of categories in a particular sample, W_{Hm} is the hail water content determined from the measurements, and W_{Hp} is the hail water content determined from n_0 and λ. The results are plotted in Fig. 7. Reasonably good agreement is evident. The differences between the measured hail water contents [Eq. (13)] and the predicted hail water contents [Eq. (14)] were within 10%.

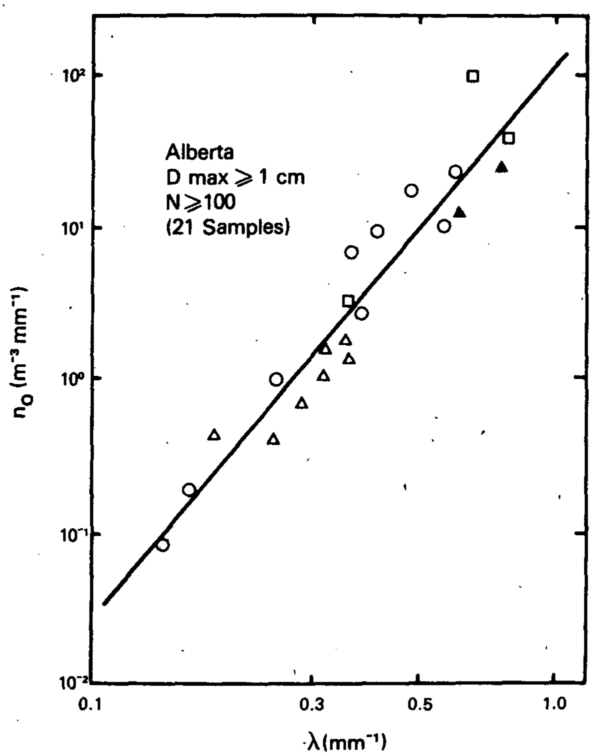

FIG. 6. Variation of the intercept parameter (n_0) as a function of the slope parameter (λ) of exponential size distributions for the discriminated 1980 time-resolved hailstone samples. (Symbols as in Fig. 5.)

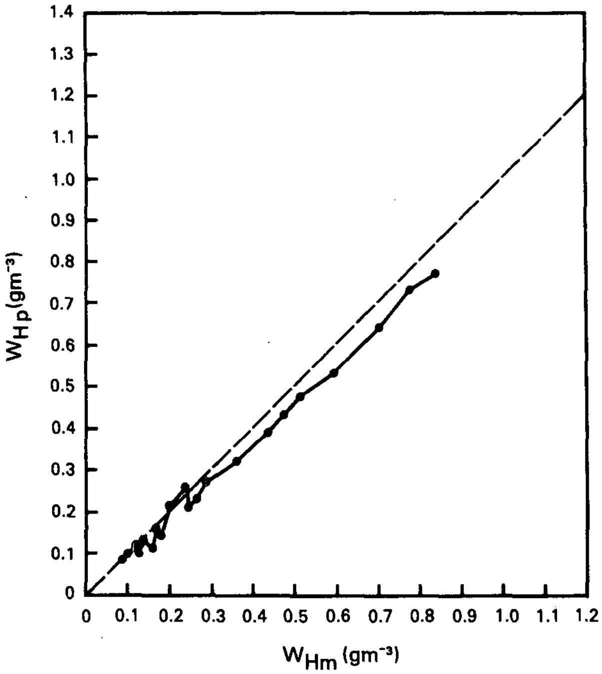

FIG. 7. Comparison of measured (W_{Hm}) and predicted (W_{Hp}) hail water content of the discriminated 1980 time-resolved hailstone samples.

d. Maximum concentrations

If hailstone size distributions are exponential and the relationship between n_0 and λ found in the preceeding section is valid, the hailstone distributions can be expressed as a single parameter function, i.e., dependent on λ only. Using various values of λ in Eq. (12), a family of hailstone size distribution functions can be obtained. For our 41 hailstone size distributions, λ was found to range from 0.1 to 2.0 mm^{-1}. A family of hailstone spectra can be drawn that would give all possible distributions of hailstone size and concentrations, if it is assumed that λ does not fall outside the range of 0.1–2.0 mm^{-1}. This family of hailstone spectra is shown in Fig. 8. An envelope can be drawn to these curves, such that the envelope gives the maximum possible concentration of hailstones in each size category. This envelope is also shown in Fig. 8, and can be derived from (12) in the form

$$n(D)_{\max} = 115\left(\frac{3.63}{D}\right)^{3.63} e^{-3.63}. \qquad (15)$$

A few hailstone samples were collected in 1979,[1] the best data being obtained from the storm of 21 July 1979. Because of the particular hail catcher used in 1979, stones smaller than 7 mm were generally not collected. The maximum concentration for each hail size category from the storm of 21 July 1979 are plotted in Fig. 9, together with the maximum possible concentration function [Eq. (15)] expressed as the solid line. The dashed lines were calculated using the upper and lower limits of the coefficient constants in Eq. (11). The maximum concentrations from the 21 July 1979 storm lie close to the maximum concentration curve; considering the scatter in the observations, the agreement is reasonably good.

e. Verification of the relationship with data from a hailstorm in Switzerland

Exponential size distributions have been fitted to the hailstone concentration and size data given by Federer and Waldvogel (1975) for hailstone samples collected from the Napf hailstorm of 6 July 1973. Samples of less than 100 stones or maximum size < 1 cm were combined with the subsequent time sequence(s) to form samples of adequate size and concentration to satisfy the criteria of $D_{\max} \geq 1$ cm and total concentration $\geq$ 100 stones. Each sample was used only once either by itself or together with other samples. This resulted in 22 revised data sets. Least-

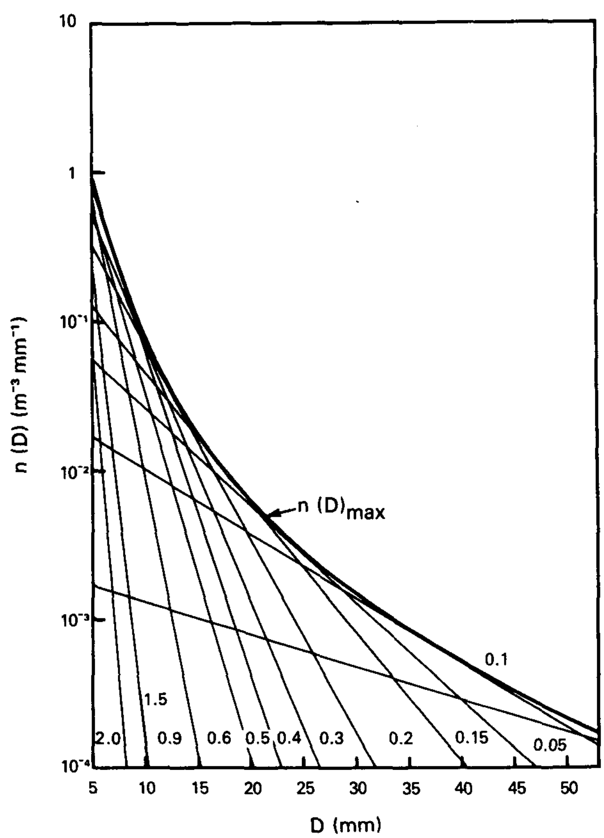

FIG. 8. Family of hailstone size distributions for $0.05 \leq \lambda \leq 2.0$ mm^{-1} and the maximum likely concentration as a function of size.

[1] All stones sampled were measured, in a manner similar to that of the hailstones collected in 1980, by N. C. Knight of NCAR, Boulder, CO.

squares regressions were performed on these data, and n_0, λ, the goodness of fit, and the correlation coefficient for each hail spectrum were obtained. All but one of the distributions (95%) have correlation coefficients of −0.88 or better. Eighteen distributions (92%) have correlation coefficients better than −0.90 and 10 (45%) have correlation coefficients better than −0.95. The remaining spectrum has a correlation coefficient of ~ −0.25. It is, therefore, not considered an exponential distribution and was rejected in the calculation of the relationship between the intercept and the slope.

The n_0 and λ so obtained for each of the 21 hailstone size distributions from the Swiss storm are plotted in Fig. 10. A power-law relationship exists between n_0 and λ. Least-squares regression analysis of the data points in Fig. 10 gives the following expression for the line of best fit:

$$\log n_0 = (2.88 \pm 0.15) + (4.18 \pm 0.38) \log\lambda. \quad (16)$$

The correlation coefficient to this line is 0.93. This seems to support the power relationship between the intercept and slope determined from the Alberta data.

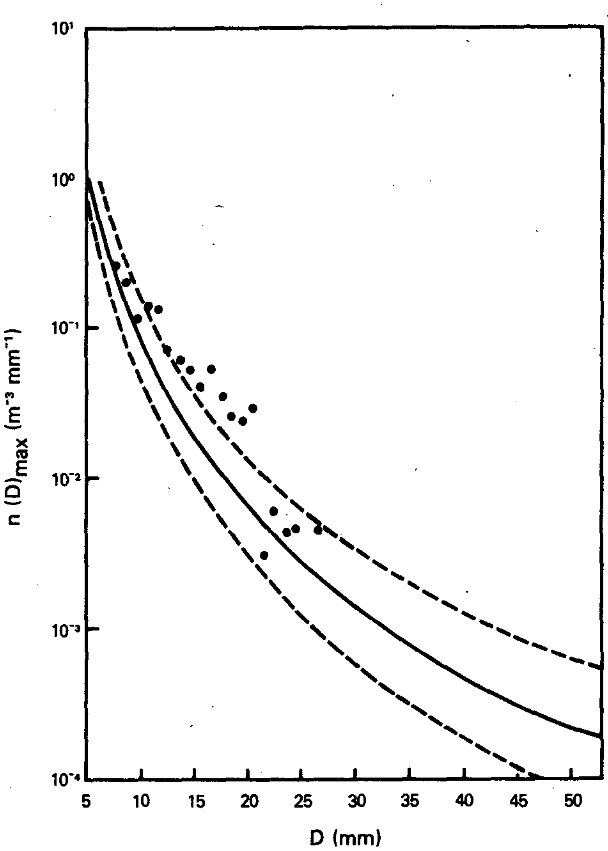

FIG. 9. Maximum possible concentration of hailstones as a function of size, from Fig. 8. The dashed lines are those calculated with the standard errors of the slope and intercept of expression (11). Also shown are data points of maximum concentration for the time-resolved hailstone samples collected on 21 July 1979.

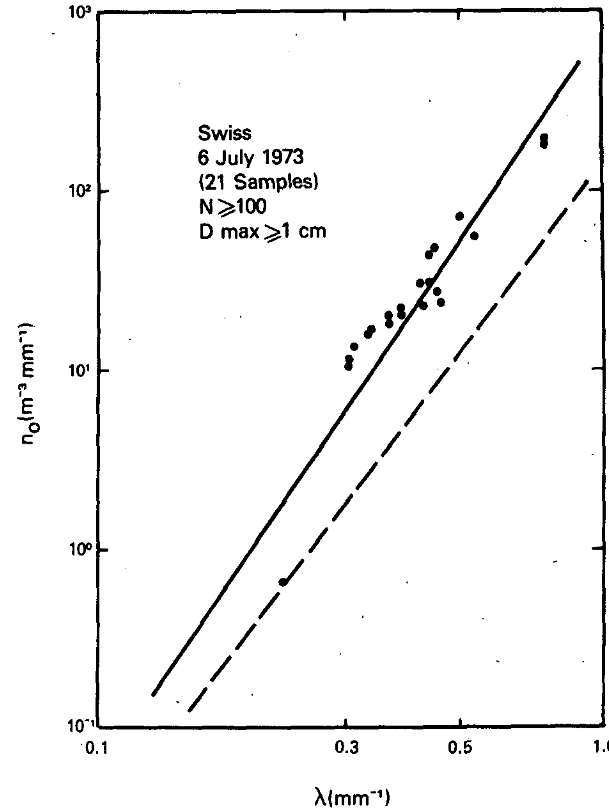

FIG. 10. Variation of the intercept parameter (n_0) as a function of the slope parameter (λ) of exponential size distributions for the Napf storm in Switzerland on 6 July 1973. The solid line is the line of best fit to the Swiss data; the dashed line is the corresponding line of best fit for the Alberta data.

Eq. (11) is also plotted as the dashed line in Fig. 10 for comparison purposes. The exponent parameters in Eqs. (11) and (16) are in good agreement within the standard deviations. Statistical testing of one-way analysis of covariance shows no significant difference between the two slopes. Thus, the data from the Swiss hailstorm support a power relationship between n_0 and λ with an exponent of value between 3.63 and 4.18. The intercepts in Eqs. (11) and (16) are somewhat different, suggesting higher concentrations of hailstones in the Swiss storm. Passarelli (1978) in his study of the equilibrium spectra of snow suggested that n_0 is proportional to λ^3. The proportionality constant, however, depended upon the thermodynamic properties of the cloud and was expected to vary from storm to storm.[2] More data, from Alberta, Switzerland and other regions, are required to determine the reasons for the existence of these relationships, their similarities and their differences.

[2] An operational seeding project for hail suppression has been underway in Alberta since 1974. Thus, all the Alberta storms from which data were collected were seeded. However, since no conclusive evidence of any seeding effect has been found to date, the seeding has been ignored in this analysis.

5. Summary and conclusions

Hailstone size distributions have been determined from 41 time-resolved hail samples collected from seven storms that occurred in Alberta in the summer of 1980. Exponential distributions have been fitted to the hailstone size and concentration data from each hail sample. A power relationship has been found between the intercept (n_0) and the slope (λ) of the size distributions. Using this relationship, a maximum concentration likely for each size category of hailstones has been determined.

The power relationship between n_0 and λ has been tested with data from a hailstorm in Switzerland. Significant differences were not found between the exponent parameters of the relationship obtained from Alberta and Swiss data but significant differences in the intercept values were found. Obviously, data from many more storms are required to confirm the relationship. It would be desirable to examine data from other regions to see if similar relationships can be determined for those regions.

It is interesting to note that the relationship between n_0 and λ implies an inverse relationship between concentration and size since n_0 is related to concentration and λ is inversely related to hailstone size. This suggests that hailstorms in a given geographical region have a somewhat similar propensity to produce hail. That is, if the thermodynamics are such as to allow only small hail to develop, the storms compensate by producing many more hailstones. This, in turn, suggests that if more hailstones could somehow be produced artificially, for instance by cloud seeding, the size of the resulting hailstones would be reduced.

The relationships shown in this study are based on ground measurements. When falling below cloud base, sedimentation and melting should greatly influence, if not control, the relationships between n_0 and λ. Thus, it would be interesting to see if a relationship exists aloft between n_0 and λ for hail spectra. However, sampling problems may inhibit such observational studies. Approximate analytical or numerical modeling may be the alternative. The relationship so determined should enable cloud modelers to use more realistic distributions in describing hail spectra in simulations of convective clouds.

Weather radar has been used for decades to follow the development of precipitation within clouds. Since the radar reflectivity Z is a function of the size and concentration of precipitation particles, the relationship between concentration and size can greatly facilitate the measurement of hailfall by radar, although it does not solve the problem completely. Of considerable influence on the backscattering cross section are the hailstone shape, orientation and wetness. These parameters, however, remain unknown. Thus the relationship found in our results merely reduces the number of unknowns.

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