NOTES AND CORRESPONDENCE

A Note on Orographically Induced Instabilities in Two-Layer Models

ERLAND KÄLLÉN[1]

Institute of Meteorology and Oceanography, University of Utrecht, Princetonplein 5, 3584 CC Utrecht, The Netherlands

16 July 1982 and 28 September 1982

ABSTRACT

In a low-order, two-layer model with orographic and zonal thermal and momentum forcing, two different types of orographically induced instabilities exist. One is of a barotropic nature with a westerly flow in the bottom layer, while the other one is close to a baroclinic instability of the basic long wave and has a very weak westerly or easterly bottom layer flow.

1. Introduction

In recent years, a number of studies dealing with nonlinear aspects of orographically forced flows have been published. Most of these studies have been concerned with barotropic models, as was the first paper in this field by Charney and De Vore (1979). A few papers have looked at baroclinic models and, in particular, two-layer models have been used by Charney and Straus (1980), and Roads (1980). The results of both of these studies indicate that the orography creates the possibility of obtaining more than one stable steady-state in a baroclinic flow, but the mechanism leading to the formation of multiple steady states appears to be different from the one studied by Charney and De Vore (1979). In the baroclinic models, the stable steady states with a large-amplitude wave are predominantly maintained by a conversion of available potential energy to kinetic energy (see Charney and Straus, 1980), while in the barotropic model it is the conversion of zonal kinetic to eddy kinetic energy which maintains the high-amplitude wave in a steady state (see Källén, 1981).

It is the purpose of this note to investigate the conditions under which orographically induced barotropic instability has a close analogy in a baroclinic model. The two-layer model of Charney and Straus (1980) is extended to include a direct momentum forcing of the bottom layer flow. In the original two-layer model formulation (Lorenz, 1963), the forcing and dissipation parameterization were arranged in such a way that in a steady state, which is maintained purely by a balance between forcing and dissipation, the bottom layer flow is zero. If the orographic influences are introduced *via* the bottom layer flow, this means that the orographic forcing will have no effect in such a steady state. In the atmosphere, westerly momentum is transported from equatorial regions to the midlatitudes in meridional cell circulations and *via* eddy transports. Through vertical momentum transfer the westerly momentum is transported to the surface in midlatitudes where it is dissipated through frictional processes, as discussed by Palmén and Newton (1969). In the low-order model of Charney and Straus (1980), neither the horizontal nor the vertical transports of westerly angular momentum from the equatorial regions to the surface in midlatitudes can be modeled accurately. As the effects of a nonzero, westerly momentum forcing in the lower levels of the midlatitude atmosphere is of importance for the atmospheric response to orographic forcing, a bottom layer momentum forcing is included in the two-level model of this investigation. The purely barotropic effect of orographic forcing will thus be retained in this model, and it will be shown that the orographic instabilities with their associated multiple equilibria, discussed by Charney and De Vore (1979) and Charney and Straus (1980), respectively, can be found in this model but in two quite different regions of forcing parameter space. A discussion of which type of instability that is most likely to be found in the atmosphere will be given.

2. The model

The model used here is a two-layer, quasi-geostrophic model on a spherical geometry. We expand the space-dependent variables in a series of spherical harmonics and truncate the expansion at a very low order. The flow components retained will describe a zonal flow with a constant angular velocity and one wave component. The model thus describes non-linear interactions only between the zonal mean flow and one wave, no wave-wave to wave interactions are included due to the truncation. We thus obtain six

[1] On leave from the Department of Meteorology, University of Stockholm, Arrheniuslaboratoriet, S-10691 Stockholm, Sweden.

nonlinearly coupled ordinary differential equations and their steady states and stability properties will be determined. The system of ordinary differential equations is structurally very similar to that given by Charney and Straus (1980) and Roads (1980) although the coefficients are not the same due to the difference in flow geometry. The parametrization of the forcing and frictional processes also differ which will be discussed more in detail below.

The governing equations for a two-level model with orographic forcing in the bottom layer and a rigid upper lid can be written as

$$\frac{\partial \zeta_1}{\partial t} = J(\zeta_1 + f, \psi_1) + \frac{f_0}{\Delta p}\omega_2 + F_1(\zeta_1, \zeta_3), \quad (1)$$

$$\frac{\partial}{\partial t}(\psi_3 - \psi_1) = J[\psi_3 - \psi_1, \tfrac{1}{2}(\psi_3 + \psi_1)] - \frac{\sigma \Delta p}{f_0}\omega_2 + F_2(\psi_3 - \psi_1), \quad (2)$$

$$\frac{\partial \zeta_3}{\partial t} = J(\zeta_3 + f + 2f_0\hbar, \psi_3) - \frac{f_0}{\Delta p}\omega_2 + F_3(\zeta_1, \zeta_3), \quad (3)$$

where indices 1 and 3 denote the top and bottom layers, respectively, while 2 refers to the interface. The height of the orography divided by the scale height of the atmosphere and multiplied by the ratio of the mean surface wind to the mean wind in the bottom layer is given by $\hbar$. F_i's denote frictional and forcing parameterizations which will be considered below and the rest of the notation is standard (see Holton, 1972).

Both Charney and Straus (1980) and Roads (1980) adopted the parameterization scheme for frictional and forcing processes suggested by Lorenz (1963). The F_i's are then given as

$$F_1 = \tfrac{1}{4}k_d'(\zeta_3 - \zeta_1), \quad (4a)$$

$$F_2 = k_d'[(\psi_3 - \psi_1)^* - (\psi_3 - \psi_1)], \quad (4b)$$

$$F_3 = -\tfrac{1}{4}k_d'(\zeta_3 - \zeta_1) - k_d'\zeta_3. \quad (4c)$$

The forcing denoted by variables with asterisks is thus only applied as a heating in the thermodynamic equation. There is a vorticity exchange between the top and bottom layers and a surface drag in the bottom layer.

To investigate the properties of this friction/forcing parameterization we examine the simplest possible linear steady solution of the model. In such a steady state (f assumed constant, orography absent) $\zeta_3 = 0$ and the temperature gradient introduced at the middle level only gives height changes and a nonzero flow in the top layer. This may be seen by adding Eqs. (1) and (3) with F_1 and F_3 given by (4a) and (4c). If the forcing is only in the zonal flow, this circulation is the so-called Hadley solution (Lorenz, 1963), and because of the zero flow in the bottom layer an addition of orographic forcing will not influence the flow for moderate forcing values as we shall see later.

To set up a nonzero flow in the bottom layer it is desirable to add a second forcing parameter which can be introduced as a direct momentum forcing in the vorticity equations. This will here be taken as a Newtonian type of forcing and the momentum exchange between the layers through the friction parameterization will be dropped. The vertical momentum exchange now takes place only through the thermodynamic coupling. The F_i's may be written as

$$F_1 = k_d(\zeta_m^* - \zeta_1), \quad (5a)$$

$$F_2 = k_d[(\psi_3 - \psi_1)^* - (\psi_3 - \psi_1)], \quad (5b)$$

$$F_3 = k_d(\zeta_m^* - \zeta_3). \quad (5c)$$

The ζ_m^* is a mean vorticity forcing which physically may be connected with transports of horizontal momentum that cannot be described by a severely truncated model.

Here we will study the effects of orographic forcing on midlatitude flow in a model which has only one wave component. The transport of westerly momentum from equatorial to midlatitude regions therefore must be externally prescribed as a mean westerly momentum forcing. This is done by letting the ζ_m^* describe a zonally homogeneous westerly momentum source. Together with the shear forcing in (5b) this will set up a forced background flow which resembles that of the midlatitude atmosphere. In a linear model this is accomplished by specifying a zonal mean flow. Here we have to give zonal mean forcings since the zonal mean flow can change due to the nonlinearity of the model.

For the derivation of the equations in the truncated spectral model, the reader is referred to Källén (1981). As pointed out by Baer (1970) a low-order, two-level model is structurally similar to a barotropic model with one extra scale of motion, so the derivation in Källén (1981) can easily be adapted to this model. We first define a mean and a difference flow as $\psi_m = \tfrac{1}{2}(\psi_1 + \psi_3)$ and $\psi_T = \tfrac{1}{2}(\psi_1 - \psi_3)$. All variables are nondimensionalized by the earth's radius a and the rotation rate Ω. The vertical velocity ω_2 is eliminated from the set of equations (1)–(3) and the spectral expansion of the streamfunction is assumed to be given by

$$\psi_m = -u_m(t)P_1(\mu) + [x_m(t)\cos l\lambda + y_m(t)\sin l\lambda]P_n^l(\mu), \quad (6a)$$

$$\psi_T = -u_T(t)P_1(\mu) + [x_T(t)\cos l\lambda + y_T(t)\sin l\lambda]P_n^l(\mu), \quad (6b)$$

where $\mu = \sin\varphi$, φ is latitude and λ is longitude.

The orography is given as

$$h = h_1(\cos l\lambda)P_n^l(\mu), \quad (7)$$

where h is related to a dimensional orographic height

z *via*

$$h = \frac{f_0}{\Omega}\frac{|\mathbf{v}_0|}{|\mathbf{v}_3|}\frac{z}{H}, \tag{8}$$

and H is the scale height of the two level atmosphere.

The equations for the truncated, nondimensionalized, six-component model can now be written as

$$\dot{u}_m = \delta_1 h_1(y_m - y_T) + \epsilon(u_m^* - u_m), \tag{9a}$$

$$\dot{x}_m = -\alpha u_T y_T + (\beta - \alpha u_m)y_m - \epsilon x_m, \tag{9b}$$

$$\dot{y}_m = \alpha u_T x_T - (\beta - \alpha u_m)x_m - \delta_2 h_1(u_m - u_T) - \epsilon y_m, \tag{9c}$$

$$(1 + \tfrac{1}{2}\kappa)\dot{u}_T = \gamma_3(x_m y_T - x_T y_m) - \delta_1 h_1(y_m - y_T) + \epsilon[\tfrac{1}{2}\kappa u_T^* - (1 + \tfrac{1}{2}\kappa)u_T], \tag{9d}$$

$$(1 + \kappa c^{-1})\dot{x}_T = -\gamma_2 u_T y_m + (\beta - \gamma_1 u_m)y_T - \epsilon(1 + \kappa c^{-1})x_T, \tag{9e}$$

$$(1 + \kappa c^{-1})\dot{y}_T = \gamma_2 u_T x_m - (\beta - \gamma_1 u_m)x_T + \delta_2 h_1(u_m - u_T) - \epsilon(1 + \kappa c^{-1})y_T, \tag{9f}$$

where

$$\alpha = \sqrt{3}l(1 - 2c^{-1}),\quad \beta = 2lc^{-1},\quad \gamma_1 = \sqrt{3}l[1 + (\kappa - 2)c^{-1}],\quad \gamma_2 = \sqrt{3}l[1 - (\kappa + 2)c^{-1}]$$

$$\gamma_3 = \tfrac{1}{4}\sqrt{3}\kappa l,\quad \delta_1 = \tfrac{1}{4}\sqrt{3}l,\quad \delta_2 = \sqrt{3}lc^{-1},\quad c = n(n + 1),\quad \kappa = \frac{2a^2 f_0^2}{\sigma(\Delta p)^2},\quad \epsilon = k_d\Omega.$$

The parameter κ is the squared ratio between the horizontal length scale and the Rossby radius of deformation and ϵ is the nondimensional dissipation rate. As we will be looking at long waves, the length scale is order radius of the earth and thus κ will be large. To solve for the steady states of this model, we first observe that the equations for the time evolution of the waves are linear in the wave amplitudes. We can thus specify the zonal mean state and solve for the wave coefficients easily. The equations for the zonal mean coefficients [(9a) and (9d)] can then be seen as two simultaneous nonlinear equations in u_T and u_m, which have to be solved given u_T^*, u_m^*, h_1 and ϵ.

We will here mainly be interested in the orographic influences on the flow, and it may be seen from all the terms involving h_1 that it is only the lower layer flow given by the differences between the mean and the sheared flow which interacts directly with the orography. The method of solution for the steady states will therefore be as follows. The lower level zonal flow is specified ($u_m - u_T$) along with the shear and orographic forcing and the dissipation rate (u_T^*, h_1 and ϵ). Eqs. (9b)–(9f) are then used to find all values of u_T (normally only one) which satisfy a steady state of these equations. Finally, the value of u_m^* corresponding to this (these) steady-state(s) is calculated *via* (9a) when $\dot{u}_m = 0$.

The stability of each steady state is determined by calculating the eigenvalues of the linearized problem around it. A steady state which has at least one eigenvalue with a real part larger than zero is unstable. The unstable steady states are furthermore distinguished by the type of instability. If the imaginary part of the eigenvalue(s) with a positive real part is zero, the instability is stationary in space and the wave "grows in place" (Charney and Straus, 1980). This type of instability is referred to as the orographic instability. In phase space this is a so-called saddle point type of instability. If the imaginary part of at least one eigenvalue with a positive real part is non zero, the wave solution is propagating and we have a so-called Hopf-bifurcation (see Marsden and McCracken, 1976). In phase space this is connected with either a stable or an unstable periodic trajectory. This latter type of instability is characteristic of waves which are baroclinically unstable and may be referred to as a propagating instability.

3. Results

The results of the steady-state and stability calculations for a range of parameter values can be found in Figs. 1 and 2. Each curve in Figs. 1 and 2 is plotted for a constant shear forcing, u_T^*. When the shear forcing is zero, the model is essentially barotropic and the curve for $u_T^* = 0$ in Fig. 1 very much resembles one of the curves in Fig. 4 of Källén (1981) if h is chosen to be above the critical value needed for a bifurcation to occur. In the region of u_m^* for which there are three steady states, the middle one is unstable while the ones with maximum and minimum zonal velocity in the bottom layer are stable. The middle unstable steady state is not of the propagating type and the instability is associated with the orography as in the barotropic model. For increasing shear forcings the fold in the steady-state curve, and thus the region of multiple steady states, is moving toward the origin, and the range of u_m^* for which we have more than one steady state is decreasing. It is interesting to note

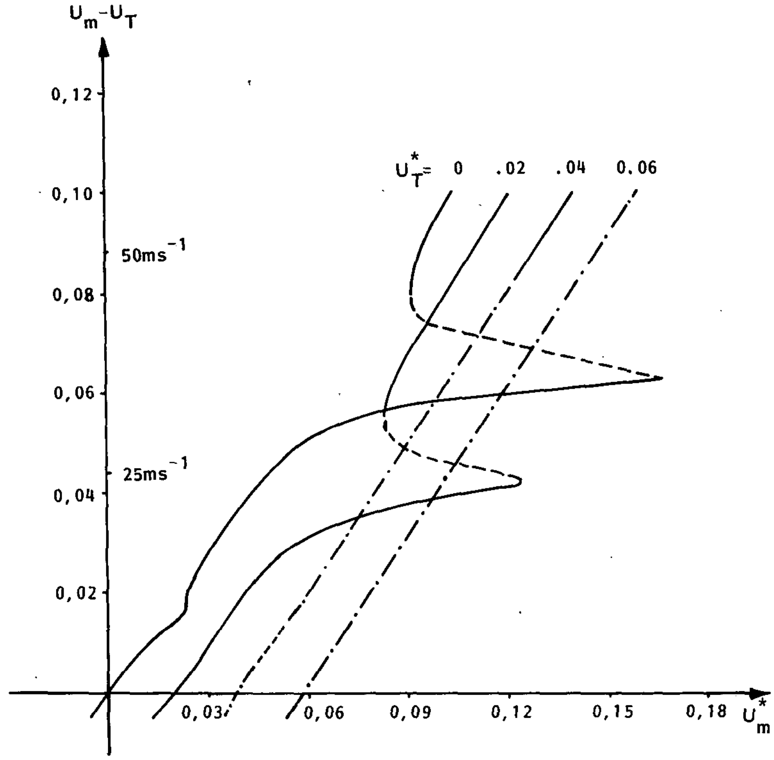

FIG. 1. Steady-state curves for the two-layer model with varying values of the shear (u_T^*) and mean (u_m^*) flow forcing. The vertical axis gives the steady-state value of the bottom layer zonal streamfunction in dimensional and nondimensional units. Dimensional units corresponds to wind speed at 45° latitude. Stability properties are indicated on the steady-state curves, solid line stable, dashed line unstable but "growing in place," and dash-dotted line is unstable and propagating (see text). Parameter values: $l = 3$, $n = 4$, $\kappa = 60$, $\epsilon = 0.02$, $h = 0.1$, $a = 6.37 \times 10^6$ m, $\Omega = 7.29 \times 10^{-5}$ s^{-1}.

that an increased baroclinicity of the flow results in the formation of multiple steady states for lower values of the bottom layer zonal flow. When u_T^* reaches a value of 0.04 the fold vanishes, and at 0.06 all steady states are baroclinically unstable. The onset of baroclinic instability is almost independent of u_m^*, since it is the shear forcing which is important here as is to be expected from baroclinic instability theory. The weak dependence on u_m^* is due to the effect of the orography. Without the orography the model becomes baroclinically unstable at around the same value of u_T^* as when the orographic forcing is present.

A comparison of these results with those of Charney and Straus (1980), and Roads (1980) can be made through a transformation of the forcing parameters. As the models differ only in some of their linear terms the nonlinear characteristics should not be very different. The models are, however, not structurally identical and the parameter values for which bifurcations occur will therefore not be the same. The Hadley solution of the two models can be directly compared by noting that u_m^* should be a constant such that $(u_m - u_T)$ is equal to zero. In Figs. 1 and 2 this will consist of vertical lines passing through the intersection between any given u_T^* curve and the u_m^* axis. In Fig. 1 it is seen that such a line has only one intersection with each curve. To investigate further how the multiple steady states obtained by Charney and Straus (1980) occur, a more detailed diagram around the region of the onset of baroclinic instability and $(u_m - u_T)$ close to zero is shown in Fig. 2. Here it may be seen that for u_T^* values just below the one needed for baroclinic instability, an instability which "grows in place" develops close to the origin. It is this instability that was analyzed in Charney and Straus (1980). For only a slight increase in u_T^*, baroclinic instability develops on the lower part of the steady-state curves, and for $u_T^* = 0.038$ the whole region has become baroclinically unstable, except for an interval which has an instability "growing in place." To investigate further how the nonlinear model behaves around these unstable steady states, time integrations would have to be performed and the results of Charney and Straus (1980) indicate that limit-cycles develop around the unstable steady states. Charney and Straus (1980) also obtained up to five steady states

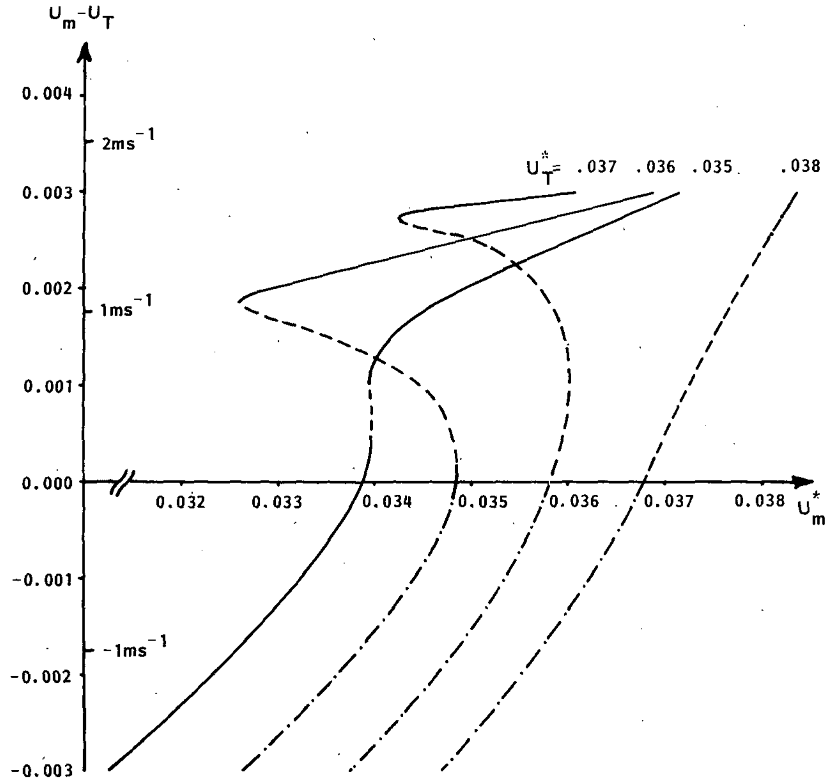

FIG. 2. As in Fig. 1, but for a different range of u_T^* and u_m^*.

for large values of the thermal forcing, while in this model the multiple steady states vanish for sufficiently large values of u_T^* when all other parameters are kept fixed. This difference is presumably due to the different friction parameterization schemes.

Thus, there are two quite different regions in parameter space where an orographically induced instability develops. In the region shown in Fig. 1, the zonal flow in the bottom layer is westerly and fairly strong on both of the stable steady-state branches. The orography thus plays an active part in transferring kinetic energy between the zonal flow and the wave (see Källén, 1981). In the region of Fig. 2, the bottom layer flow is very weak and the main part of the energy exchange takes place between the available potential and kinetic eddy energies as shown by Charney and Straus (1980).

4. Conclusions

It is the main purpose of this note to show that the barotropic, orographically induced instability found by Charney and De Vore (1979), Hart (1979), and Davey (1980) is also present in a two-layer baroclinic model with an appropriate forcing mechanism included. The more complicated behavior of the orographic instability found by Charney and Straus (1980) is also present in the model, but for a much narrower range of parameter values and close to the point of baroclinic instability. The main difference between this model and the model of Charney and Straus (1980) is the absence of any direct forcing in the bottom layer of the latter model. Roads (1980) has partly circumvented this problem by making the surface flow, which interacts with the orography, a linear combination of the top and bottom flow. The dependence on the top flow is, however, so weak that it probably does not have any significant interaction with the orography unless the model comes close to being baroclinically unstable. One of the stable, stationary solutions found by Roads has a weak easterly zonal flow in the bottom layer and therefore is associated with the stable steady states in Fig. 2 on the negative side of the $(u_m - u_T)$ axis. The Charney and De Vore type of orographic instability occurs for westerly flow across the orography.

In the atmosphere, baroclinic instability mainly takes place at wavelengths shorter than the ultra long waves which this type of model is aimed at (Saltzman, 1970). To a large extent the ultra long waves in midlatitudes receive their energy from conversions of kinetic energy rather than a conversion from available potential to kinetic eddy energy (Kanamitsu, 1981). It may be that, in some instances, baroclinic conversions take place on the length scale of ultra long waves, but seen as a time average over a month or more, observational studies seem to indicate that the ultra long waves in midlatitudes are predominantly maintained by kinetic energy conversions. If the multiple equilibrium theory, as originally proposed by Charney and De Vore (1979), is relevant to the formation of ultra long waves and anomalous flow patterns, it should be connected with a barotropic type of mechanism rather than a mechanism which is close to the limit of baroclinic instability of the ultra long waves. Classical studies by Charney and Eliassen (1949) and Bolin (1950) also show that a large part of the standing wave pattern in the Northern Hemisphere can be explained by considering only the barotropic effects of orographic forcing. It may very well be that the baroclinically unstable shorter waves feed energy to the ultra long waves and that these in turn determine the regions in which baroclinically unstable waves may form. It has been suggested by Källén (1981) that the momentum transports associated with transient baroclinic waves feed back on the ultra long waves and further help to maintain the separation between the two stable equilibrium solutions found in a barotropic model with orographic forcing.

A serious shortcoming of a severely truncated model is the neglect of interactions with components outside the truncation. For the barotropic, orographic instability mechanism it has been shown by Charney and De Vore (1979), Davey (1980), Källén (1982) and others that it is not sensitive to the number of components included. Roads (1980) also included more components in the baroclinic model and it appeared that the orographic instability which he analyzed was sensitive to the number of components included. This may be due to the fact that his instability is very close to a baroclinic instability of the basic wave. It may be argued that as the barotropic, orographic instability of Fig. 1 is further removed from the baroclinic instability of the basic wave, the results from higher resolution barotropic models may more easily be carried over to the baroclinic case. As noted above, it may be that shorter waves are baroclinically unstable in the long wave flow set up by the orographic forcing, and interactions between these baroclinically unstable short waves and the orographically induced long waves may explain some of the observed long wave variability in the atmosphere.

Acknowledgments. I would like to thank J. Oerlemans, J. D. Opsteegh, B. Reinhold and C. Schuurmans for useful comments and discussions on this paper.

REFERENCES

Baer, F., 1970: Analytical solutions to low-order spectral systems. *Arch. Meteor. Geophys. Bioklim.*, **A19**, 255–282.

Bolin, B., 1950: On the influence of the earth's orography on the general character of the westerlies. *Tellus*, **2**, 184–195.

Charney, J. G., and J. G. De Vore, 1979: Multiple flow equilibria in the atmosphere and blocking. *J. Atmos. Sci.*, **36**, 1205–1216.

——, and A. Eliassen, 1949: A numerical method for predicting the perturbations of the middle latitude westerlies. *Tellus*, **1**, 38–54.

——, and D. M. Straus, 1980: Form-drag instability, multiple equilibria and propagating planetary waves in baroclinic, orographically forced, planetary wave systems. *J. Atmos. Sci.*, **37**, 1157–1176.

Davey, M., 1981: A quasi-linear theory for rotating flow over topography: Part 2. Beta-plane annulus. *J. Fluid Mech.*, **103**, 297–320.

Hart, J. E., 1979: Barotropic quasi-geostrophic flow over anisotropic mountains. *J. Atmos. Sci.*, **36**, 1736–1746.

Holton, J. R., 1972: *An Introduction to Dynamic Meteorology*. Academic Press, 319 pp.

Källén, E., 1981: The nonlinear effects of orographic and momentum forcing in a low-order, barotropic model. *J. Atmos. Sci.*, **38**, 2150–2163.

——, 1982: Bifurcation properties of quasi-geostrophic, barotropic models and their relation to atmospheric blocking. *Tellus*, **34**, 255–265.

Kanamitsu, M., 1981: Some climatological and energy budget calculations using the FGGE III-b analyses during January 1979. *Dynamic Meteorology: Data Assimilation Methods*, L. Bengtsson, M. Ghil, E. Källén, Eds., Springer Verlag, 263–318.

Lorenz, E. N., 1963: The mechanics of vacillation. *J. Atmos. Sci.*, **20**, 448–464.

Marsden, J. E., and M. McCracken, 1976: *The Hopf Bifurcation and Its Applications*. Springer Verlag, 408 pp.

Palmén, E., and C. W. Newton, 1969: *Atmospheric Circulation Systems*. Academic Press, 603 pp.

Roads, J. O., 1980: Stable near resonant states forced by orography in a simple baroclinic model. *J. Atmos. Sci.*, **37**, 2381–2395.

Saltzman, B., 1970: Large-scale atmospheric energetics in the wavenumber domain. *Rev. Geophys. Space Phys.*, **8**, 289–302.