The Mesoscale and Microscale Structure and Organization of Clouds and Precipitation in Midlatitude Cyclones. VI: Wavelike Rainbands Associated With a Cold-Frontal Zone

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ABSTRACT

The cloud and precipitation structure and the airflow associated with wavelike rainbands in a cold-frontal zone have been investigated with Doppler radar, instrumented aircraft, rawinsondes and a network of ground stations. The rainbands were oriented perpendicular to the cold front and embedded within wide cold-frontal rainbands. The wavelike rainbands were 20–40 km long, 3–6 km wide, spaced 9–13 km apart and their tops ranged from 3–5 km in height. The radar reflectivities, convergence/divergence and airflow show regular patterns associated with the rainbands.

There is evidence that wavelike rainbands were associated with generating cells aloft. These rainbands may have been initiated by shear instability in the frontal zone, since the resonant mode for such an instability had a similar orientation, movement and spacing to those observed for the rainbands.

1. Introduction

In previous papers describing results from the University of Washington's (UW) CYClonic Extratropical Storms (CYCLES) Project, six types of mesoscale rainbands, have been identified (e.g., Houze et al., 1976; Hobbs, 1978; Matejka et al., 1980). Of these, the wavelike rainbands occur on the smallest scale and exhibit the most regular patterns (Parsons and Hobbs, 1982).

Wavelike rainbands are often located within or near wide cold-frontal rainbands. They are oriented approximately perpendicular to both the synoptic-scale cold front and the wide cold-frontal rainbands. The area covered by these rainbands is ~200 km² and the mean spacing between them is ~11–16 km. This type of wavelike rainband extends in the vertical from near the top of the cold-frontal zone to the top of the radar echoes in the vicinity of the front (typically from ~1.5 to 5.5 km). The base of this layer is generally in a region with a stable lapse rate, but potential instability may exist near the top of the rainbands.

In this paper we are concerned with a detailed case study of wavelike rainbands associated with a cold-frontal zone observed in the CYCLES Project on 6 March 1979. The structure and airflow associated with these rainbands were well documented with the NCAR CP-3 Doppler radar, the UW B-23 research aircraft, frequent rawinsonde ascents and high-resolution data from ground stations, as well as satellite and regular synoptic data.

2. Synoptic situation

On 6 March 1979 a cold front moved eastward through the CYCLES observational network at a speed of ~14 m s⁻¹. The front was associated with a cyclone centered ~1800 km to the northwest of Seattle (Fig. 1). The passage of the cold front through the CYCLES network was characterized by rather large surface pressures (>1020 mb), slight increases in surface pressure, and slight decreases in temperature and dewpoint (Fig. 2). In contrast to the cold-frontal systems discussed by Hobbs et al. (1980) and Hobbs and Persson (1982), the cold front on 6 March 1979 did not exhibit a marked windshift, pressure jump or a narrow cold-frontal rainband at the surface.

A time-height cross section of the wet-bulb potential temperature ($\theta_w$), derived from serial rawinsondes launched from Pt. Brown, is shown in Fig. 3. The close packing of the isotherms in the cold-frontal zone indicates the transition to cold, dry air. The closer packing of the isotherms at higher levels suggests that the cold front was stronger aloft. Regions of potential instability ($\partial \theta_w / \partial z < 0$) are shown by the shading in Fig. 3. The large region of potential instability in the

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2 Present affiliation: Wave Propagation Laboratory, Environmental Research Laboratories, NOAA; Boulder, 80303.
3 On some occasions, wavelike rainbands are located in the unstable cold airmass behind prefrontal surges in occlusions (Matejka et al., 1980). We will present a case study of this type of wavelike rainband in a future paper in this series.

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lower levels ahead of the front was produced by a rapid decrease in mixing ratio within a pronounced temperature inversion. We will see later that the base of the region of potential instability above the cold front (near 640 mb) was at the top of the wavelike rainbands.

A time-height cross section of temperature, wind and relative humidity through the frontal system, derived from the rawinsondes, is shown in Fig. 4. The vertical extent of the cold-frontal zone (derived from VAD Doppler radar measurements), vertical crosssections of convergence, vertical air motion and airflow, calculated from the VAD Doppler radar measurements, are shown in Fig. 5. It can be seen (Fig. 5a) that strong convergence was located within and above the leading portion of the frontal zone and that there was weaker convergence farther back in the frontal zone. A region of weak divergence was located above the frontal zone, between the two regions of convergence. The vertical air motion pattern (Fig. 5b) was obtained by integrating the convergence field, with $w = 0$ at $z = 0$. In general, the vertical motions increase with height, reaching a maximum of $\sim 1$ m s$^{-1}$. The airflow, relative to the frontal motion, shows the air generally flowing backward and upward, particularly near the top and above the frontal zone (Fig. 5c). This airflow, coupled with the lack of strong lifting at the leading edge of the front, should have resulted in the entrainment of warm-sector air and the weakening of the front with time.

3. Analysis of the rainbands from radar observations

In this section we consider the characteristics of the rainbands detected with the CP-3 Doppler radar at Pt. Brown on 6 March 1979.

a. Wide cold-frontal rainbands

Two wide cold-frontal rainbands were associated with the cold front on 6 March 1979. These rainbands were situated behind the surface cold front. Shown in Fig. 6 are the horizontal radar reflectivity patterns of these rainbands and their movements relative to
Fig. 2. Surface synoptic analyses for (a) 1500, (b) 1600, (c) 1700, and (d) 1800 PST on 6 March 1979. The station model is shown in the inset to (a).
the surface cold front. The first rainband (labeled A) was 30–40 km wide; the second rainband (labeled B), which was situated behind A, was 10–20 km wide.

It can be seen from Fig. 6 that rainband A was dissipating along its leading edge, and that it moved slower than rainband B. The two rainbands combined at about the time they reached the coast. Although both rainbands initially moved faster than the cold front, they moved slower than the front during their decay.

b. Wavelike rainbands

1) BASIC INFORMATION

The PPI radar reflectivity pattern at higher elevation angles (Fig. 7) shows wavelike rainbands perpendicular to the cold front and superimposed on the two wide cold-frontal rainbands as they merged on the coast. The wavelike rainbands were about 20–40 km long, 3–6 km wide and spaced 9–13 km apart. Their tops were at about 3–5 km.

Fig. 8 depicts several aspects of the development of these rainbands in space and time. They appeared at 1604 PST and developed until 1711 PST. Fig. 9 shows the wavelike rainbands [at 20 dB(Z)] observed by the radar at different elevation angles. It can be seen that the wavelike rainbands were observed closer to the radar at higher elevation angles. It can also be seen from Fig. 9 that the rainbands were arrayed from about 35 km south to about 130 km to the north of the radar site; they were most intense 70–130 km to the north of the radar. There was a tendency for the distance between the wavelike rainbands to decrease as their intensity increased. The more intense wavelike rainbands can be clearly discerned in the surface raingauge data from Pt. Brown (Fig. 4), and in the high-resolution precipitation data from Moclips (Fig. 10), which is located to the north of Pt. Brown (see Fig. 2a).

The more intense wavelike rainbands to the north of Pt. Brown moved in a different direction and with a slightly higher speed than the wide cold-frontal rainband B (Table 1) on which they were superimposed. The motion of the wavelike rainbands also differed from the winds aloft measured at Pt. Brown. Fig. 11 shows the wind direction and speed profiles measured by two rawinsondes from 1558–1627 PST and from 1721–1751 PST. It can be seen from Table 1 and Fig. 11 that the difference between the wind direction at 2–5 km in altitude and the motion of the wavelike rainbands was 10–20°; the speed of the wavelike rainbands was similar to the wind speed near their top (4–5 km). However, there was a substantial N–S gra-
Fig. 4. Time-height cross section of temperature, wind and relative humidity derived from serial rawinsondes launched from Pt. Brown on 6 March 1979. Shown are isotherms (°C), winds (arrows: half-feathers 2.5 m s⁻¹, full feather = 5 m s⁻¹, and dark flag = 25 m s⁻¹), relative humidity, RH (no shading RH < 30%, light shading 30 < RH < 70%, medium shading 70 < RH < 90%, darkest shading RH > 90%). Surface winds and 5 min average precipitation rates at Pt. Brown are shown below the cross-section. Also shown are the wide cold-frontal rainbands A and B and the effects of the wavelike rainbands (arrows numbered 1-5) on the rainfall at Pt. Brown.

Fig. 5. Time-height cross sections of (a) divergence (10⁻⁴ s⁻¹; positive values indicate divergence and negative values convergence), (b) vertical velocity (cm s⁻¹; positive values indicate upward motion), and (c) streamflow relative to the motion of the cold front; lengths of arrows represent 20 min displacements. Note that the large difference between the spatial scales on the horizontal and vertical axes results in an apparent magnification of x20 in the vertical air velocities indicated by the streamflow. The heavier solid lines show the cold-frontal zone as determined by the region where the Doppler radar data showed the winds to be backing with height.
Fig. 6. PPI radar reflectivity factor patterns at an elevation angle of 0.1° measured by the CP-3 radar at Pt. Brown five times on 6 March 1979. The two wide cold-frontal rainbands are labeled A and B. The approximate positions of the surface cold front are also shown. The radius of the large circle is 140 km.
gradient to the wind velocity, as evident in the 700 mb winds (Fig. 1). There was also a strong N–S gradient in the phase velocity of the wavelike rainbands [\(\sim 1\) m s\(^{-1}\) (10 km\(^{-1}\)], see Table 2]. The rainbands were accelerating as they moved northward. An extrapolation of the N–S gradient of the velocity of the rainbands to Pt. Brown indicates that they were probably moving as slow as 13 m s\(^{-1}\) when they passed over the CP-3 radar.

Fig. 7. (a) PPI radar reflectivity factor pattern showing the wavelike rainbands on 6 March 1979 at 1710:26 PST. The elevation angle of the radar was 1.7°. The radar reflectivity pattern is shown at 20 dB(Z). The wide cold-frontal rainbands A and B are indicated. The line OS, which is perpendicular to the wavelike rainbands, was used in the cross-sectional analysis. (b) As in (a) but at 1711:39 PST and at a radar elevation angle of 3.7°.

Fig. 8. Altitudes and locations of the wavelike rainbands between 1604 and 1726 PST on 6 March 1979. The widths of the rainbands are roughly represented on the abscissa. The dashed line represents regions where the wavelike rainbands could not be clearly detected by the radar.
Fig. 9. Schematic diagram to illustrate the wavelike rainbands (shaded areas) observed at 20 dB(Z) at different radar elevation angles. The various radial directions are labeled with the range of elevation angles and the time of observation.

2) Radar Reflectivity Factor in a Vertical Cross-Section Through the Wavelike Rainbands

Shown in Fig. 12 are measurements of the radar reflectivity factor in a vertical cross-section through line OS (356°) in Fig. 7a. The 20 dB(Z) contours show wavelike characteristics at altitudes of 3–5 km, and the 25 dB(Z) contours a wavelike pattern below 3 km. This is why the wavelike rainbands were observed closer to the radar with increasing elevation angle (Fig. 7 and 9). It can also be seen from Fig. 12 that the radar echo tops increased with increasing range from the CP-3 radar.

3) Air Motions Associated with the Wavelike Rainbands

The components (u) of the horizontal winds in a vertical plane perpendicular to the wavelike rainbands were obtained from the CP-3 Doppler velocity data when the radar beam was oriented perpendicular to the wavelike rainbands (along OS in Fig. 7a). The values of u were plotted in a vertical cross section and adjusted for the motion of the wavelike rainbands between radar scans. The resulting velocity field was then contoured, and horizontal velocities deduced at grid points separated by 2.8 km in the horizontal and 1/2 km in the vertical. The horizontal divergence in the vertical plane perpendicular to the wavelike rainbands was then derived. Assuming that the divergence of the wind parallel to the wavelike rainbands was negligible, the vertical velocity w of the air was obtained by integrating upward from the surface, applying the equation of mass continuity and assuming w = 0 at the surface. The speed (c = 20 m s⁻¹) of the wavelike rainbands in the OS (i.e., x-direction) was subtracted from each value of u. Values of (u - c) and w were then combined to produce a vector field representing the instantaneous airflow relative to the motion of the wavelike rainbands in the vertical plane.

Shown in Fig. 13, 14 and 15 are the derived fields of divergence, vertical velocity and airflow. It can be seen from Fig. 13 that convergence and divergence alternated along the direction perpendicular to the wavelike rainbands. The positions of the strongest

Fig. 10. 1-min average precipitation rates from the UW high-resolution raingauge located at Moclips. The extents of the wide cold-frontal rainbands A and B are indicated, and brackets and arrows indicate the contributions from the wavelike rainbands.
Table 1. Motion of the wavelike rainbands. The motion of the wide cold-frontal rainband B is shown for comparison.

<table>
<thead>
<tr>
<th>Time (PST)</th>
<th>Wavelike rainbands</th>
<th>Rainband B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Direction of origin (deg)</td>
<td>Speed (m s⁻¹)</td>
</tr>
<tr>
<td>1641–1656</td>
<td>209</td>
<td>27</td>
</tr>
<tr>
<td>1656–1710</td>
<td>208</td>
<td>27</td>
</tr>
<tr>
<td>1710–1725</td>
<td>209</td>
<td>26</td>
</tr>
<tr>
<td>Average</td>
<td>209</td>
<td>27</td>
</tr>
</tbody>
</table>

Convergences and divergences (with maximum values of about ±5.0 × 10⁻⁴ s⁻¹) were between 2.5–4 km in altitude at horizontal distances >30 km from the radar; weaker convergences and divergences (with magnitude of ~1.0 × 10⁻⁴ s⁻¹) were located below 1.5 km. From 1.5 to 2.5 km in altitude there was negligible convergence and divergence, except within 40 km of the radar. We will see later that there was a stable layer in the frontal zone between 1.5 and 2.5 km in altitude, while between 2.5 and 4 km there was a potentially unstable layer above the cold-frontal zone. The weaker convergence and divergence pattern below 1.5 km in altitude might have been the result of horizontal variations in cooling caused by melting snow (Atlas et al., 1969).

The vertical velocities shown in Fig. 14 also reveal the wavelike pattern. The maximum updrafts (~1 m s⁻¹) and downdrafts (~1.5 m s⁻¹) were located at an altitude of 4–5 km. However, it appears that the lifting extended from the frontal zone to the radar echo tops. The downward motions also extended into the frontal zone.

The airflow relative to the motion of the wavelike rainbands, shown in Fig. 15, also shows wavelike characteristics. The regions of strongest upflow and downdownflow are located between 3–5 km in altitude at a distance of 60–90 km from the radar, which corresponds to the strong updrafts and downdrafts at this location seen in Fig. 14.

4) Time-Height Cross Sections of the Radar Reflectivity Factor and the Vertical Air Velocities Measured by the Doppler Radar

Fig. 16 shows time-height cross sections of the radar reflectivity factor and the vertical air velocity associated with the cold front, as measured by the CP-3 radar at Pt. Brown when it was pointed vertically upwards.

The mean vertical air velocity $w$ over the radar was calculated from:

$$w = V_D - V_p,$$

where $V_D$ is the mean Doppler velocity measured by the vertically-pointing radar and $V_p$ the mean reflectivity-weighted terminal velocity of the particles. To estimate $V_p$ we assumed that the size distribution of the particles followed a Marshall–Palmer distribution and that the particles were sufficiently small with respect to the wavelength of the radar that Rayleigh scattering prevailed. In this case, for the region above the bright band (see Appendix):

$$V_p = \frac{a\Gamma(7 + b)}{\Gamma(7)} \left[ \frac{Z}{N_0 \Gamma(7)} K_w^2 \right]^{1/7} \left( \frac{\rho_0}{\rho} \right)^{0.4},$$

where $Z$ is the radar reflectivity factor, $|K_w|^2$ and $|K_v|^2$ are, respectively, the dielectric factors for ice crystals and water droplets, $N_0$ the total number concentration of ice crystals (obtained from our aircraft measurement), $\rho_0$ and $\rho$ the air densities at the surface and at the sampling altitude over the radar, respectively, $\Gamma$ the gamma function, and $a$ and $b$ the coefficients involved in the empirical expression for particle fallspeeds. The types of ice particles were determined from PMS 2-D data obtained aboard the B-23 research aircraft which flew through the cold front.

Fig. 11. Vertical profiles of wind direction and speed measured from Pt. Brown by rawinsondes between 1558–1627 PST (solid lines) and 1721–1751 PST (dashed lines) on 6 March 1979.

Table 2. Phase velocities of wavelike rainbands observed at different distances from the CP-3 radar located at Pt. Brown.

<table>
<thead>
<tr>
<th>Time (PST)</th>
<th>Radar elevation angle at which wavelike rainbands were detected (deg.)</th>
<th>Horizontal distance of wavelike rainbands from radar (km)</th>
<th>Horizontal phase velocity (m s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1641–1726</td>
<td>1.7–1.8</td>
<td>70–140</td>
<td>22–23</td>
</tr>
<tr>
<td>1641–1725</td>
<td>2.7</td>
<td>60–80</td>
<td>18–19</td>
</tr>
<tr>
<td>1711–1726</td>
<td>3.7</td>
<td>40–60</td>
<td>~17</td>
</tr>
<tr>
<td>1712–1727</td>
<td>4.8</td>
<td>40–50</td>
<td>~17</td>
</tr>
</tbody>
</table>
It can be seen from Fig. 16 that as the leading edge of the cold-frontal zone that coincided with the wide cold-frontal rainband A passed over the radar between 1648 to 1707 PST, and as the rear portion of the cold front passed over from 1830 to 1834 PST, the radar reflectivity varied fairly smoothly, with a maximum reflectivity of 24 dB(Z). However, as the central portion of the cold front and the potential unstable layer passed over the radar from 1718 to 1750 PST, the radar reflectivity changed more sharply both in the horizontal and the vertical directions. This time period coincided with the passage of the wave-like rainbands. On three occasions (around 1720, 1736, and 1748 PST), the top of the radar echo (defined by −8 dB(Z)) reached to altitudes of 5–6 km. The bright band can also be seen clearly in Fig. 16a, at altitudes between 2.2–2.3 km.

A time-height cross section of the vertical air velocity is shown in Fig. 16b. In the leading portion of the rainbands the updrafts were <100 cm s⁻¹. This region coincided with wide cold-frontal rainband A. During the passage of the wavelike rainbands, vigorous updrafts and downdrafts, with maximum values of ±3 m s⁻¹, were present. Interestingly, downdrafts dominated in the lower regions, but updrafts and downdrafts alternated in the higher regions (from 4–6 km). The downdrafts in the lower regions appear to be associated with the higher radar reflectivity values; they may have been associated with negative buoyancy due to melting precipitation. Strong up-

FIG. 12. Radar reflectivity factor [dB(Z)] measured at 1710 PST on 6 March 1979 in a vertical cross section through line OS in Fig. 7a.
ward velocities (up to 2–3 m s⁻¹) were associated with two regions above 5 km, where a radar echo was detected.

Three strong updrafts, with maximum values in excess of 1.5 m s⁻¹, were measured between 1718–1722, 1735–1738 and 1747–1749 PST, respectively. These were associated with the high radar reflectivity values at about 5 km in altitude shown in Fig. 16a. These three patterns are similar to the generating cells discussed by Herzegh and Hobbs (1981), although the vertical velocities were stronger in the present case.

Comparisons between the vertically-pointing radar data and the peaks in precipitation at Pt. Brown produced by the wavelike rainbands (Fig. 4) suggests that in two cases at least (wavelike rainbands 1 and 2) strong updrafts were associated with the rainbands. It is possible that the wavelike rainbands were associated with the release of potential instability aloft in the form of generating cells.

5) SUMMARY OF RADAR OBSERVATIONS

Two wide cold-frontal rainbands and a train of wavelike rainbands were observed in the cloud shield associated with the cold-frontal systems on 6 March 1979. These rainbands appeared to be independent.

The wavelike rainbands had lengths of 20–40 km, widths of 3–6 km and a mean spacing of 9–13 km.
Fig. 16. Time-height cross sections through the side of frontal cloud shield as they passed over Pt. Brown on 6 March 1979. (a) Radar reflectivity factor (dBZ); (b) vertical air-velocity (cm s⁻¹) measured by the vertically-pointing Doppler radar. Positive and negative values indicate updrafts and downdrafts, respectively.
Their phase velocities varied significantly with their location, ranging from \(~22\text{–}23\text{ m s}^{-1}\) in the north, where the rainbands were older, to \(~17\text{ m s}^{-1}\) some 100 km farther south. Also, their separation was less and their tops were higher in the north than in the south.

The wavelike rainbands were associated with a fairly regular pattern of convergence–divergence and vertical air motions. The largest lifting and convergence were located above the frontal zone. At times it appeared that the rainbands were associated with embedded convection aloft, possibly due to the release of potential instability; the lifting originated in the frontal zone and extended upward.

4. A possible mechanism for the formation of the wavelike rainbands

In the case presented in this paper and for the wavelike rainbands discussed by Parsons and Hobbs (1983), the rainbands were oriented approximately perpendicular to the alignment of the cold front, and hence nearly perpendicular to the vertical shear of the horizontal winds in the frontal zone. This suggests that the bands may have been produced by disturbances associated with the vertical shear. It is interesting to point out that the spacing and movement of wavelike rainbands are sometimes similar to that expected from the resonance modes of shear-induced waves. Testud et al. (1980) made a similar comparison for wavelike air motions in a cold-frontal zone. The Doppler radar measurements for the case described in the present paper show that the vertical motions associated with the rainbands originated near the frontal zone. Also, both the convergence and vertical air motions increased in height with increasing distance from the radar (hence with increasing “age” of the rainbands). These observations suggest that the energy associated with the wavelike rainbands propagated upward from a lower layer. Hence, the resonance modes of shear-induced waves may have been responsible for the formation of the wavelike rainbands. This possibility is investigated in more detail below.

The stability criterion for the spontaneous growth of small-scale waves in a stably stratified atmosphere with vertical wind shear can be expressed in terms of the Richardson number \(Ri\), defined as

\[
Ri = \frac{g}{\theta} \frac{d\theta}{dz} \left( \frac{d\bar{V}}{dz} \right)^2,
\]

where \(g\) is the acceleration due to gravity, \(\theta\) the potential temperature, \(\bar{V}\) the horizontal wind velocity and \(z\) vertical height. The most commonly accepted value for the onset of shear instability in dry air is \(Ri \leq 0.25\); once turbulence is established within a shear layer of dry air it should be sustained if \(Ri \leq 1.0\) (see, for example, Wallace and Hobbs, 1977). Based on the rawinsondes launched from Pt. Brown at 1558, 1721 and 1833 PST on 6 March 1979, the values of \(Ri\) over the depth of the frontal zone were 2.2, 3.5 and 2.4. Hence, in the case of dry air, instability would have been unlikely in the frontal zone.

Since the rainbands were embedded in an atmosphere near or at saturation and they were associated with precipitation, latent heat release would have affected the stability. Lalas and Einaudi (1974) defined a Richardson number for saturated air (\(Ri_{sat}\)) by considering the air to be composed of three fluids: dry air, water vapor and liquid water. Another method for estimating a Richardson number for saturated air is to replace the potential temperature in (3) by the equivalent potential temperature (\(\theta_e\)); we will indicate this Richardson number by \(Ri(\theta_e)\).

The values of \(Ri_{sat}\) and \(Ri(\theta_e)\) were calculated over 50 mb pressure increments for the frontal zone on 6 March 1979 using the rawinsonde soundings launched from Pt. Brown at 1558, 1721 and 1833 PST. The results are shown in Fig. 17. It can be seen that in all three soundings the Richardson numbers were generally <1.0, and often <0.25 in portions of the frontal zone. The average values of \(Ri(\theta_e)\) over the depths of the frontal zone for the three soundings shown in Fig. 17 are 0.8, 0.3 and 0.5. Even these

![Fig. 17. The Richardson numbers (\(Ri_{sat}\)—solid line, and \(Ri(\theta_e)\)—dashed–dotted line) in the shear zone (between dashed lines) above the CP-3 radar at Pt. Brown at (a) 1601–1604, (b) 1727–1734 and (c) 1836–1846 PST on 6 March 1979.](image-url)
values of $\text{Ri}(\theta)$ may be too large by 20–50% due to the possibility of underestimating $d\hat{V}/dz$ from the rawinsonde data (Bennetts and Hoskins, 1979). We conclude, therefore, that it is likely that conditions were suitable for shear instability within the frontal zone on 6 March 1979.

In the model of shear instability developed by Lallas and Einaudi (1976), horizontal wind velocities in the vicinity of the shear zone are represented by a hyperbolic-tangent profile in the presence of air density that decreases exponentially with height. Thus, the wind velocity profile in the vertical is given by:

$$V(z) = V_0 + \Delta V \tanh \left( \frac{Z - Z_0}{h} \right)$$

where $V$ is the horizontal wind speed along the direction of the shear vector, $Z$ the altitude above ground, $Z_0$ the altitude of the inflection point in the vertical profile of $V$, $V_0$ the value of $V$ at $Z_0$, $\Delta V$ is one-half the increase in $V$ across the shear layer, and $h$ one-half the width of the shear layer. This profile was fitted to the component of the horizontal wind perpendicular to the wavelike rainbands obtained from the rawinsondes launched from Pt. Brown on 6 March 1979 (Fig. 18). The best-fit parameters to the 1558 and 1721 PST soundings are, respectively: $V_0 = 10.5$ m s$^{-1}$, $\Delta V = 2.5$ m s$^{-1}$, $Z_0 = 1.32$ km, $h = 300$ m, and $V_0 = 7.0$ m s$^{-1}$, $\Delta V = 5.0$ m s$^{-1}$, $Z_0 = 2.26$ km, $h = 550$ m.

The best-fit parameters can be used to estimate the wavelength and phase velocities of the shear-induced modes. According to Lallas and Einaudi (1976), the first three modes are the most likely to be realized. The first mode disregards the lower boundary; the other two modes are due to resonance effects from the lower boundary. In our case, the wind profiles are

### Table 3. Comparison of the characteristics of the unstable shear-induced instability modes predicted by the theory of Lallas and Einaudi (1976) with the measurements obtained on the wavelike rainbands observed on 6 March 1979.

<table>
<thead>
<tr>
<th>Time (PST)</th>
<th>Theoretical mode</th>
<th>Domain of instability in wavelength (km)</th>
<th>Critical Richardson number</th>
<th>Rawinsonde data from Pt. Brown</th>
<th>Rawinsonde data from Quillayute (launched 1600 PST)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Predicted by theory</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1600-1605 (Rawinsonde launched from Pt. Brown 1558 PST)</td>
<td>I</td>
<td>2.6–4.2</td>
<td>0.25</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>6.3–14.5</td>
<td>0.14</td>
<td>11–12</td>
<td>19–21</td>
</tr>
<tr>
<td>1726–1736 (Rawinsonde launched from Pt. Brown 1721 PST)</td>
<td>I</td>
<td>4.1–7.7</td>
<td>0.25</td>
<td>7</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>11.5–26.6</td>
<td>0.14</td>
<td>8–9</td>
<td>—</td>
</tr>
<tr>
<td>b. Measurements</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12–19</td>
<td>See Fig. 17</td>
<td>~13*</td>
<td>22–23**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* An extension of the north–south gradient of phase speed to Pt. Brown (see Table 2).
** These phase velocities were estimated by tracking the intense rainbands located 70–140 km to the north of the radar.
such that only the first and third modes are of importance. The characteristics of these modes are shown in Table 3. It can be seen that the third mode agrees best with the observed wavelength of the wavelike rainbands. The observed movement of the rainbands and the theoretical predictions for the third mode are also in reasonable agreement.

Chimonas et al. (1980) pointed out that gravity waves generated by wind shear can attain sufficiently large amplitudes to induce condensation and that the latent heat released may reinforce the waves. This provides a possible explanation for the observed increased vigor of the wavelike rainbands with increasing "age."

5. Summary and conclusions

The characteristics of a train of wavelike rainbands in a cold-frontal zone have been investigated. The rainbands were oriented perpendicular to the cold front, and were 9–13 km apart, 3–6 km wide, 20–40 km long and had radar echo tops at 3–5 km. They moved at phase speeds of 17–23 m s⁻¹, with the older rainbands moving faster than the younger. The rainbands were associated with rather regular patterns of convergence and vertical air motions above the frontal zone. The wavelike rainbands were associated with the release of potential instability aloft in the form of generating cells.

The orientation of the wavelike rainbands was nearly perpendicular to the wind shear in the frontal zone; this suggests that they may have been caused by some type of shear instability. The spacing and movement of the rainbands was found to be generally consistent with the resonance mode theory for shear-induced instability proposed by Lalas and Einaudi (1976).

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APPENDIX

The Mean Reflectivity-Weighted Terminal Velocity of Precipitation Particles (\( \bar{V}_p \))

A common method for estimating \( \bar{V}_p \) is through the expression

\[
\bar{V}_p = c \varepsilon^2 \left( \frac{\rho_0}{\rho} \right)^{0.4},
\]

where \( c \) and \( d \) are empirical constants. For example, Herzegh and Hobbs (1981) used (A1) with \( d = 0.06 \) and \( c \) was allowed to vary with altitude.

In the present case we had sufficient radar and aircraft data to calculate \( \bar{V}_p \) directly using a technique similar to that described by Atlas et al. (1973) and Battan (1973). The technique we used is described below.

If particles are sufficiently small compared to the wavelength of the radar that Rayleigh scattering prevails, the radar reflectivity factor is given by

\[
Z = \int N_D D^6 dD,
\]

where \( N_D dD \) is the concentration of particles in the diameter range \( D \) to \( D + dD \). Also, if the sizes of the particles follow a Marshall–Palmer distribution,

\[
N_D = N_0 e^{-\lambda D}.
\]

The terminal velocity of precipitation particles \( V_p \) can be represented by

\[
V_p = a D^b,
\]

where \( a \) and \( b \) are empirical constants the values of which depend upon the nature of the particles (Locatelli and Hobbs, 1974).

The mean reflectivity-weighted terminal velocity \( \bar{V}_p \) of particles (that is, the mean Doppler velocity in still air of particles that would produce the observed \( Z \)) is given by

\[
\bar{V}_p = \frac{\int_0^\infty a D^b N_D D^6 dD}{\int_0^\infty N_D D^6 dD}.
\]

Substitution of (A3) into (A5) and integrating yields

\[
\bar{V}_p = \frac{a \Gamma(7 + b)}{\Delta \Gamma(7)},
\]

where \( \Gamma \) represents the gamma function. Substituting (A3) into (A2) and integrating yields

\[
\Lambda = \left[ \frac{N_0 \Gamma(7)}{Z} \right]^{1/7}.
\]

For ice particles above the radar bright band

\[
Z = Z_{\text{mea}} \left[ \frac{|K_{\text{ii}}|^2}{|K_{\text{ef}}|^2} \right],
\]

where \( Z_{\text{mea}} \) is the reflectivity factor measured by the radar, and \( |K_{\text{ii}}|^2 \) and \( |K_{\text{ef}}|^2 \) are the dielectric factors for ice particles and water droplets, respectively (Battan, 1973). Therefore,

\[
\Lambda = \left[ \frac{N_0 \Gamma(7) |K_{\text{ii}}|^2}{Z_{\text{mea}} |K_{\text{ef}}|^2} \right]^{1/7}.
\]

Substituting (A9) in (A6) and allowing for the effects of air density on particle fallspeeds (Foote and DuToit, 1969), we obtain
\[ V_p = \frac{a \Gamma(7+b)}{\Gamma(7)} \left[ \frac{z[K_m^2]}{N_0 \Gamma(7)[K_i^2]} \right]^{0.4} (\rho_0 \rho)^{0.4}, \]  

where \( Z \) is the reflectivity factor measured by radar \((\text{mm}^6 \text{m}^{-3})\), and \( \rho_0 \) and \( \rho \) are the air densities at the surface and at the sampling altitude over the radar, respectively.

REFERENCES


