On the Linear Theory of the Land and Sea Breeze

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ABSTRACT

Given that the earth's atmosphere may be idealized as a rotating, stratified fluid characterized by the Coriolis parameter f and the Brunt–Väisälä frequency N, and that the diurnal cycle of heating and cooling of the land relative to the sea acts as a stationary, oscillatory source of energy of frequency ω ($=2\pi$ day^{-1}), it follows from the linear theory of motion that where $f > \omega$ the atmospheric response is confined to within a distance $Nh(f^2 - \omega^2)^{-1/2}$ of the coastline, where h is the vertical scale of the heating. Where $f < \omega$, the atmospheric response is in the form of internal-inertial waves which extend to "infinity" along ray paths extending upward and outward from the coast. Near the ground, the horizontal extent of the sea breeze is given by the horizontal scale of the dominant wave mode, $Nh(\omega^2 - f^2)^{-1/2}$.

Although these concepts are familiar from the linear theory of motion in a rotating, stratified fluid, their relevance with respect to the interpretation of linear models of the land and sea breeze has not been emphasized in the literature. Hence, a critical historical review of extant linear models of the land and sea breeze is presented, and from these varied linear models, a simple model, which allows the above-described conclusions to be reached, is decocted.

1. Introduction

Among the oldest subjects studied by meteorologists is the phenomenon of the land and sea breeze. Although it's been mentioned in the literature since the 17th century (Halley, 1686), the earliest quantitative study, which also includes an extensive historical review, is that of Davis *et al.* (1889); at least two aspects of that report deserve note. First, the "convectional" theory of the land and sea breeze (see Section 2) therein finds its clearest articulation, and second, the typically observed veering of the onshore wind in midlatitudes in late afternoon is attributed to the rotation of the earth. Although there were some earlier attempts toward analytical modeling of certain aspects of the phenomenon [beginning with Jeffreys (1922)], the first analytical models to include the effects of the earth's rotation did not appear in the literature until 1947 [Haurwitz, 1947; Schmidt, 1947 (this paper includes a good bibliography of the analytical work up to that date)]. The number of works which use linear theory to study the sea breeze, with Coriolis effects included, is small enough to list here; after the above-cited works of Haurwitz and Schmidt, came Pierson (1950), Defant (1951), Haurwitz (1959), Walsh (1974) and Sun and Orlanski (1981). Except for the paper by Sun and Orlanski (1981), the question, "What determines the horizontal scale of the land and sea breeze circulation?" has received only scant attention. Especially under the conditions of constant stability, the linear theory does offer a definite simple answer, but as far as this writer can determine, it is not available in the literature. Relatedly, the fundamental difference which the linear theory predicts between the case where $f > \omega$ and that where $f < \omega$ in patterns of flow and phase relations among the variables has not been emphasized.

The argument develops roughly as follows. Imagine a coastal region under fair weather conditions in which the land is being heated and cooled with a frequency ω ($=2\pi$ day^{-1}). Without being concerned about the exact manner in which the heat is transferred to the air from the ground, it may be said that the atmosphere in the vicinity of the coast (because it is only the horizontal gradient of the temperature which produces circulation) is being forced at the frequency ω. If the atmosphere is idealized as a rotating, stratified fluid (with the local rotation rate and measure of stratification given by f, the Coriolis parameter, and N, the Brunt–Väisälä frequency, respectively), then the response of the atmosphere will depend critically on whether f is greater than or less than ω (e.g., Eckart, 1960, Chap. 10). When $f > \omega$ (latitudes greater than 30°) no waves are excited and the atmospheric response takes the form of an elliptically shaped pattern of flow in the vertical plane, centered on the coast, having an aspect ratio (vertical/horizontal scale) given by $(f^2 - \omega^2)^{1/2}N^{-1}$. The hor-

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izontal scale of the circulation, therefore, is $Nh(f^2 - \omega^2)^{-1/2}$, where, it will be argued, h is the vertical length scale associated with the heating. Walsh (1974, p. 2017) detected such a relationship in his model, but obtained no analytical expression. When $f < \omega$, the atmospheric response is in the form of internal-inertial waves. According to the current model, the sea breeze circulation extends to "infinity" along two ray paths, originating at the coast, which make an angle of approximately $(\omega^2 - f^2)^{1/2}N^{-1}$ to the horizontal. Near the ground, and in the vicinity of the coast, the horizontal scale of the flow is that which corresponds to the horizontal scale of the dominant wave mode which is $Nh(\omega^2 - f^2)^{-1/2}$. Thus, there is a fundamentally different behavior predicted by the linear theory depending on whether f is greater than or less than ω. Furthermore, the linear theory predicts that for $f > \omega$ the circulation in a plane normal to the coast is in phase with the heating, but is almost 180° out of phase with the heating when $f < \omega$. That the atmospheric response to the diurnal cycle of heating is in the form of internal-inertial waves and that the horizontal scale is determined from the frequency and vertical wavelength of the dominant wave mode are discussed by Sun and Orlanski (1981). But, because the focus of that study is on the role played by the instability due to the diurnal variation of the stratification (see Orlanski, 1973) which, according to the theory is most clearly manifest in the tropics, the stark difference in the response of the basic atmosphere (which I define as inviscid with N = constant) between the case where $f > \omega$ and that where $f < \omega$ is of secondary concern.

The plan of this report is to first review the above-cited papers concerning the linear theory of the land and sea breeze. Then, in Section 3 a simple linear theory for the land and sea breeze when $f > \omega$ is developed which emphasizes the fact that there is an internal radius of deformation which determines the scale of the motion. The linear version of the Bjerknes circulation theorem is derived and used to explain why the circulation in a plane normal to the coast is, in this case, exactly in phase with the heating. Section 4 treats the case where $f < \omega$ and emphasizes the wave-like nature of the flow pattern. The circulation theorem is used to explain why the circulation now is almost 180° out of phase with the diurnal cycle of heating and cooling. Section 5 shows how these results are mitigated by the inclusion of linear (Rayleigh) friction.

There will be little discussion here of real data or of how one may justify the use of linear theory to describe the land and sea breeze. Discussions of this nature may be found in the above-cited literature. The attention is focused here on the linear theory of the land and sea breeze itself with the goal of filling a few gaps in its historical development.

2. Review of the linear theory of the land and sea breeze

Consider a Cartesian system of coordinates in which the ground-plane is at $z = 0$ and the coastline is at $x = 0$ with land for $x > 0$ and sea for $x < 0$. It will be assumed the motion is independent of y, the distance along the coast. All of the linear models mentioned above may be viewed within the context of the shallow, anelastic approximation to the equations of motion (see Ogura and Phillips, 1962), *viz.*,

$$\frac{\partial u}{\partial t} - fv = -\frac{\partial \phi}{\partial x} + F^x, \tag{1}$$

$$\frac{\partial v}{\partial t} + fu = + F^y, \tag{2}$$

$$\frac{\partial w}{\partial t} - b = -\frac{\partial \phi}{\partial z} + F^z, \tag{3}$$

$$\frac{\partial b}{\partial t} + N^2 w = Q, \tag{4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{5}$$

where u, v and w are the components of the velocity vector in the x, y, and z directions, respectively; $\phi \equiv c_p\theta_0(P/P_0)^{R/c_p}$, where P, P_0, R and c_p are the pressure, ground pressure, gas constant for air and the specific heat at constant pressure, respectively. The Brunt–Väisälä frequency $N \equiv [(g/\theta_0)(\partial\bar{\theta}/\partial z)]^{1/2}$, where g is the acceleration due to gravity. The quantity b, termed the buoyancy, is $g\theta'/\theta_0$ and the potential temperature $\theta = \theta_0 + \bar{\theta}(z) + \theta'(x, z, t)$, where θ_0 is a constant reference potential temperature. The terms F^x, F^y, F^z and Q represent irreversible processes. By the appropriate choice of F^x, F^y, F^z and Q, any of the sets of basic equations used in the previously-cited studies (with the exception of Sun and Orlanski; see below) can be recovered. In all of these studies, it is required that

$$w(x, 0, t) = 0; \tag{6}$$

however, additional boundary conditions are assigned depending on the way in which the irreversible processes are represented (e.g., if $F^x \equiv \nu\partial^2 u/\partial z^2$, where ν is a positive constant, one may require $u(x, 0, t) = 0$).

To obtain an understanding of the motivation for the various approximations which have been used, consider the "convectional" theory of the land and sea breeze as stated by Buchan (1860, quoted by Davis *et al.*, 1889), which, remarkably, is still apposite. The cause of the land and sea breeze is that the land is "heated to a much greater degree than the sea during the day, by which the air resting on it being also heated, ascends, and the cooler air of the sea breeze flows in to supply its place. But during the night the temperature of the land and the air above it falls below that of the sea, and the air thus becoming heavier and denser flows over the sea as a land breeze." There are a number of ways these physical effects could be represented by the system of equations (1)–(5), and the ways the various authors have chosen to do so are described, in chronological order, below.

Haurwitz (1947) specifies the pressure gradient force normal to the coastline, $-\partial\phi/\partial x$, as a periodic function of time (of frequency ω), independent of both x and z [this is tantamount to setting $w = 0$ in (3)–(5) and requiring $\partial F^z/\partial x = \partial Q/\partial x = 0$] and he lets $(F^x, F^y) = -\alpha(u, v)$, where α is a positive constant. Thus, the process described by Buchan (1860) is represented by an assumed behavior for $\phi(x, z, t)$. With Coriolis and frictional effects included, Haurwitz obtains realistic wind hodographs (v vs. u with t as a parameter); however, no information concerning the scale of the motion can be obtained in this approach. Although Haurwitz includes Coriolis effects in his linear model, he neglects them in his application of the circulation theorem to the physical problem because, he says they "[do not] affect the argument" (p. 2). It is demonstrated below that the inclusion of Coriolis effects radically affects the argument when $f \geqslant \omega$.

Schmidt (1947) attacks the problem in almost the same way; instead of prescribing $-\partial\phi/\partial x$ as a function of t only, he prescribes it as a sinusoidal function of x and t that exponentially decays with height. [This is tantamount to setting $w = 0$ in (3) and (4), but not in (5), and then prescribing Q in a manner which will produce the assumed function $\phi(x, z, t)$.] Again, $(F^x, F^y) = -\alpha(u, v)$. Even in Schmidt's approach, nothing can be said concerning the spatial scale of motion except that it must be as it is prescribed through $\phi(x, z, t)$.

Pierson's (1950) analysis is along the same lines as Haurwitz's and Schmidt's; Pierson prescribes $-\partial\phi/\partial x$ as a function of z and t and takes $(F^x, F^y) = \nu(\partial^2 u/\partial z^2, \partial^2 v/\partial z^2)$ where ν is the eddy viscosity. Although there is, in Pierson's analysis, a mechanism which determines the vertical scale (viscous diffusion), nothing can be said of the scale of motion in the horizontal direction. It may be shown that because the pressure gradient is prescribed and not allowed to vary according to the changes affected by the motion field, the mechanism which allows an internal adjustment of the horizontal scale is eliminated.

Defant (1951) first solved for the flow in the x–z plane with $f \neq 0$; he took $F^x = F^y = F^z = 0$ and $Q = \nu\partial^2 b/\partial z^2$. The temperature is specified at the ground by the function

$$\theta(x, 0, t) = \theta_{\max} e^{i\omega t} \sin lx, \tag{7}$$

and the horizontal scale of the resulting circulation pattern is $2\pi/l$. Defant says that this solution, ". . . is based on the assumption that θ is proportional to $\sin lx$ [and that this] naturally yields a continuous chain of circulations. If a number of such circulations with variable l resulting from the Fourier series are superimposed on one another, we obtain a single circulation of definite horizontal extent." This is true, but he then goes on to add the *non sequitur,* "Thus, this extent becomes solely a function of the form of the land–water temperature difference." As will be shown below, there is a definite internal radius of deformation, in equations similar to those considered by Defant, which determines the horizontal extent of the motion. That such an internal scale should exist was briefly mentioned by Haurwitz (1959, pp. 11–12), but no expression for it was derived.

Walsh (1974) takes $(F^x, F^y, F^z, Q) = \nu\nabla^2(u, v, w, b)$, where ∇^2 is the Laplacian operator in the x–z plane, and requires that u, v, and w vanish at $z = 0$ and remain bounded as $|x|, z \rightarrow \infty$. The temperature is specified at the ground by

$$\theta(x, 0, t) = \sin\omega t \begin{cases} \theta_{\max}, & x > 0 \\ -\theta_{\max}, & x < 0. \end{cases} \tag{8}$$

Walsh solved the linear system of equations through an eigenfunction expansion technique; among the results obtained is the behavior of the solution as f and N are varied. Walsh concluded (p. 2017) that the result ". . . suggests an analogy between the circulation's horizontal extent and the Rossby deformation radius NH/f, where H is the depth of the disturbance." Further, Walsh's Fig. 3, which contains a graph of $u(x, z, t)$ vs. x at a height far above the ground for $f/\omega = 1.5$, 1.0 and 0.5, indicates an oscillatory behavior when $f/\omega = 0.5$, a behavior of a more damped nature when $f/\omega = 1.5$, and a behavior which exhibits a very large radius of influence when $f/\omega = 1$.

Sun and Orlanski (1981) take F^x, F^y, F^z and Q to represent Fickian diffusion with different coefficients of eddy diffusivity for the horizontal and vertical terms which have a prescribed variation with x, z and t. They take $N^2 = N^2(x, z, t)$ and thus have the term $ug\theta_0^{-1}\partial\bar{\theta}/\partial x$ which is absent from (4). The resulting equations are solved numerically. As mentioned in the Introduction, the emphasis of that study is on the effects produced by the variation with time of N. Although this aspect of the problem is outside the purview of the present work, there is one result of that study that is of direct concern here. It is recognized by Sun and Orlanski (1981) that the basic response of the atmosphere to the diurnal heating and cooling in the tropics is in the form of internal–inertial waves whose horizontal scale may be identified with the horizontal scale of the sea breeze. Because the emphasis of the present study is on the essential difference between the type of flow, predicted by linear theory, when $f > \omega$ (no waves) and when $f < \omega$ (waves), there will, necessarily, be some overlap with Sun and Orlanski's work. However, because the situation examined here is much simpler [N = constant, $F^x = F^y = F^z = 0$, $Q = Q(x, z, t)$], one should view the part of this report which concerns the wave-like response ($f < \omega$) as complementary to theirs.

If one reasons (along with Defant, 1951) that the most essential function of irreversible processes is to transfer heat from the ground to the air above it (recall Buchan's description), then one should let $F^x = F^y = F^z = 0$ but have $Q \neq 0$. However, if one considers that the actual process by which heat is transferred to the air from the ground to be one of such great complexity that, in a first approximation, one may let $Q = Q(x, z, t)$, a known function, then a simple problem results. Under these conditions, Eqs. (1)–(5) may be combined into a single equation for the streamfunction ψ ($u \equiv \partial\psi/\partial z$ and $w \equiv -\partial\psi/\partial x$), *viz.*,

$$\left(\frac{\partial^2}{\partial t^2} + N^2\right)\frac{\partial^2\psi}{\partial x^2} + \left(\frac{\partial^2}{\partial t^2} + f^2\right)\frac{\partial^2\psi}{\partial z^2} = -\frac{\partial Q}{\partial x}. \quad (9)$$

It is assumed that $Q \approx e^{-i\omega t}$, so that $\psi \approx e^{-i\omega t}$ and (9) becomes

$$(N^2 - \omega^2)\frac{\partial^2\psi}{\partial x^2} + (f^2 - \omega^2)\frac{\partial^2\psi}{\partial z^2} = -\frac{\partial Q}{\partial x}. \quad (10)$$

Because $N \approx 10^{-2}$ s^{-1} and $\omega \simeq 0.73 \times 10^{-4}$ s^{-1}, $N^2 \gg \omega^2$. Eq. (10) may then be written as

$$N^2\frac{\partial^2\psi}{\partial x^2} + (f^2 - \omega^2)\frac{\partial^2\psi}{\partial z^2} = -\frac{\partial Q}{\partial x}, \quad (11)$$

[this is tantamount to having made the hydrostatic approximation in (3)]. Three cases immediately present themselves:

1) $f > \omega$: Eq. (11) is elliptic; the response to the forcing is confined to its neighborhood.
2) $f < \omega$: Eq. (11) is hyperbolic; the response to the forcing will be in the form of internal–inertial waves propagating away from the region of forcing.
3) $f = \omega$: Eq. (11) is singular and the effects of friction must be included.

The emphasis in this study will be on cases 1) and 2); the behavior at $f = \omega$ will be examined only briefly in conjunction with the discussion on the effects of friction in Section 5.

3. The case where $f > \omega$ (latitudes greater than 30°)

Suppose $N^2(z)$ = constant so that the non-dimensional coordinates,

$$\zeta \equiv \frac{z}{h}, \quad \xi \equiv \frac{(f^2 - \omega^2)^{1/2}}{N}\frac{x}{h}, \quad \tau \equiv \omega t, \quad (12)$$

can be defined. The non-dimensional dependent variables are defined as

$$\left.\begin{array}{l}\tilde\psi \equiv \psi h^{-2}\omega^{-1}, \quad (\tilde u, \tilde v, \tilde w) \equiv (u, v, w)h^{-1}\omega^{-1} \\ \tilde b \equiv bh^{-1}\omega^{-3}; \quad \tilde\phi \equiv \phi h^{-2}\omega^{-2}, \quad \tilde Q \equiv Qh^{-1}\omega^{-3}\end{array}\right\}, \quad (13)$$

where h is a scale of length yet to be specified. With these definitions, (11) may be written as

$$\frac{\partial^2\tilde\psi}{\partial\xi^2} + \frac{\partial^2\tilde\psi}{\partial\zeta^2} = -\beta\frac{\partial\tilde Q}{\partial\xi}, \quad (14)$$

where $\beta \equiv \omega^2 N^{-1}(f^2 - \omega^2)^{-1/2}$. Eq. (14) is Poisson's equation, the solution of which may be written, away from any boundaries as (see Morse and Feshbach, 1953, Chap. 7)

$$\tilde\psi(\xi, \zeta, \tau) = -\frac{\beta\sin\tau}{4\pi}\iint \ln[(\xi - \xi')^2 + (\zeta - \zeta')^2] \times \frac{\partial\tilde H}{\partial\xi'}(\xi', \zeta')d\xi' d\zeta', \quad (15)$$

where $\tilde Q(\xi, \zeta, \tau)$ has been set equal to $\tilde H(\xi, \zeta)\sin\tau$. By letting $\tilde Q = \tilde H\sin\tau$, it is assumed the diabatic heating at any point is in phase with that at any other point. This parameterization is predicated on the rapidity with which heat is transferred (presumably by turbulence) in comparison to the diurnal time period.

Consider the response of the fluid to a point-source of $\partial\tilde H/\partial\xi$ located at the origin (which, for the purpose of this example, is taken to be far from the boundary), *viz.*,

$$\frac{\partial\tilde H}{\partial\xi} = \delta(\xi)\delta(\zeta), \quad (16)$$

where $\delta(x)$ is the Dirac delta function. Then (15) becomes

$$\tilde\psi(\xi, \zeta, \tau) = -\frac{\beta\sin\tau}{4\pi}\ln(\xi^2 + \zeta^2). \quad (17)$$

Thus, the streamfunction is constant on circles of radius $(\xi^2 + \zeta^2)^{1/2}$ in this system of coordinates. However, in physical space, the streamfunction is

$$\psi(x, z, t) = -\frac{h^2\omega\beta}{4\pi} \times \sin\omega t \ln\left[\left(\frac{f^2 - \omega^2}{N^2}\right)\frac{x^2}{h^2} + \frac{z^2}{h^2}\right] \quad (18)$$

and so ψ is constant on ellipses whose ratio of the length of the major axis to that of the minor axis is given by

$$\frac{\lambda_H}{\lambda_V} = \frac{N}{(f^2 - \omega^2)^{1/2}}, \quad (19)$$

where λ_H and λ_V represent the horizontal and vertical scale of the motion, respectively. Using the values for N and ω given above and calculating f [$=2\omega$

$\times$ sin(latitude)] at 45° latitude (which is 10^{-4} s^{-1}) gives $\lambda_H/\lambda_V \approx 146$—indicating a very flat ellipse. Two other points can be made from this simple example. From Eq. (18) it follows that the horizontal and vertical components of the velocity field are proportional to $N^{-1}(f^2 - \omega^2)^{-1/2}$ and $N^{-3}(f^2 - \omega^2)^{1/2}$, respectively. Thus, the intensity of the flow is inversely proportional to N. The relative weakness of the nighttime land breeze as compared to the daytime sea breeze has been attributed to the greater stability in the former case (Walsh, 1974; Mak and Walsh, 1976). The second point is that as $f \to \omega$, the horizontal length scale and the horizonal velocity become unbounded and the vertical velocity goes to zero. This behavior, mitigated by the effect of friction, is evident in Walsh's Fig. 3b.

The solution to (14) in the presence of a solid boundary at $\zeta = 0$ is obtained by the method of images (Morse and Feshbach, p. 812); it is

$$\tilde{\psi}(\xi, \zeta, \tau) = -\frac{\beta \sin\tau}{4\pi} \int_0^\infty \int_{-\infty}^\infty \ln\left[\frac{(\xi - \xi')^2 + (\zeta - \zeta')^2}{(\xi - \xi')^2 + (\zeta + \zeta')^2}\right] \times \frac{\partial \tilde{H}}{\partial \xi'}(\xi', \zeta') d\xi' d\zeta'. \quad (20)$$

An example of the type of flow obtained for a smooth heating profile is now given. Suppose that

$$H(x, z) = A\left(\frac{\pi}{2} + \tan^{-1}\frac{x}{x_0}\right)e^{-z/z_0}, \quad (21)$$

where the scale of the land–sea contrast in heating is denoted by x_0 and the vertical scale of the heating is denoted z_0. Because both x_0 and z_0 are specified externally, it would be quite a coincidence if they satisfied (19); in general they do not. Hence, there are two possibilities for the emergence of a third, internal, length scale from the solution of (14). Either $\lambda_H = x_0$, and so from (19) $\lambda_v = (f^2 - \omega^2)^{1/2}N^{-1}x_0$ or, $\lambda_v = z_0$, and so, $\lambda_H = Nz_0(f^2 - \omega^2)^{-1/2}$. The actual scale the solution will exhibit will be the greater of either the internal or the external scale. So, considering that $x_0 \approx 10^3$ m and $z_0 \approx 500$ m leads, in the first case, to $\lambda_v \simeq 6.8$ m which is much less than z_0. However, in the second case, $\lambda_H \approx 7.3 \times 10^4$ m which is much greater than x_0. Therefore it is the horizontal scale which is set internally. Since the external scale z_0 will determine the vertical scale of the solution, it is natural to set $h = z_0$. Then, Eq. (21) and its derivative become, in non-dimensional terms,

$$\tilde{H}(\xi, \zeta) = \tilde{A}\left(\frac{\pi}{2} + \tan^{-1}\frac{\xi}{\xi_0}\right)e^{-\zeta}, \quad (22)$$

$$\frac{\partial \tilde{H}}{\partial \xi}(\xi, \zeta) = \tilde{A}\frac{\xi_0}{\xi^2 + \xi_0^2}e^{-\zeta}, \quad (23)$$

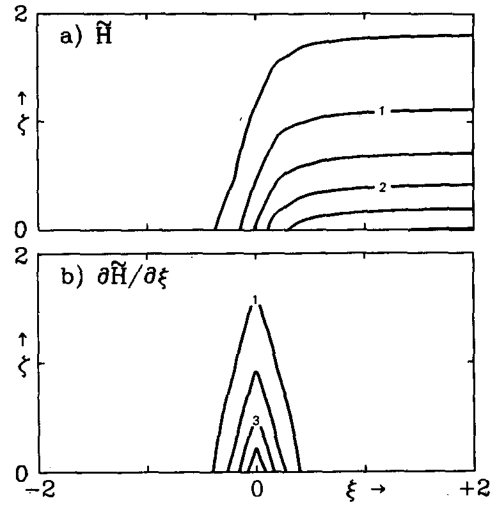

FIG. 1. (a) The heating function $\tilde{H}(\xi, \zeta)$ given by (22) with $\tilde{A} = 1$ and $\xi_0 = 0.2$; and (b) $\partial\tilde{H}(\xi, \zeta)/\partial\xi$ given by (23).

where $\tilde{A} \equiv Ah^{-1}\omega^{-3}$; Fig. 1 displays $\tilde{H}$ and $\partial\tilde{H}/\partial\xi$ for $\tilde{A} = 1$ and $\xi_0 = 0.2$. Substituting (23) into (20) gives

$$\tilde{\psi}(\xi, \zeta, \tau) = -\frac{\beta\xi_0\tilde{A}}{4\pi}\sin\tau \int\int \ln\left[\frac{(\xi - \xi')^2 + (\zeta - \zeta')^2}{(\xi - \xi')^2 + (\zeta + \zeta')^2}\right] \times \frac{e^{-\zeta'}}{\xi'^2 + \xi_0^2} d\xi' d\zeta'. \quad (24)$$

The double integration in Eq. (24) is performed numerically and the results for $\tilde{\psi}(\xi, \zeta, \pi/2)$ with $\beta = 7.27 \times 10^{-3}$, $\tilde{A} = 10^3$ and $\xi_0 = 0.2$ are displayed in Fig. 2. Also displayed in Fig. 2 are the horizontal velocity $\tilde{u}$ normal to the coast and the vertical velocity $\tilde{w}$ at $\tau = \pi/2$. It is evident from Fig. 2b that the horizontal extent of the flow is of order unity, or, in physical space, of order $Nh(f^2 - \omega^2)^{-1/2}$.

To explore a little further the relative importance of the external scale x_0 and the internal scale $Nh(f^2 - \omega^2)^{-1/2}$, the limiting case where $x_0 \to 0$ is considered. The ξ-dependence of the forcing in (23) becomes, in the limit as $\xi_0 \to 0$, $\pi\delta(\xi)$ [in the distributional sense, see Stakgold (1979, p. 107)]. The solution for the horizontal velocity at $\zeta = 0$, in this limit, is

$$\tilde{u}(\xi, 0, \tau) = \beta\tilde{A}\sin\tau\{-\mathrm{Ci}(\xi)\cos\xi + [\tfrac{1}{2}\pi - \mathrm{Si}(\xi)]\sin\xi\}, \quad (25)$$

where Ci(x) and Si(x) are the cosine and sine integrals of x, respectively (e.g., Abramowitz and Stegun, 1970, p. 231). Fig. 3 contains a graph of $\tilde{u}(\xi, 0, \pi/2)$ vs ξ from (25) and from Fig. 2b. Even though the solution, in the limit as $\xi_0 \to 0$, is singular at $\xi = 0$, it is, away from the vicinity of $\xi = 0$, nearly identical to the solution with $\xi_0 = 0.2$. It may be concluded that the strength of the wind near $\xi = 0$ depends on $\beta\tilde{A}$ and

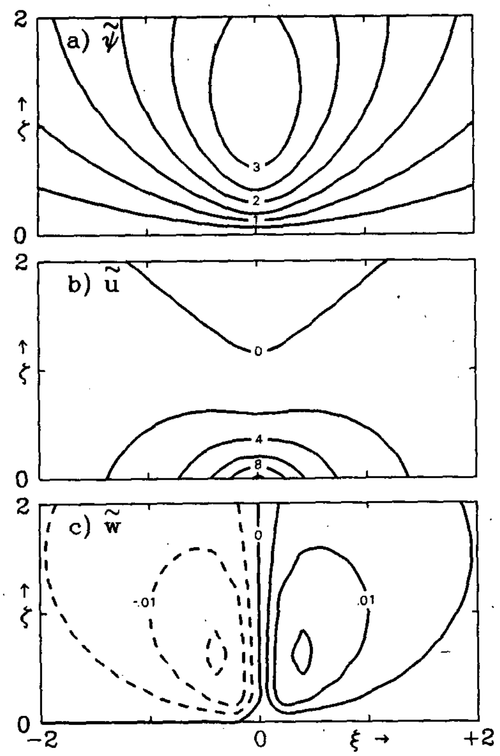

FIG. 2. (a) The streamfunction $\tilde{\psi}(\xi, \zeta)$ at $\tau = \pi/2$ for $f > \omega$ [Eq. (24)]; (b), as in (a) except for $\tilde{u}(\xi, \zeta)$; (c), as in (a) except for $\tilde{w}(\xi, \zeta)$. Dashed lines indicate negative values in all figures.

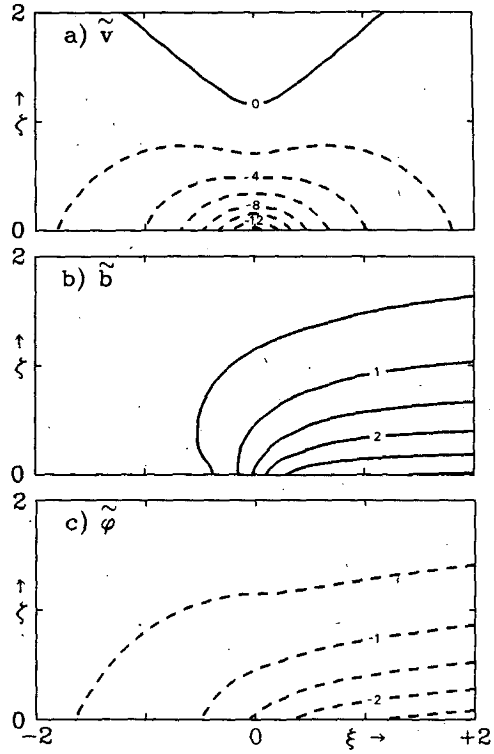

FIG. 4. (a) The velocity parallel to the coast $\tilde{v}(\xi, \zeta)$ at $\tau = \pi$; (b) the buoyancy ($\div 10^3$), $\tilde{b}(\xi, \zeta)$ at $\tau = \pi$; (c) the pressure ($\div 10^3$), $\tilde{\phi}(\xi, \zeta)$ at $\tau = \pi$.

the external scale x_0, but the strength of the wind away from the coast depends only on $\beta\tilde{A}$ and its dependence on x scales with the internal length, $Nh(f^2 - \omega^2)^{-1/2}$.

The velocity $\tilde{v}$ parallel to the coast, the buoyancy $\tilde{b}$ and pressure $\tilde{\phi}$ at $\tau = \pi$ are displayed in Fig. 4. This solution (displayed in Figs. 2 and 4) is similar in its basic features to those obtained in numerical models of the land and sea breeze (e.g., Fig. 1 of Anthes, 1978); there is a flat elliptical circulation in the x–z plane with an aspect ratio of $\sim 10^{-2}$; the wind near the ground veers with time and blows parallel to the coast, the air over the land is warmer, and the pressure is lower by the time the heating has ended.

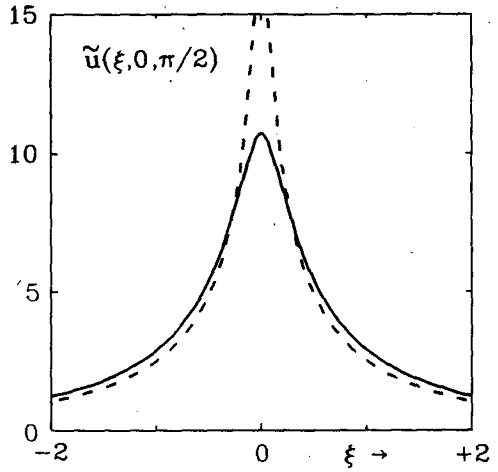

FIG. 3. $\tilde{u}(\xi, 0, \pi/2)$ vs ξ for $\xi_0 = 0$ [Eq. (25)], dashed line, and $\xi_0 = 0.2$, solid line.

The time dependence of this solution [Eq. (20)] is more clearly understood by considering the linear version of the Bjerknes circulation theorem (1898, 1901). The circulation C is defined as

$$C \equiv \int_{x=-\infty}^{\infty} [u(x, 0, t) - u(x, \infty, t)]dx. \qquad (26)$$

(The vertical branches may be neglected in the hydrostatic approximation.)

The equation for C is obtained from (1) and (3) as

$$\frac{\partial C}{\partial t} = f \int_{x=-\infty}^{\infty} [v(x, 0, t) - v(x, \infty, t)]dx + \int_{z=0}^{\infty} [b(\infty, z, t) - b(-\infty, z, t)]dz. \qquad (27)$$

Now taking $\partial/\partial t$ of (27) and substituting the expression for $\partial v/\partial t$ and $\partial b/\partial t$ from (2) and (3), respectively, yields

$$\left(\frac{\partial^2}{\partial t^2}+f^2\right)C=\int_{z=0}^{\infty}\{[Q(\infty,z,t)-N^2w(\infty,z,t)]$$
$$-[Q(-\infty,z,t)-N^2w(-\infty,z,t)]\}dz. \quad (28)$$

In the present case, $|w(x, z, t)| \rightarrow 0$ as $|x| \rightarrow \infty$, hence the terms which involve $w(\pm\infty, z, t)$ in (28) vanish. From (28) it follows that, if $Q(x, z, t) = H(x, z) \sin\omega t$, then

$$C=\frac{\sin\omega t}{(f^2-\omega^2)}\int_{z=0}^{\infty}[H(\infty,z)-H(-\infty,z)]dz. \quad (29)$$

Thus, because $f > \omega$, C and Q are in phase.

Several features of the circulation [Eq. (29)] deserve note.

1) The circulation is independent of the scale of the land–sea contrast x_0.

2) The circulation is independent of the stratification N. Although it was demonstrated above that the strength of the horizontal velocity is inversely proportional to N, its horizontal scale is proportional to N; hence, the horizontal integral of the horizontal velocity, the circulation, is independent of N.

3) The circulation is inversely proportional to $(f^2 - \omega^2)$. Thus as $f \rightarrow \omega$, *ceteris paribus,* the amplitude of the circulation increases.

The basic balances which govern the behavior of C can be better understood by considering the individual terms in (27) in a general way.

We consider the first term on the right-hand side of (27). The definition

$$F(t)\equiv f\int_{x=-\infty}^{\infty}[v(x,0,t)-v(x,\infty,t)]dx \quad (30)$$

enables (2) (multiplied by f and integrated from $x = -\infty$ to ∞) to be written as

$$\frac{\partial F}{\partial t}=-f^2C. \quad (31)$$

Substituting (29) into (31) then yields

$$F=\frac{f^2}{f^2-\omega^2}\times\frac{\cos\omega t}{\omega}\int_{z=0}^{\infty}[H(\infty,z)-H(-\infty,z)]dz. \quad (32)$$

The second term on the right-hand side of (27) may be evaluated in a similar manner. The definition

$$B(t)\equiv\int_{z=0}^{\infty}[b(\infty,z,t)-b(-\infty,z,t)]dz, \quad (33)$$

together with (4), after some manipulation, yields

$$B(t)=-\frac{\cos\omega t}{\omega}\int_{z=0}^{\infty}[H(\infty,z)-H(-\infty,z)]dz. \quad (34)$$

From (32) and (34), observe that $|F| > |B|$. Hence the circulation is dominated by the Coriolis effect.

Consider now, in physical terms, the behavior of C, F and B vs t. At sunrise ($t = 0$), $B < 0$ because of the cooling which occurred during the night past; the land breeze, initiated at the sunset of the previous day ($\omega t = -\pi$), has been entirely deflected by the Coriolis effect and blows toward the north. Because this northward wind is continually acted upon by the Coriolis effect, and the effect is not sufficiently counteracted by the high pressure (due to negative buoyancy) over land ($F > B$; $C_t > 0$), the wind turns toward the land. By noon ($\omega t = \pi/2$), the buoyancy has increased from its negative value to zero ($B = 0$) while the onshore breeze has intensified ($\partial C/\partial t = 0$; C is maximum) and the Coriolis effect on the circulation is minimized because $v = 0(F = 0)$. However, as the afternoon progresses the onshore wind is deflected to the south and, even though the temperature contrast is acting to increase the circulation ($B > 0$), the dominating Coriolis effect ($-F > B$) acts to decrease C to zero by sunset ($\omega t = \pi$). Anthes (1978, Fig. 2) evaluated the terms in the nonlinear version of (27), for the numerical simulation alluded to above, and found the Coriolis effect is a significant effect which decelerates the circulation in the late afternoon.

It is demonstrated in the following section that these phase relationships, and concomitantly, the structure of the circulation in the ξ–ζ plane, are completely different when $f < \omega$.

4. The case where $f < \omega$ (latitudes less than 30°)

If the variable ξ and the parameter β are redefined as

$$\xi\equiv\frac{(\omega^2-f^2)^{1/2}}{N}\frac{x}{h} \quad\text{and}\quad \beta\equiv\frac{\omega^2}{(\omega^2-f^2)^{1/2}N}, \quad (35)$$

then (11) can be written as

$$\frac{\partial^2\tilde{\psi}}{\partial\xi^2}-\frac{\partial^2\tilde{\psi}}{\partial\zeta^2}=-\beta\frac{\partial\tilde{Q}}{\partial\xi}. \quad (36)$$

The solution which satisfies (37) and the condition that energy propagate away from the source at infinity (the radiation condition) is

$$\tilde{\psi}(\xi,\zeta,\tau)=\mathrm{Re}\,\mathrm{FT}^{-1}\left[e^{-i\,\mathrm{sgn}(k)k\zeta}\int_0^{\zeta}\mathrm{FT}\left(\frac{\partial\tilde{Q}}{\partial\xi}\right)\sin k\zeta' d\zeta'\right.$$
$$\left.+\sin k\zeta\int_{\zeta}^{\infty}\mathrm{FT}\left(\frac{\partial\tilde{Q}}{\partial\xi}\right)e^{-i\,\mathrm{sgn}(k)k\zeta'}d\zeta'\right], \quad (37)$$

where the symbols FT and FT^{-1} denote the Fourier transform of a function and its inverse, and k is the Fourier transform parameter (e.g., Carrier *et al.*, 1966, p. 303); the shorthand notation Re denotes the real part of. That (37) is the solution to (36) may be verified by direct substitution. Instead of performing an analysis parallel to that in the preceding section [which would consist of examining first the response predicted by (36) for point source in an infinite fluid], I will, for the sake of brevity, proceed directly to the examination of the solution (37) where again $\tilde{Q}(\xi, \zeta, \tau) = \tilde{H}(\xi, \zeta) \sin\tau$ and $\tilde{H}$ is given by (23). It is

$$\tilde{\psi}(\xi, \zeta, \tau) = -\beta\tilde{A} \int_0^\infty \frac{\cos k\xi e^{-\xi_0 k}}{1 + k^2} dk \times \{\sin(k\zeta + \tau) - e^{-\zeta} \sin\tau\}. \quad (38)$$

Fig. 5 displays $\tilde{\psi}(\xi, \zeta, \tau)$ for $\tau = 0, \pi/2$ and π for $\xi_0 = 0.2$. Although the heating function is identical to that which produces the flow pattern displayed in Fig. 2, the structure of the present response is radically different. Not only is the structure in the (ξ–ζ) plane very different, but it is also evident that $\tilde{\psi}$ is no longer in phase with $\tilde{Q}$. In fact, it will be demonstrated that the circulation is almost 180° out of phase with $\tilde{Q}$. But first, the structure of the solution in the (ξ–ζ) plane is examined.

The most prominent feature of the solution is the concentration of the motion along the lines given by the equation $\xi = \pm\zeta$. That these lines correspond to "rays" of internal–inertial waves can be seen as follows. Away from the region of forcing, the response of the fluid is of the form of plane waves, that is, of the form $\exp[i(k_x x + k_z z - \sigma t)]$, where σ, k_x and k_z are the frequency, and horizontal, and vertical wavenumbers, respectively. They are related to each other by the dispersion formula (for hydrostatic internal–inertial waves),

$$\sigma^2 = f^2 + N^2 k_x^2/k_z^2. \quad (39)$$

Energy is carried away from the source along ray paths, defined by the equations (e.g., Eckart, 1960, Chap. 10)

$$\frac{dx}{dt} = \frac{\partial\sigma}{\partial k_x} \quad \text{and} \quad \frac{dz}{dt} = \frac{\partial\sigma}{\partial k_z}. \quad (40)$$

In the present case N is a constant and these equations may be integrated to obtain the equation of the ray path, emanating from the origin, *viz.*,

$$x = \frac{\pm Nz}{(\sigma^2 - f^2)^{1/2}}, \quad (41)$$

or, if $\sigma = \omega$, in terms of the non-dimensional independent variables,

$$\xi = \pm\zeta. \quad (42)$$

Because the forcing is symmetric in x (see Fig. 1b) the energy will be found along the two lines $\xi = \pm\zeta$ as is evident in Fig. 5.

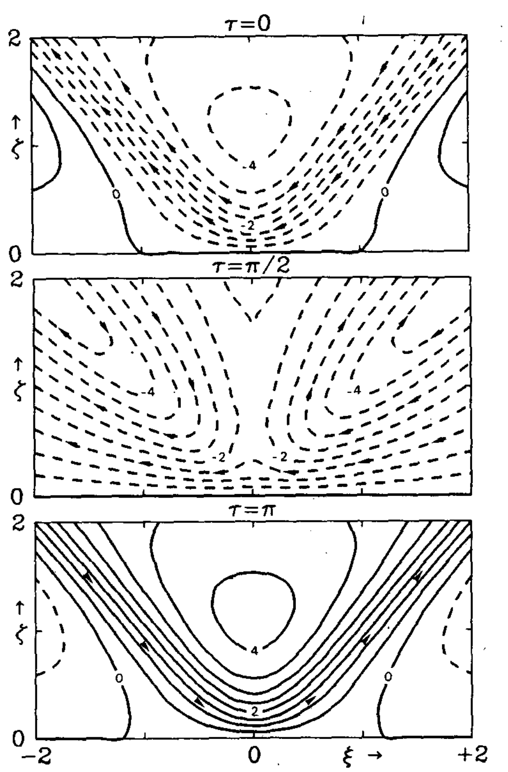

FIG. 5. The streamfunction $\tilde{\psi}(\xi, \zeta, \tau)$ at $\tau = 0, \pi/2$ and π for $f < \omega$ for the heating function displayed in Fig. 1.

Fig. 5 indicates that the amplitude of the motion is confined to a horizontal distance of order unity away from any point along the ray path, or in physical space a distance of order $Nh(\omega^2 - f^2)^{-1/2}$. Near the ground the ray paths intersect and, as is evident in the pattern for $\tilde{\psi}$, there is a region of influence extending a distance $Nh(\omega^2 - f^2)^{-1/2}$ from either side of the coast. These ideas are illustrated nicely by the experiments of Mowbray and Rarity (1967).

The study by Nitta and Esbensen (1974) of diurnal wind variations over the tropical ocean was discussed by Sun and Orlanski (1981). The measurements, although made several hundred kilometers away from the coast, indicate a strong influence of the land–sea breeze. Moreover, the amplitude of the disturbance was found to be maximum at $z \simeq 2500$ m and would thus be consistent with the idea that the response of the tropical atmosphere should be concentrated along ray paths extending upward and outward from the coast at a shallow angle.

Examination of the $\tilde{\psi}$ fields of Fig. 5 reveals the perverse result that the land breeze persists for most of the daytime ($0 \leqslant \tau \leqslant \pi$)! To understand why this is so we again consider the linearized circulation theorem (27). Now, because the motion field extends far from the source region, the adiabatic heating is not exactly zero as $|x| \to \infty$ as it is when $f > \omega$. However, it may be shown to be small compared with the diabatic heating. Thus, (29) shows that the circulation is 180° out of phase with the heating when $f < \omega$. The reason this happens is that in the circulation equation the Coriolis effect [Eq. (32)] is now in phase (instead of 180° out of phase as it is when $f > \omega$) with the buoyancy term. Therefore (27) dictates that the circulation lag the buoyancy by 90°. Eq. (4) demands that buoyancy lag the diabatic heating by almost 90° (the effect of the adiabatic heating induces a small lead), hence the circulation lags the heating by almost 180°!

Obviously, these results are counter-intuitive; however, they are the logical outcome of the model as formulated. That the results for the case where $f > \omega$ are not nearly so strange as those for $f < \omega$ can be attributed to the fact that rotation affects the motion in a manner similar to friction (which must always be acting to some degree) when $f > \omega$. When $f < \omega$ friction must be explicitly included to bring the predicted behavior more in accord with common experience. But it should be noted that Sun and Orlanski (1981) have adduced a number of observational studies in the tropics (which indicate that the most marked period of convective activity occurs in the late afternoon or early evening with largest amounts of rainfall occurring shortly before midnight) in support of the idea that wave dynamics plays an important role.

5. Some effects of linear friction

It is curious that, although Haurwitz's (1947) paper was the first to include the effects of the earth's rotation in an analytical model, Coriolis effects were neglected in his discussion of the application of the circulation theorem to the land and sea breeze. As mentioned in Section 2, Haurwitz (1947) believed Coriolis effects would not affect his argument concerning the circulation. However the 180° phase difference between the case where $f > \omega$ and that where $f < \omega$ is evident in Haurwitz's solution [see his Eqs. (20a) and (21a)]. Haurwitz reasoned that frictional effects must be important because the circulation theorem without friction predicts the circulation to increase as long as the land is warmer than the sea, and thus the maximum onshore wind should occur near sunset instead of at the observed mid-afternoon time. As noted by Schmidt (1947) the inclusion of friction was also advocated by Godske (1934) as a way of reconciling with the data V. Bjerknes *et al.* (1933) used in their application of the Bjerknes circulation theorem (1898) to the sea-breeze phenomenon. Thus, the early workers were concerned because the circulation theorem predicts the circulation wave will lag the temperature wave, and so, occur too late in the day. Understandably, frictional effects were invoked to explain the observation that the circulation and temperature wave are nearly in phase. However, according to the current model, comparison of (29) and (34) indicates the circulation wave *leads* the temperature wave when $f > \omega$. Although the current model is highly idealized, it does demonstrate in a simple way how the earth's rotation can have an effect similar to that of friction on the circulation in that both can prevent the circulation wave from lagging the temperature wave.

A few elementary effects of friction can be investigated [following Haurwitz (1947) and Schmidt (1947)] by letting $(F^x, F^y) = -\alpha(u, v)$, $F^z = 0$ (consistent with the continued use of the hydrostatic approximation) and $Q = H(x, z)\sin(\omega t) - \alpha b$. Because the benefits of a full analysis may not be greater than the costs, my remarks are confined to the behavior of the circulation as a function of time.

Eq. (28) becomes, neglecting adiabatic heating and including linear friction,

$$\left[\left(\frac{\partial}{\partial t} + \alpha\right)^2 + f^2\right]C = \sin\omega t \int_0^\infty [H(\infty, z) - H(-\infty, z)]dz. \quad (43)$$

It follows that

$$C = \frac{\int_0^\infty [H(\infty, z) - H(-\infty, z)]dz}{[(f^2 + \alpha^2 - \omega^2)^2 + 4\alpha^2\omega^2]^{1/2}} \times \sin(\omega t - \chi_1), \quad (44)$$

$$\text{where } \chi_1 = \begin{cases} \tan^{-1}\dfrac{2\alpha\omega}{f^2 + \alpha^2 - \omega^2}, \\ \quad \text{if } f^2 + \alpha^2 - \omega^2 > 0 \\ \tan^{-1}\dfrac{2\alpha\omega}{f^2 + \alpha^2 - \omega^2} + \pi, \\ \quad \text{if } f^2 + \alpha^2 - \omega^2 < 0. \end{cases} \quad (45)$$

Similarly,

$$B = \frac{\int_0^\infty [H(\infty, z) - H(-\infty, z)]dz}{\alpha^2 + \omega^2} \times \sin(\omega t - \chi_2), \quad (46)$$

where

$$\chi_2 = \tan^{-1}\frac{\omega}{\alpha}. \quad (47)$$

The quantities χ_1 and χ_2 are the phase lags of C and B with regard to Q, respectively; the quantity $(\chi_1 - \chi_2)$ is the phase lag of C with regard to B. From (45) and (47) for $\alpha \to 0$, the following may be inferred. For $f < \omega$, $\chi_1 = 180°$, $\chi_2 = 90°$ and thus $\chi_1 - \chi_2 = 90°$. This is the case envisioned by the early work-

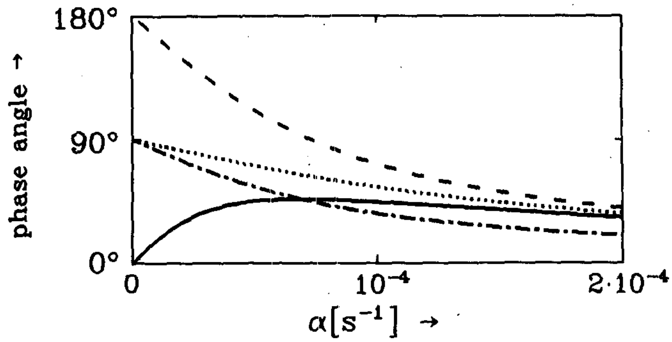

FIG. 6. The phase lag of the circulation relative to the heating as a function of the linear friction parameter α [Eq. (45)], for $f = 0$ (dashed line), $f = \omega$ (dotted line) and $f = 10^{-4}$ s^{-1} (solid line). The phase lag of the temperature wave with respect to the heating is given by the dashed-dotted line [Eq. (47)].

ers; the circulation wave lags the temperature wave by 90°. For $f > \omega$, $\chi_1 = 0°$, $\chi_2 = 90°$ and thus $\chi_1 - \chi_2 = -90°$; here the circulation wave leads the temperature by 90° for reasons explained in Section 3. This behavior is opposite to that for $f < \omega$ and was not recognized by the early workers. When $f = \omega$, the response is singular when $\alpha = 0$; however, for small but non-zero α, Eq. (45) shows that $\chi_1 = 90°$ [(47)] still gives $\chi_2 = 90°$ so that $\chi_1 - \chi_2 = 0°$; C and B are in phase in this limit. Fig. 6 displays $\chi_1(\alpha;\ f = 0, \omega, 10^{-4})$ and $\chi_2(\alpha)$. The behavior of $\chi_2(\alpha)$ is straightforward; increasing α tends to decrease the phase lag of the temperature with respect to the heating. When $f = 0$, Fig. 6 shows that an increase in α decreases the phase lag χ_1 at any alpha, and moreover, the phase lag of C with respect to B, while always positive, decreases with increasing α. This decreasing phase lag with α is what both Haurwitz (1947) and Schmidt (1947) describe. When $f = 10^{-4}$, an increase of α near $\alpha = 0$ increases the phase lag χ_1 and increases the phase difference, $\chi_1 - \chi_2$. Again, this behavior is opposite to that for $f < \omega$. Beyond a certain amount of friction χ_1 decreases with increasing α and becomes asymptotically equal to the curve for $f < \omega$. For $f = \omega$, increasing α decreases χ_1. For large α, Fig. 6 [or (45)] indicates that the phase angle decreases to zero, and that any distinction between the wave-like and no-wave regimes becomes lost. Further it may be shown that, in this limit, the ratio of the horizontal to vertical scale is $N\alpha^{-1}$. Thus in the limit of large friction, effects of the earth's rotation disappear.

6. Summary

Some notes on the linear theory of the land and sea breeze have been made. Of primary concern is the fundamentally different type of response of the atmosphere to a diurnally varying heat source between the cases where $f < \omega$ and that where $f > \omega$. When $f > \omega$, the response is that of an elliptical pattern of flow of aspect ratio $(f^2 - \omega^2)^{1/2}N^{-1}$. That the circulation in the x–z plane is in phase with the heating is a direct consequence of the dominant role played by the Coriolis effect. When $f < \omega$, the response is of the form of internal–inertial waves which radiate outward and upward from the coastline at an angle given by $(\omega^2 - f^2)^{1/2}N^{-1}$. In addition to this radically different spatial response, the circulation is found to lag the heating by approximately 180°. The effects of friction are estimated only insofar as they affect the circulation's phase relationship with the heating. When $f < \omega$, an increase in friction decreases the circulation's phase lag from its frictionless value of 180°. However, when $f > \omega$ an increase in α will increase the circulation's phase lag from its frictionless value of 0°. Beyond a certain amount of friction, both cases became indistinguishable from one another.

These comments are presented in conjunction with a critical historical survey of the linear theory of the land and sea breeze. Based on the results of the work reviewed, a simple model is developed which allows fundamental relations concerning the scale and time-dependence, inherent in the earlier model solutions, to stand out more clearly.

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