On the Interpretation of Eddy Fluxes During a Blocking Episode

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ABSTRACT

Using twice daily synoptic charts, objectively analyzed at the National Meteorological Centre, horizontal eddy fluxes of temperature and quasi-geostrophic potential vorticity are computed for the month of July 1976, when a blocking anticyclone was centered over western Europe. The local time-averaged eddy variance equations are used to provide a dynamical basis for interpreting the spatial pattern of eddy fluxes, and their relation to mean gradients. It is shown that a rotational non-divergent flux can be identified, the cross-gradient component of which balances the mean flow advection of eddy variance. The remaining flux is the dynamically significant one which helps maintain the block and can be understood in terms of a response to sources and sinks of eddy variance.

1. Introduction

The availability of routinely produced global analyses has made it possible for meteorologists to compute, for the first time, local dynamical balances at high resolution over the globe and document the spatial distribution of fluxes by transient eddies. Of particular interest has been the importance of transfer by synoptic scale systems in the mean vorticity and thermodynamic budgets. For example, Holopainen and Oort (1981) conclude that eddy transfer by synoptic scale systems tends to maintain long wave features such as the Siberian anticyclone, the North Pacific and North Atlantic lows. There is growing evidence that eddy transfer is important in the dynamics of blocking. This idea, first suggested by Green (1977), has been supported in diagnostic study by Illari (1983), Hoskins *et al.* (1983) and numerical simulation by Shutts (1983a).

There is also an interest in the eddy flux of quantities by weather systems because eddy fluxes are the subject of parameterization schemes. It is far from certain that a parameterization appropriate to the zonal average eddy heat and potential vorticity flux, such as Green (1970), will be applicable to the local, time-averaged flux as assumed in White and Green (1982) or Shutts (1983b), for example. This is because the hypothesis of down-gradient transfer of heat and potential vorticity is based on integral considerations which cannot make statements about the local eddy mean flow interaction. In the baroclinically unstable westerlies, average eddy heat fluxes must have a component down the mean temperature gradient in order to release potential energy. But although a baroclinic eddy must transfer heat down-gradient to grow, the transfer in its mature and decaying phase is less certain but equally important. Maps of eddy fluxes show that at the end of the storm tracks the sense of the eddy heat flux may not be constrained by the energetics.

Relations between the eddy flux of heat and the local temperature gradient have been sought in observational studies by Clapp (1970), Tucker (1977), Lau (1978) and Lau and Wallace (1979). Lau and Wallace (1979), for example, show that at the end of the storm tracks eddy heat fluxes can be directed along or (particularly at upper levels) even up the local temperature gradient, suggesting that the transfer properties of mature and decaying systems are different from the transfer properties of their growing phase. Evidence from eddy-resolving ocean circulation models (Holland and Rhines 1980) also indicate that decaying eddies may transfer conserved properties up-gradient. The interpretation of the dynamical significance of these up-gradient fluxes, though, is not straightforward because the eddy fluxes can have large rotational non-divergent parts which mask the dynamically important divergent fluxes.

It would be of considerable help in the understanding of the transfer properties of synoptic scale systems, and hence to the parameterization of those transfers in low resolution climate models, if it were possible to interpret the sense of the local eddy flux, or a part of it, in terms of local enstrophy conversion. Marshall and Shutts (1981, hereafter MS) have suggested such a way

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of rationalizing eddy fluxes with reference to the eddy variance equations. They identify a purely rotational flux, the cross-gradient component of which balances the mean flow advection of eddy variance, and hence can be associated with the spatial growth and decay of eddies. The sense of the remaining flux is determined by the distribution of sources and sinks of eddy variance.

Here, National Meteorological Centre (NMC) analyses are used to compute eddy fluxes at the end of the Atlantic storm track during the European drought of 1976. The separation of the fluxes into parts which balance terms in the eddy variance equations, as suggested by MS, is found to provide a helpful dynamical basis from which to view the pattern of eddy fluxes.

2. Data

The horizontal distribution of the eddy variances, eddy fluxes and mean quantities during the July 1976 European drought are examined using the twice daily NMC analyses of geopotential height, wind, temperature and humidity, on a 2.5° horizontal grid at 12 levels in the vertical. The statistics are evaluated on a 20 × 20 grid extending from 22.5°N to 70°N and from 22.5°W to 25°E covering western, northern and central Europe, centered on a blocking anticyclone. A blocking configuration is chosen since it provides an interesting context in which to study eddy fluxes, and also in the hope that the fluxes themselves may give insights into the blocking mechanism. For a detailed synoptic description and a discussion of the maintenance of the monthly-mean blocked flow, see Illari (1983).

The horizontal eddy flux of a quantity S is calculated using

$$\overline{\mathbf{V}'S'} = \overline{\mathbf{V}S} - \bar{\mathbf{V}}\bar{S}$$

and the eddy variance

$$\overline{S'^2} = \overline{S^2} - \bar{S}^2,$$

where the overbar represents a monthly average and a prime the deviation from the average. A monthly average was chosen to be long enough to include several transient systems, yet short enough so as not to smooth out the anomalous circulation pattern.

3. Rotational eddy fluxes and the storm tracks

Due to zonal asymmetries in baroclinicity induced by orographic and diabatic forcing, the growth and decay of eddies becomes organized so that eddies grow preferentially over the west coast of the oceans and move downstream to decay over the east coast, producing the observed storm tracks over the oceans.

Major cyclonic activity is associated with pronounced height variability. Therefore the variance of geopotential height $\overline{h'^2}$ gives an indication of the path followed by the storms. The distribution of $\overline{h'^2}$ at 400 mb over Europe is shown in Fig. 1a and the mean height of the 400 mb surface in Fig. 1b. There is a maximum of $\overline{h'^2}$ in the northwest, in the region of decaying eddies at the end of the Atlantic storm track. The spatial distribution is consistent with synoptic observations that the passage of depressions was interrupted by a blocking ridge centered over the British Isles: depressions were concentrated to the west of the block with a few being steered around, mainly in the northern branch of the split jet.

The eddy heat flux at 400 mb is plotted against $\bar{T}$ in Fig. 1c. The sense of the flux is not obviously related to the mean temperature contours (upstream of the temperature ridge there are up-gradient fluxes and downstream, down-gradient fluxes) but a prominent component swirling around the storm tracks is evident. Lau (1978) first recognized that eddy fluxes often rotate around the storm tracks and Lau and Wallace (1979) attributed this to the geostrophic nature of the weather systems as follows.

If the eddy motion is geostrophic and equivalent barotropic then, in pressure coordinates

$$T' \propto -\frac{\partial h'}{\partial p} \propto h',$$

where the constant of proportionality between the eddy temperature T' and the departure in the height h' of the pressure surface from its mean position, depends on the vertical structure of the eddy: it is positive if it is assumed that (appropriate to a warm high or cold low modification of the mean flow) h' increases with height. Because $\mathbf{V}' \propto \mathbf{k} \times \nabla_h h'$ then

$$\overline{\mathbf{V}'T'} \propto \mathbf{k} \times \nabla_h \overline{h'^2},$$

where $\mathbf{k}$ is a unit vector pointing vertically upwards. Hence a contribution of the eddy heat flux is expected to rotate anticyclonically around the storm tracks.

Similarly, Lau and Wallace argued that the eddy relative vorticity flux $\overline{\mathbf{V}'\zeta'}$ would have a component

$$\overline{\mathbf{V}'\zeta'} \propto -\mathbf{k} \times \nabla_h \overline{h'^2}$$

rotating cyclonically because of the proportionality between $-\zeta'$, the eddy relative vorticity, and h'.

It seems, then, that because synoptic systems are geostrophic, their eddy fluxes will have a rotational contribution that may obscure the dynamically important divergent fluxes. It is necessary to be aware of this when interpreting maps of eddy fluxes.

Lau and Wallace (1979) isolated the dynamically important divergent flux by mathematically splitting the vector field of eddy fluxes into rotational and irrotational parts by solving Poisson equations for the flux streamfunction Ψ, and the flux potential χ, with

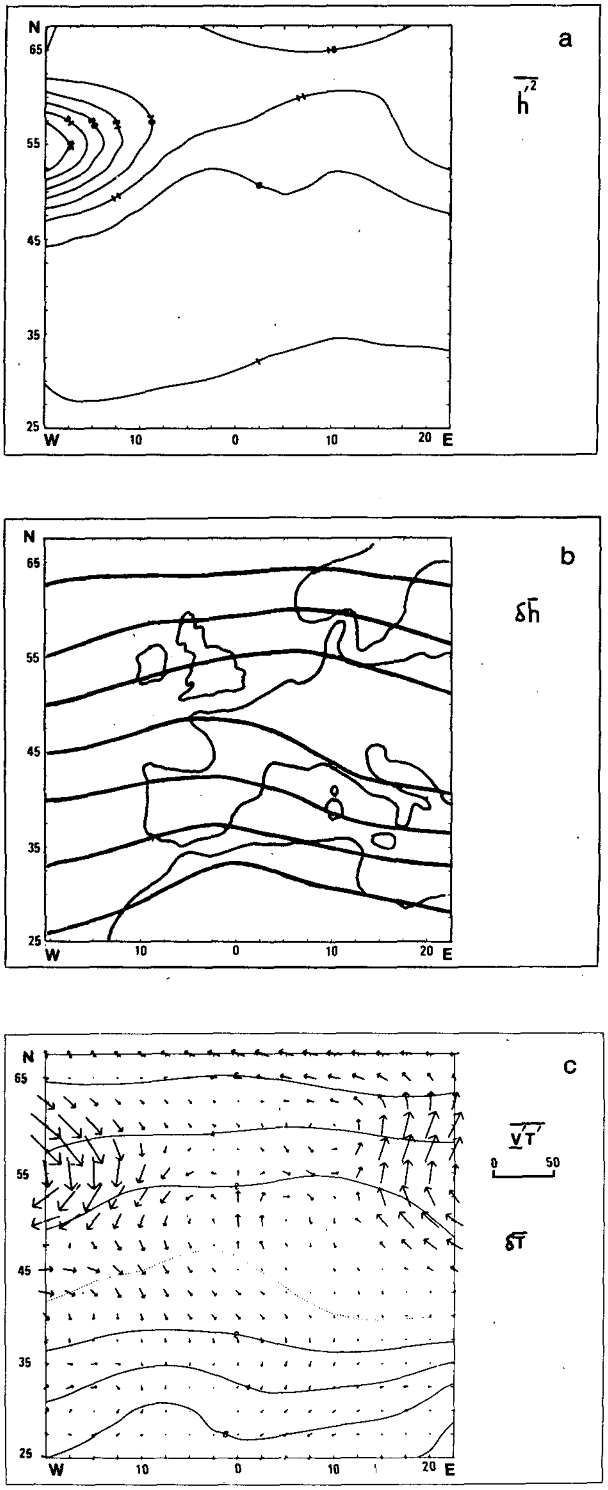


FIG. 1. (a) Temporal variance of the 400 mb geopotential height for July 1976 (C.I. = 5×10^3 m^2). (b) The deviation in the height of the monthly mean 400 mb surface from the hemispherically averaged monthly mean height (C.I. = 6 dam). The major land masses are outlined by the thin black line. (c) Eddy heat flux $\overline{\mathbf{V}'T'}$ (m s^{-1} K) at 400 mb, plotted against $\delta\bar{T}$ (C.I. = 2 K) the monthly mean temperature departure from the hemispherically averaged mean temperature. The dotted line is the zero contour.

boundary conditions $\Psi = 0$ and $\chi = 0$ applied to 20°N, where eddy activity is small. Instead MS use the eddy variance equations to identify a rotational flux that balances the mean flow advection of eddy variance. This approach goes beyond a purely mathematical device; rather it associates contributions of the flux with terms in the eddy variance equations, and hence a separation is made on dynamical grounds.

4. The eddy potential energy equation

It is the eddy potential energy equation which relates the eddy flux of heat across the horizontal temperature gradient to the advection and conversion of eddy potential energy. It may be instructively derived as follows.

The thermodynamic equation in pressure co-ordinates is:

$$\frac{\partial T}{\partial t} + \mathbf{V}\cdot\nabla_h T - S_p\omega = H, \tag{1a}$$

where

$$S_p = \frac{RT}{pC_p} - \frac{\partial T}{\partial p} = -\frac{T}{\theta}\frac{\partial \theta}{\partial p}$$

is the static stability and

- T temperature
- θ potential temperature
- $\mathbf{V}$ horizontal wind
- R gas constant
- C_p specific heat at constant pressure.

In Eq. (1a) the rhs term,

$$H = \frac{\dot{q}}{C_p},$$

is the diabatic heating with $\dot{q}$ the rate of heating per unit mass due to radiation and latent heat release.

$$\omega = \frac{Dp}{Dt}$$

is the vertical velocity in pressure coordinates.

Introducing eddy transfer by splitting the variables into time-mean and eddy components, and then time averaging, the thermodynamic equation (1a) takes the form:

$$\frac{\partial \bar{T}}{\partial t} + \bar{\mathbf{V}}\cdot\nabla_h\bar{T} + \nabla_h\cdot(\overline{\mathbf{V}'T'}) - \bar{S}_p\bar{\omega} = \bar{H}. \tag{1b}$$

Multiplying Eq. (1a) by T and Eq. (1b) by $\bar{T}$ gives equations for the time rate of change of $\frac{1}{2}T^2$ and $\frac{1}{2}\bar{T}^2$ respectively. On subtraction we arrive at an equation for the eddy temperature variance $\frac{1}{2}\overline{T'^2}$, proportional to the eddy potential energy:

$$\frac{\partial}{\partial t}\frac{\overline{T'^2}}{2} - \underbrace{\bar{T}\nabla_h\cdot(\overline{\mathbf{V}'T'})}_{\text{conversion of mean flow potential energy}} + \underbrace{\nabla_h\cdot\left(\overline{\mathbf{V}\frac{T'^2}{2}} + \overline{\mathbf{V}'T'}\bar{T}\right)}_{\text{advection of potential energy}}$$

$$- \underbrace{\overline{\omega'T'}S_p}_{\text{conversion to eddy kinetic energy}} = \underbrace{\overline{H'T'}}_{\text{diabatic sources and sinks}}. \quad (2)$$

Equation (2) is more usually written in the form

$$\frac{\partial}{\partial t}\frac{\overline{T'^2}}{2} + \nabla_h\cdot\left(\overline{\mathbf{V}\frac{T'^2}{2}}\right) + \overline{\mathbf{V}'T'}\cdot\nabla_h\bar{T} - \overline{\omega'T'}S_p = \overline{H'T'}, \quad (3)$$

where an advection and a conversion term have been combined into a heat flux across the gradient using the relation:

$$\overline{\mathbf{V}'T'}\cdot\nabla_h\bar{T} = \nabla_h\cdot(\overline{\mathbf{V}'T'}\bar{T}) - \bar{T}\nabla_h\cdot(\overline{\mathbf{V}'T'}).$$

This derivation of the eddy potential energy equation emphasizes that the eddy flux across the gradient represents both conversion and advection of potential energy. As pointed out in Holland and Rhines (1980), Eq. (3) provide useful relationships which as we show in the next section help interpret the sense of eddy fluxes.

5. A non-divergent heat flux associated with mean flow advection of eddy potential energy

Marshall and Shutts have used Eq. (3) as a basis from which to separate out dynamically distinct contributions of the flux. The separation is based on the realization that if the time-mean flow is highly balanced in the sense that diabatic heating and eddy forcing cause only small departures from mean flow $\bar{\psi}$ along mean temperature contours $\bar{T}$, so $\Psi = \bar{\psi}(\bar{T})$, then it follows that:

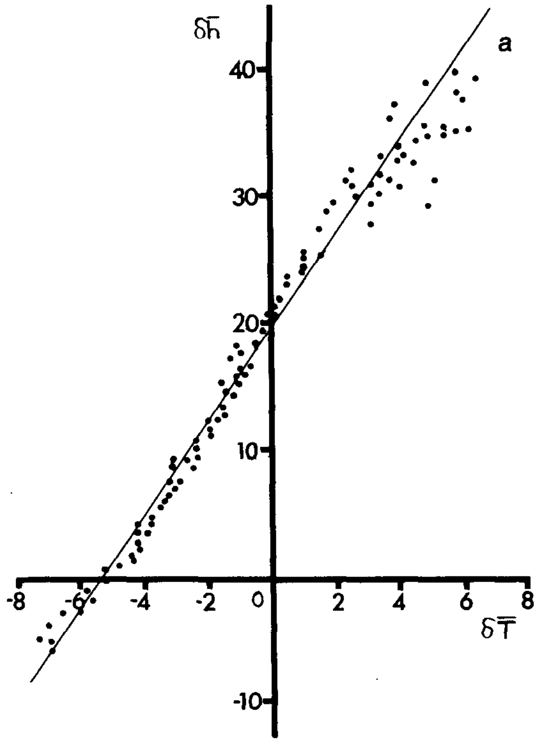
$$\bar{\mathbf{V}}\cdot\nabla_h\frac{\overline{T'^2}}{2} + (\overline{\mathbf{V}'T'})_R\cdot\nabla_h\bar{T} = 0, \quad (4)$$

where

$$(\overline{\mathbf{V}'T'})_R = \frac{1}{2}\mathbf{k}\times\nabla_h\left(\frac{d\Psi}{d\bar{T}}\overline{T'^2}\right)$$

can be called an advection flux since its cross-gradient component balances mean-flow advection of $\overline{T'^2}$. The subscript R is to emphasize that the flux is rotational.

In the original derivation of (4), MS overlooked that it has wider validity if the $d\bar{\psi}/d\bar{T}$ multiplier is taken inside the horizontal gradient operator (D. Andrews, personal communication, 1982). Then $(\overline{\mathbf{V}'T'})_R$ is non-divergent even if the $\bar{\psi}(\bar{T})$ relation is nonlinear.



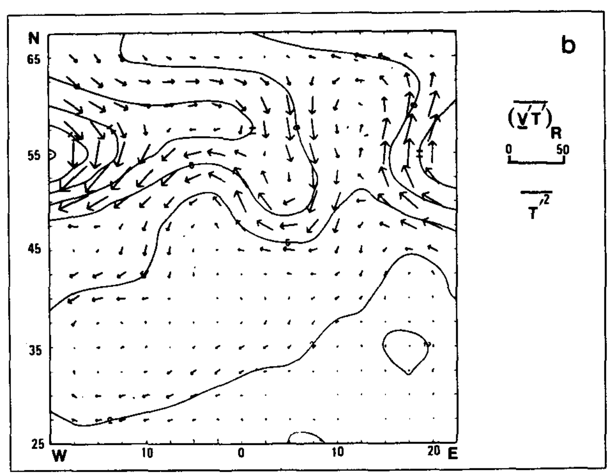


FIG. 2. (a) Scatter diagram of $\delta\bar{T}$ (K) and $\delta\bar{h}$ (in dam) at 400 mb, showing their functional relationship. (b) advection eddy heat flux $(\overline{\mathbf{V}'T'})_R$ (ms^{-1} K), Eq. (4), plotted against $\overline{T'^2}$ (C.I. = 3 K^2). The constant of proportionality $d\bar{\psi}/d\bar{T}$ is calculated from the gradient of the straight line drawn through the points of Fig. (2a). This flux is non-divergent.

It should be emphasized that the remaining eddy flux is not irrotational; our procedure is not the usual separation into rotational, non-divergent and irrotational, divergent parts. Eq. (4) only identifies a rotational non-divergent flux associated with the advection of $\frac{1}{2}\overline{T'^2}$. It is not the same as the rotational flux defined in Lau and Wallace (1979). From Eq. (4) the $(\overline{\mathbf{V}'T'})_R$ flux vector-rotates in the sense to balance flow advection, whereas the sense of rotation of the Lau-Wallace flux depends on the vertical structure of the eddies. Further, the reference contours for the flux defined here are the $\overline{T'^2}$ contours rather than the $\overline{h'^2}$. In general the $\overline{T'^2}$ and $\overline{h'^2}$ contours will not coincide.

Figure 1c shows large southwards heat fluxes at the end of the Atlantic storm track. Equation (4) suggests that this up-gradient flux could be largely rotational in response to flow advection of $\overline{T'^2}$ into the region. To investigate this hypothesis an attempt is made to compute the rotational flux, Eq. (4), from our data.

The extent to which the separation is successful depends only on the closeness of the functional relationship between $\bar{\psi}$ and $\bar{T}$. If $\bar{\psi}$ is plotted against $\bar{T}$, it is the degree to which the cluster of points collapse onto a line which is the measure of this functional relationship. The mean height of the 400 mb surface is plotted against the mean temperature at grid-points over our region in Fig. 2a. Because there is a close functional relationship, the mean flow advection of $\overline{T'^2}$ can be closely balanced by a rotational, non-divergent flux. Figure 2b shows $(\overline{\mathbf{V}'T'})_R$ (with the constant calculated from the slope of Fig. 2a) plotted against the $\overline{T'^2}$ contours. Comparison with the total flux, Fig. 1c, show that major features are due to the rotational flux. It is clear that, at this level, the eddy heat flux is dominated by a component following the $\overline{T'^2}$ contours (rather than the storm tracks $\overline{h'^2}$, Fig. 1a).

As an illustration of the success of the separation, the terms in Eq. (4) and their sum are plotted in Fig. 3. The sum Σ_T in Fig. 3c is much smaller than either of the two terms, showing that we have succeeded in isolating a rotational component which closely balances the mean flow advection. A comparison of Fig. 3a with Fig. 3b shows that the large southwards heat flux at the end of the Atlantic storm track is predominantly rotational and points up-gradient to balance advection of $\overline{T'^2}$ by the mean flow to the end of the storm track:

$$\bar{\mathbf{V}}\cdot\nabla_h\left(\frac{\overline{T'^2}}{2}\right) + (\overline{\mathbf{V}'T'})_R\cdot\nabla_h\bar{T} \approx 0.$$

$$(-) \qquad\qquad (+)$$

Similarly to the northeast of our region large rotational fluxes are directed down-gradient to balance the advection of $\overline{T'^2}$ out of the region by the mean flow.

$$\bar{\mathbf{V}}\cdot\nabla_h\left(\frac{\overline{T'^2}}{2}\right) + (\overline{\mathbf{V}'T'})_R\cdot\nabla_h\bar{T} \approx 0.$$

$$(+) \qquad\qquad (-)$$

The remaining flux $\overline{\mathbf{V}'T'} - (\overline{\mathbf{V}'T'})_R$, the conversion flux, is shown in Fig. 4a. It contains all of the dynam-

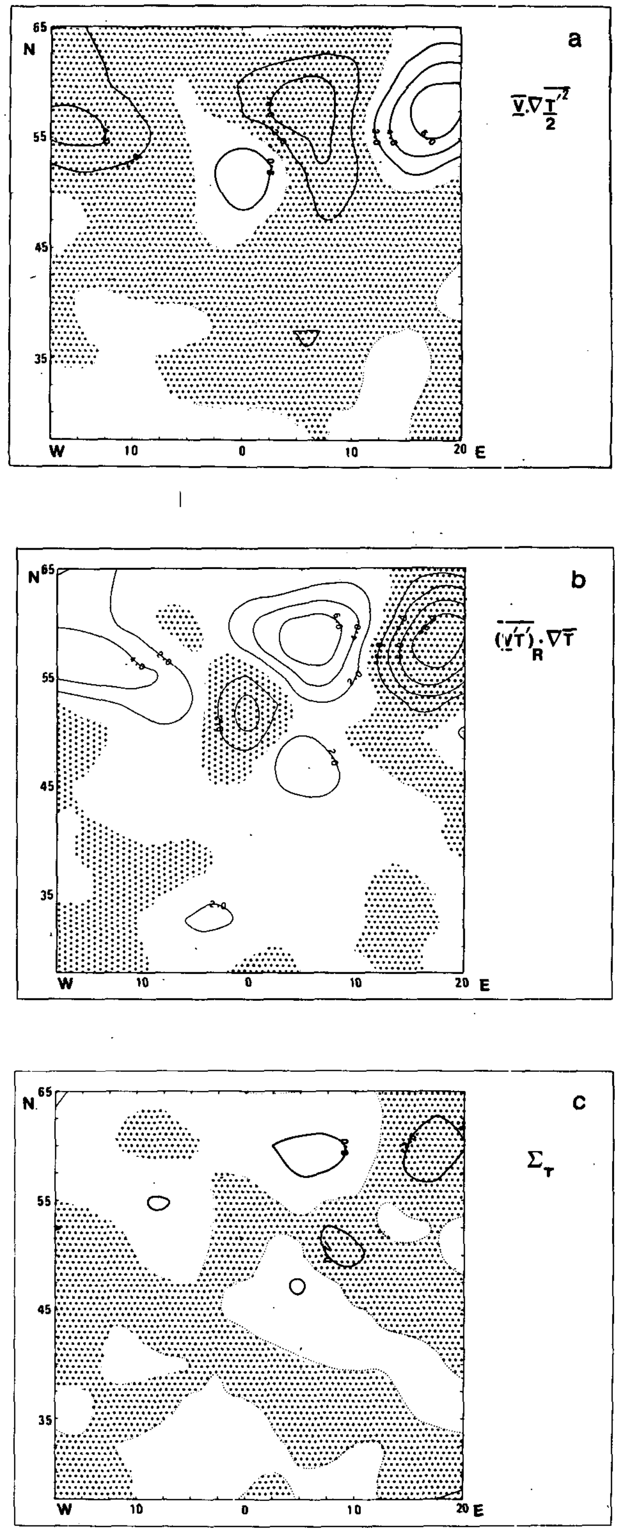


FIG. 3. The spatial distribution of the terms in Eq. (4) showing that mean flow advection of eddy potential energy is closely balanced by rotational fluxes across the gradient (C.I. = 2×10^{-5} K^2 s^{-1}): (a) $\bar{\mathbf{V}}\cdot\nabla_h\overline{T'^2}/2$) (b) $(\overline{\mathbf{V}'T'})_R\cdot\nabla_h\bar{T}$, and (c) their sum Σ_T. Shaded regions are negative, unshaded regions positive.

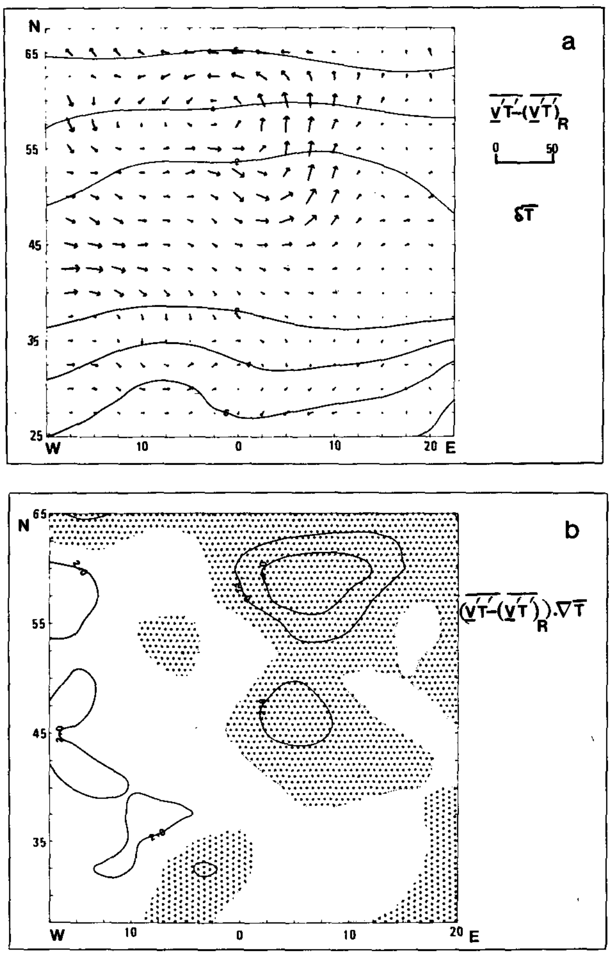


FIG. 4. (a) The "conversion" flux $\overline{\mathbf{V}'T'} - (\overline{\mathbf{V}'T'})_R$ (m s^{-1} K) plotted against $\delta\bar{T}$ (C.I. = 2 K day^{-1}) at 400 mb showing northward heat fluxes over the temperature ridge. (b) Spatial distribution of the conversion term $(\overline{\mathbf{V}'T'} - (\overline{\mathbf{V}'T'})_R)\cdot\nabla_h\bar{T}$ (C.I. = 2 × 10^{-5} °K^2 s^{-1}) showing a sink of eddy potential energy over, and to the north of the temperature ridge.

ically important divergent flux. It is directed more down-gradient and is much less rotational than the total flux Fig. 1c. This pattern of eddy fluxes is more understandable in terms of a response to sources and sinks of eddy potential energy. The conversion term $[\overline{\mathbf{V}'T'} - (\overline{\mathbf{V}'T'})_R]\cdot\nabla_h\bar{T}$, Fig. 4b, is significantly larger than the sum Σ_T and so, in the absence of data inaccuracies, must be balanced by sources and sinks of eddy potential energy:

$$[\overline{\mathbf{V}'T'} - (\overline{\mathbf{V}'T'})_R]\cdot\nabla_h\bar{T} \approx \overline{H'T'} + \overline{\omega'T'}\bar{S}_p.$$

A prominent feature of Fig. 4a is the down-gradient flux centered on, and to the northeast of the temperature ridge. The conversion term, Fig. 4b, is significantly negative here implying that there is a sink of eddy potential energy

$$\overline{H'T'} + \overline{\omega'T'}\bar{S}_p < 0.$$

Two possible sinks are conversion of eddy potential energy to eddy kinetic energy in baroclinic instability $\overline{\omega'T'}\bar{S}_p < 0$, and radiative damping of eddies to space $\overline{H'T'} < 0$. Supposing that the eddies are thermally damped $H' = -\lambda T'$, our data shows that a λ^{-1} of ten days if required to balance the conversion term over the temperature ridge. A time constant of this magnitude implies an easily attainable eddy cooling rate of less than 1 degree per day, suggesting that radiative cooling to space could be an important sink of eddy potential energy. Conversion of eddy potential energy to eddy kinetic energy, $\overline{\omega'T'}\bar{S}_p < 0$, may also be important in the baroclinically active northern branch of the split jet, particularly in the region of renewed cyclonic activity downstream of the ridge.

So it is likely that the warm blocking high is a sink of eddy potential energy and that this is offset through conversion from the mean flow by divergent down-gradient fluxes. This northwards heat flux can be interpreted as the signature in the eddy flux statistics which in Lagrangian terms is achieved through large meridional displacement of warm air parcels, caught up in synoptic systems and brought into the ridge from the south. Intermittent southerly winds on the western flanks of blocks, associated with the cold fronts of depressions as they move up to the ridge from the west, are a synoptic feature of blocking which was noted as early as Berggren *et al.* (1949). In the total eddy flux, Fig. 1c, this dynamically important flux is masked by the large southwards rotational flux of Fig. 2b.

It is interesting to look at the maps of eddy heat fluxes in relation to the eddy heat flux divergence pattern. Although $\overline{\mathbf{V}'T'} - (\overline{\mathbf{V}'T'})_R$ still contains a rotational component, the flux divergence pattern at 400 mb. Fig. 5, can be more readily associated with this flux, rather than the total flux shown in Fig. 1c. Evidently the $(\overline{\mathbf{V}'T'})_R$ defined in Eq. (4) accounts for a large part of the rotation.

Eddy heat fluxes calculated at other levels also contain advective parts moving around the $\overline{T'^2}$ contours, particularly in the upper troposphere where advection fluxes dominate in response to large mean flow advection of $\overline{T'^2}$. In the lower troposphere the $\bar{\psi}(\bar{T})$ relationships are not so tight and identification of an advection flux through Eq. (4) is less meaningful. At these levels eddy heat fluxes are less rotational and directed more down-gradient. Again this is consistent with our association of rotational fluxes with flow advection. In the lower troposphere where mean flow is weaker, the advection term in the eddy potential energy equation is smaller, and the down-gradient conversion fluxes are no longer overwhelmed by a rotational advective component.

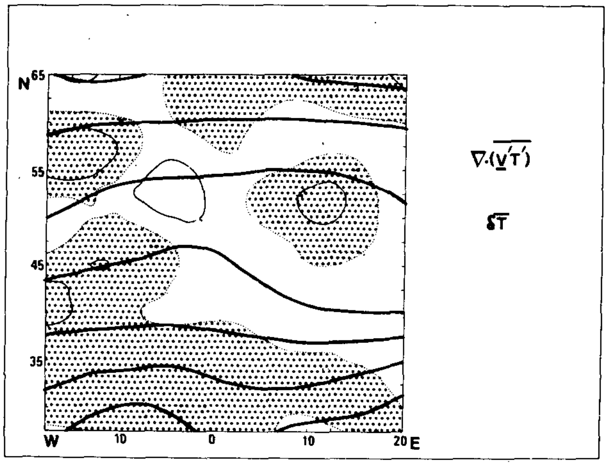


FIG. 5. The eddy heat flux divergence $\nabla_h \cdot (\overline{V'T'})$ at 400 mb (C.I. = 1 K day^{-1}) and the monthly mean temperature $\delta\bar{T}$ (C.I. = 2 K). There is convergence in shaded regions and divergence in unshaded regions.

6. A non-divergent potential vorticity flux associated with mean flow advection of eddy enstrophy

In Illari (1983) the maintenance of the monthly mean flow is studied in terms of potential vorticity, as well as relative vorticity and temperature. For large scale flow the full potential vorticity, as defined by Ertel (1942), can be approximated by

$$P = (\zeta + f)\frac{\partial \theta}{\partial p}$$

since Ro $\ll$ 1 and RiRo$^2 \gtrsim$ 1 where Ro is the Rossby number and Ri is the Richardson number. This approximation has been used in diagnostic study by Lau and Wallace (1979) and Savijarvi (1978), but because it is significantly advected by vertical motion, as well as in the horizontal, interpretation of maps of P and its eddy fluxes is difficult. Instead, Illari (1983) prefers to replace P by the potential vorticity appropriate to quasi-geostrophic motion q where

$$q = \zeta + f - f_0 \frac{\partial}{\partial p}\left(\frac{\delta T}{S_p}\right)$$

and δT is the deviation from the basic temperature profile $T(p)$, ζ is the relative vorticity and f is the Coriolis parameter. The quantity q is conserved in horizontal adiabatic frictionless motion:

$$\frac{D}{Dt_h}(q) = 0. \qquad (5)$$

The connection between Ertel, P and q has been discussed in, e.g., Charney and Stern (1962), Green (1970), and Kuo (1972). A diagnostic study in terms of q rather than P has recently been carried out by Holopainen *et al.* (1982).

Precise details of the method of calculation of q from data are given in Illari (1983). Here it is sufficient to note that the major problem arises from the computation of the static stability term which requires the evaluation of vertical derivatives of T near tropopause level. The calculation is based on vertical profiles of monthly mean temperatures horizontally averaged over the domain of interest.

The eddy quasi-geostrophic potential vorticity flux $\overline{\mathbf{V}'q'}$ at 300 mb is plotted against $\bar{q}$ and $\overline{q'^2}$ in Fig. 6. The eddy fluxes are of particular interest here, just below the tropopause, because the flux divergence is large and makes an important contribution in the maintenance of $\bar{q}$. A flux rotating around the $\overline{q'^2}$ contours is evident, leading to regions of q-flux up and down the mean potential vorticity gradient. Again it is instructive to look at the appropriate eddy variance equation—the eddy enstrophy equation. In the steady state it can be written, neglecting the triple correlation term:

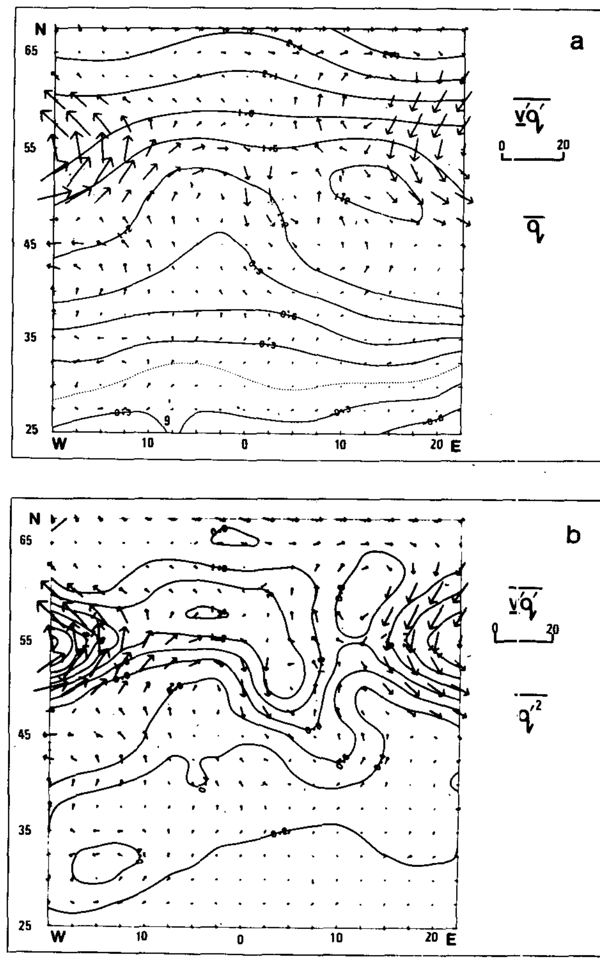


FIG. 6. The eddy quasi-geostrophic potential vorticity flux $\overline{\mathbf{V}'q'}$ at 300 mb plotted against (a) $\bar{q}$ (C.I. = 0.3 × 10^{-4} s^{-1}) and (b) $\overline{q'^2}$ (C.I. = 0.2 × 10^{-8} s^{-2}).

$$\bar{\mathbf{V}}\cdot\nabla_h\frac{\overline{q'^2}}{2} + \overline{\mathbf{V}'q'}\cdot\nabla_h\bar{q} = \overline{S'_q q'} \qquad (6)$$

advection of eddy enstrophy — flux across the gradient — source/sink of eddy enstrophy

where S_q is the source/sink term in the potential vorticity equation (5).

If the mean streamfunction $\bar{\psi}$ is a function of $\bar{q}$ then, as before (see MS)

$$(\overline{\mathbf{V}'q'})_R\cdot\nabla_h\bar{q} + \bar{\mathbf{V}}\cdot\nabla_h\left(\frac{\overline{q'^2}}{2}\right) = 0, \qquad (7)$$

where the advection q flux is

$$(\overline{\mathbf{V}'q'})_R = \frac{1}{2}\mathbf{k}\times\nabla_h\left(\frac{d\bar{\psi}}{d\bar{q}}\overline{q'^2}\right).$$

Figure 6b shows the large rotational component rotating in the correct sense to balance the mean flow advection of $\overline{q'^2}$, as suggested by Eq. (7).

In Fig. 7a $\bar{\psi}$ is plotted against $\bar{q}$. The linear functional relationship between $\bar{\psi}$ and $\bar{q}$ is not as close as it is between $\bar{\psi}$ and $\bar{T}$. However a rotational flux, with the multiplying constant calculated from the straight line fitted to the points, is plotted in Fig. 7b. There is a striking similarity between this plot and the plot of the total flux $\overline{\mathbf{V}'q'}$, Fig. 6b, showing that the q flux at 300 mb is almost all rotational flux.

The terms in Eq. (6) and their sum Σ_q are plotted in Fig. 8. The rotational flux balances the mean flow advection in most of the region. In particular, the regions of very large up- and down-gradient flux are a response through non-divergent fluxes to flow advection $\overline{q'^2}$. These fluxes do not result in any interaction between the eddies and the mean flow. However, the balance is not good in the region of minimum $\bar{q}$, presumably because the $\bar{q}$ contours are closed, whereas the mean $\bar{\psi}$ contours are not, thus breaking the functional relationship between $\bar{\psi}$ and $\bar{q}$.

The remaining flux $\overline{\mathbf{V}'q'} - (\overline{\mathbf{V}'q'})_R$, Fig. 9a, is much smaller in magnitude than $\overline{\mathbf{V}'q'}$ and is directed both up and down-gradient. Because the dynamically important $(\overline{\mathbf{V}'q'} - (\overline{\mathbf{V}'q'})_R)\cdot\nabla_h\bar{q}$ term, plotted in Fig. 9b, is a rather small term in Eq. (6) and comparable with the residual Σ_q, it is difficult to draw any firm conclusions about the importance of the enstrophy source/sink term in shaping the pattern of the flux. Nevertheless a notable feature of Fig. 9 is the large southward down-gradient flux just upstream of the closed $\bar{q}$-contour. A numerical simulation of blocking by Shutts (1983a) suggests that large southward vorticity fluxes upstream of a block may be a dynamically important signature in the eddy vorticity fluxes. In a barotropic model, Shutts is able to generate a blocked flow as a *resonant* response to eddy forcing (the eddies in the model are generated by a wave maker). The mean flow

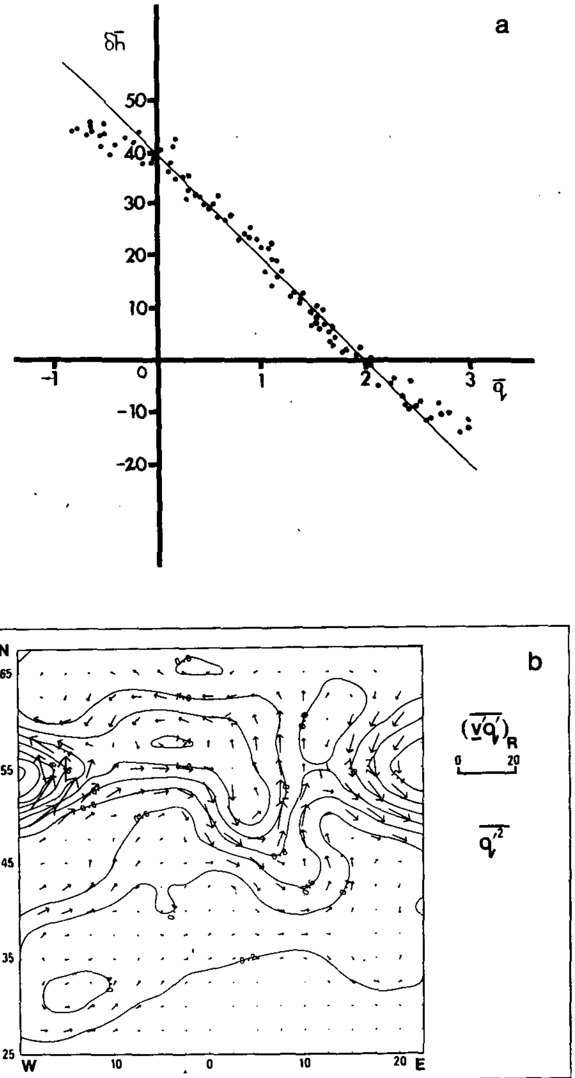


FIG. 7. (a) Scatter diagram of $\bar{q}$ in 10^{-4} s^{-1}) and $\delta\bar{h}$ (in dam) at 300 mb showing their functional relationship. (b) the advection eddy q flux, $(\overline{\mathbf{V}'q'})_R$ (10^{-4} m s^{-2}), Eq. (7), plotted against $\overline{q'^2}$ (C.I. = 0.2×10^{-8} s^{-2}) at 300 mb. The constant of proportionality $d\bar{\psi}/d\bar{q}$ is calculated from the gradient of the straight line drawn through Fig. 7(a).

has a close $\bar{\psi}(\bar{q})$ relationship, and so a meaningful separation of the eddy flux can be made. After subtracting the rotational component, southward fluxes centered on the split jet can be seen. In the model this is interpreted as a down-gradient flux balancing enhanced dissipation of enstrophy caused by the strong deformation of the eddies as they propagate in the split jet. It is very likely that our blocked flow is a region of enhanced enstrophy cascade. Indeed the distortion and change in scale of the synoptic systems as they move in the split jet can be seen in the synoptic charts. However, because the flow advection in Eq. (6) is such a large term, and the functional relationship between $\bar{\psi}$ and $\bar{q}$ not sufficiently close, such an association can be only tentatively drawn. It is tempting to associate the southward fluxes just upstream of the $\bar{q}$ minimum in Fig. 9a with a sink of eddy enstrophy. This flux is achieved through the large meridional displacement of air from the south in synoptic systems. Even in the absence of eddies, parcels would undergo minimal displacement about the point of the jet-stream split, but the strong synoptic impression is that the blocking anticyclone is supplied by warm air of low potential vorticity, brought from the south by transient systems on steep northward trajectories cutting *across* monthly mean $\bar{\psi}$ contours. This contention seems even more reasonable on consideration of the Austausch coefficient for the conversion flux of Fig. 9a, a measure of the rate of dispersion of fluid parcels

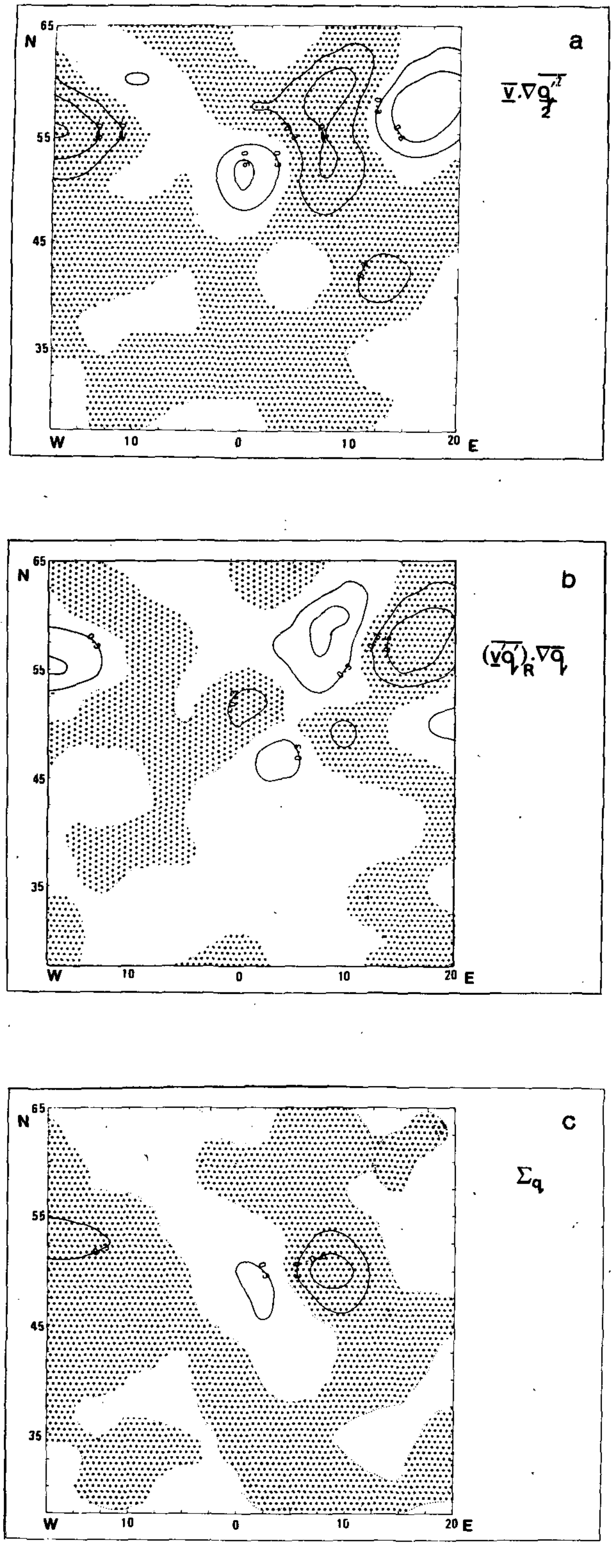


FIG. 8. The spatial distribution of the terms in Eq. (6) at 300 mb (C.I. = 3×10^{-15} s^{-3}). (a) $\bar{V}\cdot\nabla_h(q'^2/2)$, (b) $(\overline{V'q'})_R\cdot\nabla_h\bar{q}$ and (c) their sum Σ_q. Shaded regions are negative, unshaded regions positive.

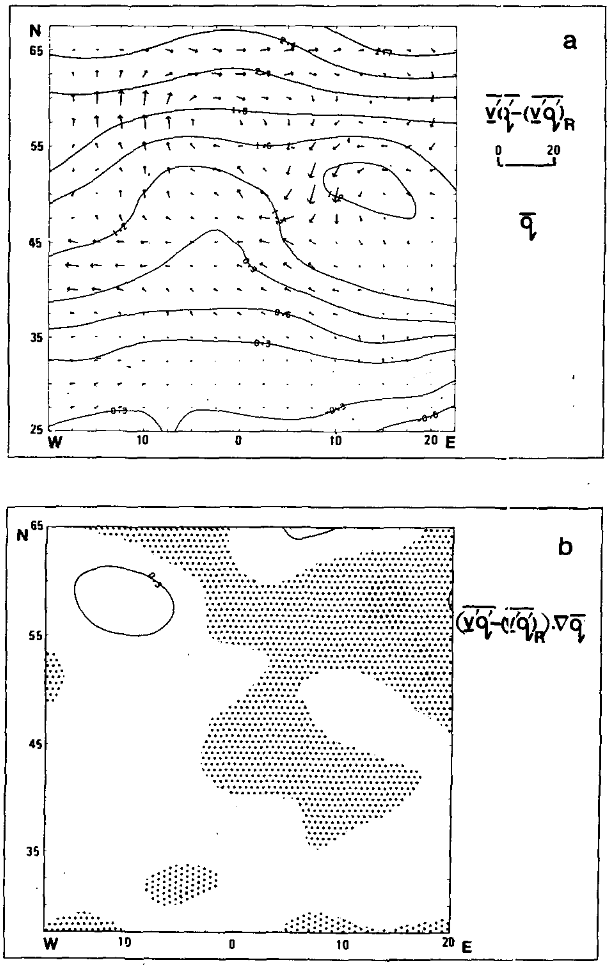


FIG. 9. (a) "conversion" flux $\overline{V'q'} - (\overline{V'q'})_R$ (10^{-4} m s^{-2}) plotted against $\bar{q}$ (C.I. = 0.3×10^{-4} s^{-1}) showing southward fluxes upstream of the $\bar{q}$ minimum. (b) Spatial distribution of $(\overline{V'q'} - (\overline{V'q'})_R)\cdot\nabla_h\bar{q}$ (C.I. = 3×10^{-15} s^{-3}).

$$-\frac{(\overline{\mathbf{V}'q'} - (\overline{\mathbf{V}'q'})_R)\cdot\nabla_h\bar{q}}{|\nabla_h\bar{q}|^2}.$$

This is strongly positive up-stream of the $\bar{q}$ minimum not only because the fluxes are large and southwards here, but also because of the slack $\bar{q}$ gradient, supporting the idea that parcels undergo large meridional displacements on the western flank.

The eddy q flux divergence is shown in Fig. 10 and, as with the heat flux, can be more readily understood in terms of Fig. 9a with the large non-divergent part subtracted. The southwards flux centered on the split jet leads to an eddy potential vorticity flux divergence to the north and a convergence to the south, which as discussed in Illari (1983), tends to balance the mean flow advection of $\bar{q}$. This down-gradient divergent flux tends to destroy the gradients, making the block a region of almost uniform $\bar{q}$. The closed $\bar{q}$ contours of the block are suggestive of a "homogenization" of the $\bar{q}$ by eddies, Rhines and Young (1982); an expulsion of the $\bar{q}$ contours to the north and south leaving a plateau in between. This indeed is a likely end-state if the diabatic forcing in the potential vorticity equation, restoring the $\bar{q}$ gradients, is sufficiently weak to be overwhelmed by the systematic down-gradient transfer of q by eddies induced by an enhanced enstrophy cascade in the split jet. A time sequence of instantaneous q maps would give valuable insight into this process.

7. Discussion

Simple representations of the zonal mean transfer properties of synoptic scale systems presently used in the climate models, such as Green (1970), may not be adequate for the time-mean local flux. This is because the schemes are based largely on the intuition gained from instability analyses of zonal flows, which give detailed information only about the growing phase, and may therefore be inadequate to describe the transfer properties of a field of growing and decaying eddies. It must be said, however, that the inappropriateness of a local diffusive parameterization for the potential vorticity and temperature has been overstated because maps of eddy fluxes are often misinterpreted. Instability analyses of longitudinally varying flows such as Frederickson (1979) support the idea of a local application of the zonal mean theory. This view is reinforced by the present study, for it has been shown that the extra advection term in the time-mean eddy variance equation (which vanishes in the zonal mean) is associated with a non-divergent flux.

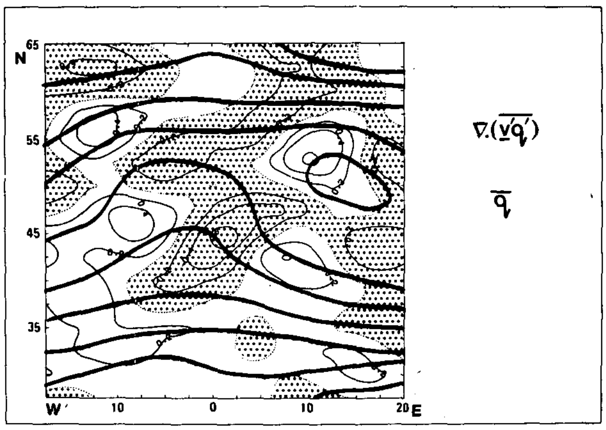


FIG. 10. The eddy flux divergence $\nabla_h\cdot(\overline{\mathbf{V}'q'})$ at 300 mb (C.I. $= 2\times10^{-10}$ s^{-2}) and the monthly mean $\bar{q}$ (C.I. $= 0.3\times10^{-4}$ s^{-1}). There is convergence in shaded regions and divergence in unshaded regions.

Here, as suggested in MS, we have successfully identified in data a rotational eddy flux which, through the eddy variance equation, can be associated with the spatial growth and decay of eddies, but cannot drive near flow. Such an approach separates the eddy flux on dynamical grounds rather than, e.g., by definition into components parallel and perpendicular to the mean gradient (as in Clapp, 1970; Tucker, 1977; Savijarvi, 1978) or mathematically into rotational and divergent parts (as in Lau and Wallace, 1979).

This separation into "advection" and "conversion" fluxes depends only on the closeness of the functional relationship between $\bar{\psi}$ and $\bar{q}$ (or $\bar{\psi}$ and $\bar{T}$). Any such relationship might be expected to be broken in regions where the eddy flux divergence is large—precisely where there is most interest in the eddy fluxes themselves. This could detract from the usefulness of the approach. In general, however, a rather close functional relationship can be expected, especially between $\bar{\psi}$ and $\bar{q}$, because the $\bar{q}$'s are the reference contours along which, but for diabatic effects and eddies, the large-scale mean flow moves. Particularly at upper levels, where mean flow is strong and diabatic effects small, $\bar{\psi}$ cannot deviate far from the $\bar{q}$ contours. It is at these upper levels that large rotational fluxes are observed obscuring the dynamically important fluxes.

Our rationalization of the spatial distribution of eddy fluxes in a blocking episode has, in accord with the numerical experiments of Shutts (1983a), led us to identify southward eddy potential vorticity fluxes and northward eddy heat fluxes centered on the split jet as dynamically important signatures in the eddy flux statistics. The southward eddy potential vorticity fluxes, which can be interpreted as a westward force in the local, time-mean zonal momentum equation, decelerate the flow tending to split the jet.

A blocking configuration was chosen primarily because of an interest in the blocking mechanism, but it is probably a rather complicated (though interesting) flow configuration in which to study eddy fluxes, because the split jet may cause the eddy transfer to be anomalous. Furthermore, a blocked flow in which eddy forcing is abnormally large could put an extra strain on the $\bar{\psi}(\bar{q})$ relationship. Some caution should be exercised not to overly generalize our results before other geographical regions have been studied. Nevertheless our computations suggest the following three general statements about the spatial distribution and sense of eddy fluxes.

First, an advection flux defined in Eqs. (4) and (7) can be identified, which follows the eddy variance contours and whose cross-gradient component balances the major part of the mean flow advection of eddy variance.

Second, the advection flux is non-divergent: eddies respond to flow advection of eddy variance through nondivergent fluxes. These fluxes are dynamically unimportant as far as the mean is concerned but, particularly at upper levels where flow advection is strong, they swamp the irrotational fluxes which effect the time-mean flow.

Third, the conversion flux contains all the dynamically important flux, is more down-gradient and, since it is less rotational, it bears a closer relationship to the flux divergence pattern. It can be more readily interpreted in terms of a response to sources and sinks of eddy variance than the total flux.

An obvious extension of our study is a global investigation of the relative importance of the advection and conversion terms in the eddy variance equations in determining the pattern of eddy fluxes associated with the storm tracks. Scatter diagrams of $\bar{\psi}$ against $\bar{q}$ would indicate the extent to which $\bar{q}$ is the reference for the mean flow and hence the appropriateness of the separation. Finally it would be of interest to study how the relative contribution of advection and conversion fluxes to the total flux, depends on the length of the averaging period and the temporal frequency of the eddy motion.

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REFERENCES

Berggren, R., B. Bolin and C-G. Rossby, 1949: An aerological study of zonal motion, its perturbations and breakdown. *Tellus,* **1,** 14–37.

Charney, J. G., and M. Stern, 1962: On the stability of internal baroclinic jets in a rotating atmosphere. *J. Atmos. Sci.,* **19,** 159–162.

Clapp, P. F., 1970: Parameterization of macroscale transient heat transport for use in a mean-motion model of the general circulation. *J. Appl. Meteor.,* **9,** 554–56.

Ertel, H., 1942: Ein neuer hydrodynamischer Wirbelsete *Meteor. Z.,* **59,** 277–281.

Frederikson, J. S., 1979: The effects of long planetary waves on the regions of cyclogenesis: Linear theory. *J. Atmos. Sci.,* **36,** 195–204.

Green, J. S. A., 1970: Transfer properties of the large scale eddies and the general circulation of the atmosphere. *Quart. J. Roy. Meteor. Soc.,* **96,** 157–185.

——, 1977: The weather during July 1976: Some dynamical considerations of the drought. *Weather,* **32,** 120–128.

Holland, W. R., and P. B. Rhines, 1980: An example of eddy-induced ocean circulation. *J. Phys. Oceanogr.,* **10,** 1010–1031.

Holopainen, E. O., and A. H. Oort, 1981: On the role of large-scale transient eddies in the maintenance of the vorticity and enstrophy of the time-mean atmospheric flow. *J. Atmos. Sci.,* **38,** 270–280.

——, L. Rontu and N-C. Lau, 1982: The effect of large-scale transient eddies on the time-mean flow in the atmosphere. *J. Atmos. Sci.,* **39,** 1972–1984.

Hoskins, B., I. James and G. White, 1983: The shape, propogation and mean flow interaction of transient weather systems. *J. Atmos. Sci.,* **40,** 1595–1612.

Illari, L., 1983: Diagnostic study of a warm blocking anticyclone. To be submitted to *J. Atmos. Sci.*

Kuo, H. L., 1972: On the generalised potential vorticity equation for quasi-geostrophic flow. *Pure Appl. Geophys.,* N, 171–175.

Lau, N-C., 1978: On the three-dimensional structure of the observed transient eddy statistics of the Northern Hemisphere wintertime circulation. *J. Atmos. Sci.,* **35,** 1900–1923.

——, and J. M. Wallace, 1979: On the distribution of horizontal transports by transient eddies in the Northern Hemisphere wintertime circulation. *J. Atmos. Sci.,* **36,** 1844–1861.

Marshall, J. C., and G. L. Shutts, 1981: A note on rotational and divergent eddy fluxes. *J. Phys. Oceanogr.,* **11,** 1677–1680.

Rhines, P. B., and W. R. Young, 1982: Homogenization of potential vorticity in planetary gyres. *J. Fluid Mech.,* **122,** 347–36.

Savijarvi, H. I., 1978: The interaction of the monthly mean flow and large-scale transient eddies in two different circulation types, Part II. *Geophysica,* **14,** 207–229.

Shutts, G. L., 1983a: The propagation of eddies in diffluent jetstreams: eddy vorticity forcing of "blocking" flow fields. *J. Roy. Meteor. Soc.* (in press).

——, 1983b: Monthly-mean climatology and the parameterization of travelling weather systems. *Advances in Geophysics,* Academic Press.

Tucker, G. B., 1977: An observed relation between the macroscale local eddy flux of heat and the mean horizontal temperature gradient. *Quart. J. Roy. Meteor. Soc.,* **103,** 157–168.

White, A. A., and J. S. A. Green, 1982: A non-linear atmospheric long wave model incorporating parameterization of transient baroclinic eddies. *Quart. J. Roy. Meteor. Soc.,* **108,** 55–85.