NOTES AND CORRESPONDENCE

Comments on "Scalar Diffusion in the Convective Boundary Layer"

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13 November 1984 and 25 January 1985

Wyngaard (1984) has proposed a relatively simple parameterization of diffusion across the convective boundary layer based on the large eddy simulation (LES) results obtained by him and his colleagues (Wyngaard and Brost, 1984; Moeng and Wyngaard, 1984). A central feature of this parameterization is the incorporation of the difference between the effective eddy diffusivity for scalar diffusion down from the top of the convective layer and that for diffusion up from the bottom. This asymmetry exhibited by the LES results for the scalar transport in the buoyantly produced turbulence driven by surface heating cannot be simulated using first-order K theory.

In this comment we wish to address two questions related to the asymmetric diffusion. First, what level of turbulence closure is required to exhibit the asymmetry between bottom-up and top-down diffusivity? Second, how sensitive is Wyngaard's parameterization to the details of the asymmetric diffusion? Not surprisingly, we find that a level of closure which includes some diffusion of the second-order scalar correlations will produce the asymmetry. More surprising, we find that Wyngaard's parameterization is not sensitive to the asymmetric diffusion. In fact, it appears that precise symmetry could be imposed and make little practical difference in his end results.

Under the horizontally homogeneous, quasi-steady conditions assumed for the LES, an expression for K in buoyantly produced turbulence may be written symbolically as:

$$K = -\overline{w'c'}/(\partial C/\partial z)$$
$$= \tau_1\left[\overline{w'w'} - \left(\frac{g}{T}\overline{c'\theta'} - \frac{\partial}{\partial z}\overline{w'^2c'}\right)\Big/\partial C/\partial z\right], \quad (1)$$

where the time scale τ_1 is defined equal to

$$\overline{c'w'}\Big/\overline{\frac{c'}{\rho}\frac{\partial p'}{\partial z}}.$$

Under the same conditions the potential temperature-species correlation appearing in Eq. (1) may be written as:

$$\overline{c'\theta'} = \tau_2\left[-\overline{w'\theta'}\frac{\partial C}{\partial z} + K\frac{\partial\theta}{\partial z}\frac{\partial C}{\partial z} - \frac{\partial}{\partial z}\overline{w'c'\theta'}\right] \quad (2)$$

with τ_2 equal to the ratio of $\overline{c'\theta'}$ to the rate of dissipation of $\overline{c'\theta'}$. When Eqs. (1) and (2) are combined

$$K = \tau_1\left[\overline{w'w'} + \tau_2\frac{g}{T_0}\left(\overline{w'\theta'} + \frac{\partial\overline{w'c'\theta'}/\partial z}{\partial C/\partial z}\right) + \frac{\partial\overline{w'w'c'}/\partial z}{\partial C/\partial z}\right]\left[1 + \tau_1\tau_2\frac{g}{T_0}\frac{\partial\theta}{\partial z}\right]^{-1}. \quad (3)$$

Equation (3) is an exact expression for K for the quasi-steady horizontally homogeneous, buoyant convective layer as long as the τ's remain exact. If the τ's are taken as properties of the turbulence, independent of the C distribution and the third-order diffusion terms are ignored, then it is clear that K will also be a property of the turbulence only.

Figures 8 and 10 of Moeng and Wyngaard (1984) for the τ's as given by their LES results suggest that for this problem the τ's may be taken as properties of the turbulence. Thus, the asymmetry appears to be imposed by the turbulent transport of either $\overline{c'\theta'}$ or $\overline{w'c'}$.

The answer to our first question is that a level of closure that incorporates turbulent diffusion of the second-order species correlations is required to produce a proper asymmetry in the top-down bottom-up diffusion.

Figure 1 compares Wyngaard and Brost's bottom-up and top-down diffusivities with values we obtain from the second-order closure model of Lewellen (1977) which uses a simple gradient diffusion model for the turbulent transport of both $\overline{c'\theta'}$ and $\overline{w'c'}$. The most obvious difference between the LES results and the second-order closure results is the singularity which appears in K_b near $z = 0.65z_i$ in the SOC results, but doesn't appear in the LES results. However, this is apparently more a result of uncertainties in the LES result than it is a true difference between the LES and the SOC results. Moeng and Wyngaard (1984), in a recalculation of the LES, present results for the normalized gradient g_b which go through zero a little above $z = 0.6z_i$. Thus, their more recent results would give a singularity in K_b near the same value of z as given by the SOC results. Since the turbulent transport terms are divided by the concen-

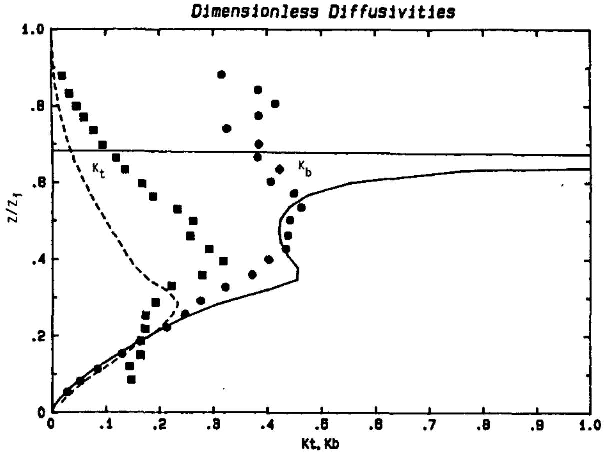

FIG. 1. Comparison of the dimensionless scalar diffusivities as obtained by the LES of Wyngaard and Brost (1984) with those from a simple second-order closure model.

tration gradient in Eq. (3), relatively small errors in the turbulent transport can lead to large errors in K in the middle of the convective layer where the gradient is very small. We conclude that the asymmetry between K_b and K_t exhibited by the SOC model is qualitatively similar to that of the LES. Presumably we could use terms of the type proposed by Zeman and Lumley (1976) to adjust the modeled turbulent transport terms and make our profiles of K_t and K_b come closer to the LES. As more detailed LES results become available this may be desirable.

The next question we address is how important is this K asymmetry in Wyngaard's proposed parameterization? We approach this question by repeating Wyngaard's analysis with the top-down dimensionless gradient g_t multiplied by a factor K_1. The parameterizations provided in Eqs. (21) and (23) are then modified by replacing cw_1 and w_e with cw_1/K_1 and w_e/K_1, respectively. The only other change is Eqn. (29) for Δ_t. It becomes

$$\Delta_t = (2.0/K_1)[(\alpha w_e/w_* K_1)^{-1/3} - 1]. \quad (4)$$

These changes force two types of changes in the resulting unmixed layer profile. First, h_1 is moved a little further from z_i as K_1 is decreased, and second the gradient of C in the interfacial region is increased (or decreased) as K_1 is increased (or decreased). To make this sensitivity more specific, consider Wyngaard's boundary conditions for humidity-like scalars. Then his Eqs 26–31 yield

$$\overline{cw_1} = w_e\left[(C_S - C_2) - (C_S - C_0) \times \left(1 + \frac{\Delta_b w_S}{w_*}\right)\right]\left(1 + \frac{w_e\Delta_t}{w_*}\right). \quad (5)$$

If the top-down and bottom-up diffusivities were made symmetric by making $K_1 \approx 0.4$, then $\overline{cw_1}$ would be decreased by $\approx$20% for typical PBL conditions by virtue of $w_e\Delta_t/w_*$ increasing from $\approx$0.2 to $\approx$0.4. However, since w_e cannot in general be estimated to within an accuracy of 20%, this change is of little practical significance until better parameterizations of the entrainment across the inversion are available.

Wyngaard's Section 4 is devoted to arguing that his closure is valid for a wide variety of typical conditions. We believe that this is true, but not because the LES results for g_t are valid for all these conditions. It is true because the critical features which allow the parameterization to work are that K go smoothly to a reasonable representation of the surface layer dynamics in the lower part of the layer and that K go smoothly to a small value within the capping layer. We believe the LES results for g_t in the upper part of the boundary layer are likely to be sensitive to such things as strength of the inversion and nonlinearity in the flux profile, but this sensitivity is of little consequence until a much better parameterization of the inversion layer is available.

In summary, the asymmetry in scalar diffusion within the quasi-steady, homogeneous, convective layer is an interesting feature which can be used to test diffusion models. Second-order closure has a definite advantage over lower level closure in simulating this feature. However, further improvements in the parameterization of entrainment across the inversion are necessary before accurate simulation of this interesting feature is very significant in practical mixed-layer parameterizations such as that proposed by Wyngaard.

Acknowledgment. This work was supported by the Naval Environmental Prediction Research Facility under Contract N00228-83-C-3073.

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