The Structure, Energetics and Propagation of Rotating Convective Storms. Part II: Helicity and Storm Stabilization

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ABSTRACT

Rotating "supercell" thunderstorms are shown to be characterized by high helicity, the vector inner product of velocity and vorticity, which is obtained both from the mean flow in which they are embedded and from buoyancy enrichment. Some unique properties of supercell helical flow are described, including a tendency for trajectory rotation to be reversed from parcel vorticity. A simple helical (Beltrami) flow model resembles gross supercell structure and also provides a prediction of storm motion. Since theory, closure model calculations and numerical simulations indicate that helicity suppresses turbulent dissipation, it is suggested that supercells owe their noted stability and long life to this effect. Enhanced predictability of such storms is then expected and is apparently seen in some results of Williamson and Klemp. It is concluded that rotating storm structure and propagation must involve a compromise between the energetic effects discussed by Lilly in Part I of this study and those considered here, but that the helicity effects seem to be dominant in long-lived storms.

1. Introduction and scope

This article continues the analysis, begun in Part I (Lilly, 1986; hereafter referred to as Part I) of the evolution, structure and energetics of rotating convective storms, or "supercells." The aspect pursued here is a hypothesis that these storms owe their long life, stability, and apparent predictability to the helical nature of their circulation. In section 2 it is shown, from model and observational results, that helicity, the vector inner product of vorticity and velocity, is typically large in both the storms and their environment, and can be exchanged between them. In section 3 it is shown that the gross structure and movement of supercell storms can be modeled as a purely helical (Beltrami) flow. Section 4 presents a review of the evidence from turbulence theory that helicity suppresses the inertial range energy cascade and tends to isolate the large energy- and helicity-containing scales from the inertial range and dissipation scales. The implication is that rotating thunderstorms, which contain and produce helicity, are less susceptible to nonlinear stirring and dissipation than are nonhelical turbulent flow fields, such as ordinary thunderstorms. In section 5, additional interpretations are made from the results of Part I and this article.

Since completing this work I have become aware of a series of papers by Levich, Tsonber and colleagues which strongly support and elaborate on the importance of helicity within the general framework of turbulent flow. The energy preservative aspect of helicity, discussed further in section 4, is invoked as an explanation for the high intermittency of turbulent energy dissipation by Levich and Tsonber (1983a,b), Tsonber and Levich (1983) and Levich et al. (1984). It is proposed that, since helical eddies resist dissipation, they will survive longer than other turbulent eddies and will tend to dominate the flow statistics. Thus most large eddies will be characterized by high helicity and low dissipation, while most dissipation will occur in the transitional and short-lived regions of low helicity. This hypothesis has now been tested against results of a numerical simulation of the Navier-Stokes equations (Pelz et al., 1985) with favorable results, as described in section 4.

The possible effects of helicity on atmospheric motions have been considered by Levich and Tzetkov (1984, 1985), where the term "helical cyclogenesis" is introduced. It is hypothesized that the mean square of helicity is subject to an upscale "cascade", somewhat similar to that of two-dimensional turbulent energy. Following this approach the authors suggest that several kinds of mesoscale disturbances, including tropical cloud clusters, mesoscale convective complexes and squall lines, develop from convective-scale energy sources which organize themselves into large scales through the previously discussed upscale transfer process. It is also noted that supercell storms show helical flow structure.

2. Helicity and its relevance to rotating storms

Helicity, here denoted by \( H \), is defined as

\[ H = \mathbf{V} \cdot \omega, \quad \omega = \nabla \times \mathbf{V}. \tag{1} \]

As a covariance of the velocity and vorticity vectors,
helicity may have either sign, and its magnitude and sign may not be invariant to a Galilean transform. A dimensionless correlation, obtained by dividing $H$ by the local scalar magnitudes of velocity and vorticity, is called relative helicity, $\text{RH}$, i.e.,

$$\text{RH} = \frac{H}{\nu \omega}.$$  (2)

The concept and the name of helicity were introduced (as best as I can determine) by Betchov (1961) and the subject has been pursued fairly actively in the fluid dynamic and magnetohydrodynamic literature for about the last 12 years. The significance and treatment of helicity, particularly with respect to magnetohydrodynamics, were surveyed by Moffatt (1978). Moffatt reserved the term “helicity” to refer to a volume-integrated quantity, using “helicity density” for the local variable. In the interests of brevity, however, and considering a similar notational ambivalence with respect to energy, I will use helicity for both local and integrated quantities, distinguishing between them when necessary.

Both simulations and observations of long-lived rotating storms show strong correlations between velocity and vorticity. Figure 1, taken from Klemp, Williamson, and Ray (1981) (referred to as KWR, henceforth) shows the hodograph of winds observed in the vicinity of a tornado-producing storm in Oklahoma and used as the environmental flow for numerical simulations of it. The wind shear, as observed from the moving storm (point X) is mostly in direction, not speed, so that the mean horizontal velocity and vorticity fields are nearly parallel. Figure 2 shows maps of the vertical components of vorticity and velocity at various levels for the KWR model simulation and for two observation times of the real storm. The centers of the corresponding maxima are quite close and the correlations appear high, especially at the middle levels.

Brandes (1984) carried out additional analyses of this storm. Figure 3 shows his maps of the horizontal velocity and vorticity vectors at the 1.3 km level. The agreement between vector fields is striking.

Weisman and Klemp (1982) carried out a set of simulations for a range of values of buoyant instability and unidirectional shear. Figure 4 shows cross sections of vertical velocity and vorticity at the 4.6 km level, with updraft and downdraft regions shown shaded for three different nominal shear values and two time intervals after start of the simulation. Only the larger two shear values produce splitting and rotating storms, and they do so starting between 30 and 80 minutes after initiation. For each of the split storm cases the vorticity and velocity again appear to be well correlated.

More detailed analyses have been carried out on a set of simulation data provided by M. Weisman, which have a nominal shear of 30 m s$^{-1}$ and a similar structure to the events depicted in the middle and lower panels of Fig. 4. The initial mean flow consisted of a unidirectional shear profile, with maximum southerly shear at the surface, decreasing to zero above 7 km. The hodograph for this profile is shown by the solid line on Fig. 5, given in coordinates relative to the storm motion. The dashed line hodograph corresponds to the actual mean flow developed by the model at the time of storm analysis, 7200 seconds after initiation. The difference between these flow profiles, of order 2 m s$^{-1}$, has maxima at the surface and near the 5 and 10 km levels. The upper and lower level maxima are probably caused mostly by the difference in velocities of the anvil and cold air outflows from those of the environment. The 5 km maximum may be associated with the development of a strong midlevel circulation.

The solid and dotted curves of Fig. 6 show the profiles of relative helicity corresponding to these mean flow profiles. The relative helicity of a mean shear flow is given by

$$\text{RH}(\mathbf{V}) = \frac{\mathbf{k} \cdot (\partial \mathbf{V} / \partial z \times \mathbf{V})}{|\partial \mathbf{V} / \partial z|}.$$  (3)

where the overbars are horizontal averages. If $\alpha$ is the wind direction, measured clockwise, this may also be written as

$$\text{RH}(\mathbf{V}) = -\frac{\mathbf{V} \partial \alpha / \partial z}{[(\mathbf{V} \partial \alpha / \partial z)^2 + (\partial \mathbf{V} / \partial z)^2]^{1/2}}.$$  

Thus, RH is positive or negative unity where $\mathbf{V}$ is a local maximum or minimum. This occurs at about the 2.5 km level for the initial mean flow, and at nearly the same place for the final state. In addition, however, three other levels have unit positive or negative helicity in the final state. When the speed and direction shear both vanish, RH is undefined. For the initial Weisman–Klemp profiles, however, $\mathbf{V}$ is constant and $\partial \mathbf{V} / \partial z > 0$, so that $\text{RH} = -\mathbf{V} / \mathbf{V}$. Very minor changes in the mean

![Figure 1](image1.png)

**FIG. 1.** The composite smoothed hodograph of the winds in the environment of a rotating severe storm in central Oklahoma on 20 May 1977. This hodograph was used to initialize the simulations shown by Klemp et al. (1981), from which the figure was obtained.
flow profiles lead to large changes in relative helicity in the upper levels of Fig. 6.

The heavy and light dashed lines on Fig. 6 depict the relative helicity of the total and disturbance flow, respectively. Relative helicity of the total flow is here defined as

\[ \text{RH}(V) = \frac{\omega \cdot \mathbf{V}}{(\omega^2 \mathbf{V}^2)^{1/2}} \]  

while the relative helicity of the disturbance flow is defined as

\[ \text{RH}(V') = \frac{\omega' \cdot \mathbf{V}}{(\omega'^2 \mathbf{V}^2)^{1/2}}. \]  

By these definitions, there is no requirement for the total relative helicity to lie between that of the mean flow and disturbance. At most levels that is the case,
model, since in a real three-dimensional turbulent fluid the total enstrophy is dominated by scales close to the viscous cutoff, a few centimeters. They indicate, however, that the simulated large-eddy structure of the rotating storm model is strongly helical.

The influence of the largest eddies can be further demonstrated by computing helicity from smoothed velocity fields. Figure 7 shows the unfiltered disturbance relative helicity (dotted curve), and that computed from a velocity field subject to a four-point lateral smoothing once (dashed curve) and three times (solid curve). Except near the top and bottom, the helicity field is evidently dominated by the larger spatial scales.

Helicity, like kinetic energy, is a quasi-conserved property of the flow, and consideration of the conservation equations aid in understanding its observed structure and evolution. I start with Boussinesq or anelastic equations of three-dimensional inviscid motion in the form

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \nabla \pi - \mathbf{k} b = 0,$$  \hspace{1cm} (6)

where $\pi = (p - \bar{p})/\rho$ and $b$ is a measure of buoyancy, such as $b = g(\theta_0 - \theta_3)/\theta_0$, with $\theta_0$ virtual potential temperature. Upon taking the curl of (6), the vorticity equation is obtained as

$$\frac{\partial \omega}{\partial t} + \mathbf{V} \cdot \nabla \omega + \omega \nabla \cdot \mathbf{V} - \omega \nabla V - \nabla b \times \mathbf{k} = 0.$$  \hspace{1cm} (7)

By forming the inner product of (6) with $\omega$ and of (7) with $\mathbf{V}$ and summing, I obtain the helicity equation, i.e.,

$$\frac{\partial H}{\partial t} + \nabla \cdot (\mathbf{V} H) + \nabla \cdot [\omega(\pi - V^2/2)]$$

$$= \mathbf{b} + \mathbf{k} \cdot (\mathbf{V} \times \nabla b).$$ \hspace{1cm} (8)

If the boundaries are infinite, periodic, or allow for no normal components of velocity and vorticity, volume-integrated helicity is conserved in the absence of buoyancy effects. As pointed out by a referee of an earlier version of this paper, however, the last term on the left leads to a contribution to the generation of volume-integrated helicity equal to $\iint (\pi - V^2/2) dx dy$ evaluated at the lower boundary minus the expression at the upper boundary. In a strong cyclonic vortex with low central pressure at the surface this term constitutes a helicity sink and perhaps is related to the decay of storm circulation often observed with tornado formation (Brandes, 1984).

The two buoyancy-containing terms on the right of (8) produce equal contributions to a volume integral, since the last can be rewritten as

$$\mathbf{k} \cdot \nabla b = \nabla \cdot (\mathbf{k} \times \mathbf{V} b) + \mathbf{b}.$$ \hspace{1cm} (9)

The two forms have locally different interpretations, however. The product of buoyancy and vertical vorticity is analogous to the kinetic energy generation by
buoyancy flux. It is also analogous to vortex stretching, in that it can only exist when vorticity is already present. The last term on the right of (8) is locally effective when the flow is normal to a horizontal buoyancy gradient, e.g., flow along a cold air outflow boundary. At such a boundary, the cold air tends to undercut the warm air and produces vorticity in the direction parallel to the boundary, i.e., along the streamlines. Klemp and Rotunno (1983) show evidence that such a region along the edge of a forward-flank downdraft is a local source of vorticity for tornadoes.

By analogy to energy conservation one may define potential helicity in the form:

$$\text{potential helicity} = -2b \int \xi dt,$$

(10)

where the integral is taken along a trajectory. When buoyancy is conserved, the total time derivative of (10) is $-2b\xi$, which, when integrated over a volume, is the negative of the right side of (8), similarly integrated. This Lagrangian time integral of vorticity is perhaps less easy to interpret physically than the analogous
In convective storm dynamics the vertical pressure derivative is found (Schlesinger, 1984) to be generally smaller than buoyancy but partially cancels it in the maximum updraft regions. Thus Eq. (11) suggests that the primary source of vertical helicity is the $b\xi$ term, while the $\xi \partial \tau / \partial z$ term transfers a portion of it to horizontal helicity.

It is also useful to consider the exchange of helicity between mean and disturbance-flow components. If the mean vertical and lateral boundary terms can be neglected, the equation for mean horizontal velocity is simply

$$\partial \mathbf{V}_H / \partial t = -(\partial / \partial z)(\mathbf{w} \cdot \nabla \mathbf{V}_H),$$

and that for mean horizontal vorticity, $\omega_H = k \times \partial \mathbf{V}_H / \partial z$, is

$$\partial \omega_H / \partial t = -(\partial^2 / \partial z^2)(k \times \mathbf{w} \cdot \nabla \mathbf{V}_H).$$

A mean flow may have helicity if its shear has a component normal to the flow direction. The mean flow helicity is then governed by the equation

FIG. 5. Mean flow hodographs, relative to storm motion, for a Weisman–Klemp simulated storm with a nominal 30 m s$^{-1}$ mean shear. The solid vertical line represents the initial state hodograph while the dashed line is the hodograph observed at 7200 s with the difference indicated by vectors. The altitude of each hodograph point is labeled in kilometers.

Boussinesq expression for potential energy, $-b \int w dt = -bz$. One might visualize each vortex tube as a wound-up spring, which unwinds as it rises from buoyancy.

Conservation equations for separate components of helicity can also be obtained. The vertical component is of special interest. By multiplying the vertical component of (6) by $\xi$ and that of (7) by $w$, one obtains a relation for $H_z = \xi w$ in the form:

$$\partial (\xi w) / \partial t + \nabla \cdot (\nabla \xi w - \omega w^2/2) + \xi \partial \tau / \partial z = b\xi.$$

The last term on the left can be interpreted as the transformation from vertical to horizontal helicity, as it also appears with opposite sign in the equation for horizontal helicity. Note that half of the buoyancy generation term contributes directly to vertical helicity and the other half to the horizontal component.

FIG. 6. Relative helicities computed from the mean flow shown on Fig. 5 and the corresponding disturbance motion fields. The relative helicities are defined by Eqs. (3), (4), and (5).
boundary, however, since the flux terms in (11) vanish when \( w = 0 \).

Figure 6 shows that in the outflow layer the mean and disturbance helicity become negative. I believe this result is produced by reversal of sign of the \( \partial \) correlation at the top of the updraft where buoyancy is reversed but vorticity tends to conserve its sign. Since \( w \) does not immediately reverse its sign, the negative helicity must appear as a negative product of downstream velocity and vorticity in the anvil outflow. This would appear to require lateral asymmetries in the anvil outflow.

3. Beltrami flows and their applications

Beltrami flows are defined by the requirement that the vorticity and velocity vectors are parallel, i.e.,

\[
\omega = \kappa \mathbf{V}. \tag{15}
\]

Thus they have unit relative helicity. The proportionality factor \( \kappa \) need not be a constant, but for an incompressible or anelastic flow it is constrained by continuity to be proportional to mean density along a streamline, since application of the divergence operator to (15) leads to

\[
\nabla \cdot \omega - \frac{\kappa}{\rho} \nabla \cdot (\mathbf{V}) = \mathbf{V} \cdot \nabla \left( \frac{\kappa}{\rho} \right) = 0. \tag{16}
\]

As stated earlier a horizontal mean flow with a rotating hodograph is purely helical, e.g.,

\[
\bar{u} = -M \cos \left( \int \kappa dz \right), \quad \bar{v} = M \sin \left( \int \kappa dz \right) \tag{17}
\]

where \( \kappa = \kappa(z) \) and \( M \) is the (constant) velocity amplitude. The mean wind direction, say \( \bar{\alpha} \), turns with height at the rate

\[
\frac{\partial \bar{\alpha}}{\partial z} = -\kappa. \tag{18}
\]

A somewhat more complex Beltrami flow is a series of roll convective elements with jets at the centers of circulation. For example, suppose that the rolls lie along the \( y \)-axis and the fluid is incompressible. Then the velocity components are \( u = \partial \psi / \partial z \) and \( w = -\partial \psi / \partial x \), with \( \psi(x, z) \) a streamfunction. Choose \( \psi \) to be periodic in both \( x \) and \( z \), e.g.,

\[
\psi = A \cos kx \sin mz. \tag{19}
\]

If we now assume that a roll-tangential velocity component is proportional to the stream field, e.g.,

\[
v = -\kappa \psi, \tag{20}
\]

then the flow will satisfy the Beltrami conditions with \( k^2 = k_x^2 + m_z^2 \), the total wave number squared. This flow is schematically illustrated in Fig. 8.

A three-dimensional swirling Beltrami flow can be
Fig. 8. Schematic illustration of a purely helical (Beltrami) flow, consisting of overturning rolls with longitudinal jets at the circulation centers, as described by Eqs. (19) and (20).

derived for a Boussinesq fluid by first defining a vector streamfunction $\psi$ such that

$$\mathbf{V} = \nabla \times \psi.$$  \hspace{1cm} (21)

Then if

$$\psi = \kappa n S - n \times \nabla S,$$  \hspace{1cm} (22)

with $n$ a constant vector, $\kappa$ a constant and $S(x, y, z)$ a scalar function, $\mathbf{V}$ can be shown to be a Beltrami flow if $S$ is harmonic with $\kappa$ the wave number, i.e.,

$$\nabla^2 S + \kappa^2 S = 0.$$  \hspace{1cm} (23)

Specifically I choose $n = k$, the unit vertical vector, and

$$S = S_0 \cos kx \cos y \sin mz.$$  \hspace{1cm} (24)

Then the velocity fields are

$$\mathbf{V} = \nabla_H \frac{\partial S}{\partial z} - \kappa k \times \nabla_H S + \kappa k^2 S,$$  \hspace{1cm} (25)

where $\kappa^2 = k^2 + l^2$ and $\kappa^2 = \kappa_H^2 + m^2$. This flow consists of a checkerboard of alternate swirling updrafts and downdrafts. Each updraft cell contains convergence at the bottom, divergence at the top, and a rotating updraft in the middle. A combination of a rotating mean shear flow like that of Eq. (17), with $\kappa$ constant, and this swirling updraft bears, except for the periodicity, a qualitative resemblance to the flow in a rotating thunderstorm like those of Figs. 2–4.

Flow trajectories have been calculated from the disturbance flow fields given by (24) and (25) plus the mean flow profile

$$\bar{u} = M \sin \kappa (z - h/2), \quad \bar{v} = M \cos \kappa (z - h/2),$$  \hspace{1cm} (26)

with $h$ the height of the domain of interest, by use of the equations

$$dx/dt = u, \quad dy/dt = v, \quad dz/dt = w.$$  \hspace{1cm} (27)

The integrations were done using a Runge–Kutta scheme. Accuracy was checked by repeating some calculations with one-half and twice the time step, for which differences were generally less than about 0.01% of the integrated distances. The results are presented on Fig. 9, which shows horizontal projections of various three-dimensional trajectories, with labels provided at points where they pass through even kilometer levels. The values of the parameters chosen are

$$S_0 = \frac{2.25}{\pi^2} \times 10^9 \text{ m}^2 \text{ s}^{-1}, \quad M = 15 \text{ m s}^{-1},$$

$$\pi/k = \pi/l = \pi/m = h = 15 \text{ km}.$$  

These values are chosen to roughly replicate the features of the storm studied by KWR with the reference frame given by the moving storm. The maximum vertical velocity is $(k^2 + l^2)S_0 = 20 \text{ m s}^{-1}$. Note that $\kappa = \sqrt{3} m$, so that the assumed mean wind vector turning from the bottom to the top of the updraft is $\sqrt{3} \pi = 312^\circ$, somewhat greater than the total turning of the KWR hodograph shown on Fig. 1. Each trajectory is centered at $y = 0$ and the level of maximum vertical velocity, $z = h/2 = 7.5 \text{ km}$, and at $x$ values of $3, 1.5, -1.5, -3, -4.5, \text{ and } -6 \text{ km}$ (trajectories $A'E'$, respectively). Integrations are carried out both backwards and forwards from these points, with the results symmetric about the $x$-axis. All trajectories are stopped when they reach an edge of the updraft region, $|x|$ or $|y| = 7.5 \text{ km}$.

Trajectories $A'E'$ turn anticyclonically everywhere, with increasing vertical excursions and increasingly tighter horizontal curvature as parcels cross the maximum updraft level further to the left. Trajectories $D'$ and $E'$ cross themselves as the curvature becomes still tighter near the center level. Note that the maximum vertical excursion occurs for trajectory $D'$, rather than for trajectories crossing the maximum updraft. This is because the cyclonic rotation opposes the northward mean flow to the left of the center, so that a parcel is held in a strong updraft region longer. A stagnation point in the horizontal velocity field occurs at $x = -5 \text{ km}, y = 0, z = h/2$, and to the left of that point the horizontal velocity is southward along the $x$-axis. A trajectory passing through the stagnation point would be cusped in the $x$-$y$ plane. Trajectory $F'$ passes to the left of the stagnation point and undergoes a tight cyclonic turning between the long regions of anticyclonic curvature. Trajectories to the left of $F'$ will be qualitatively similar, while one passing through the left boundary at $x = -7.5 \text{ km}$ simply slides along that boundary at a constant altitude.

Flow helicity can be seen by comparison of adjacent trajectories, for example $B'$ and $C$. The latter starts to the right of and lower than the former, then crosses under it, rises to the same level on the left side, continues to rise and cross over again, finishing to the right and higher. Thus parcels following the two trajectories rotate around each other clockwise as they progress downstream. Similar relationships occur for each pair except $E'$ and $F'$, for which helicity is still evident but no crossings occur.

Thus we see that when the vertical velocity amplitude is smaller than or about the same as the total mean
flow shear, the trajectories of the above combination of helical flows exhibit a characteristic reverse twist. Their horizontal projections rotate in a direction opposite to that anticipated by the sign of their vertical vorticity. This is because each updraft stream and vortex tube, as it rises, tends to follow the mean flow. The mean flow hodograph, however, rotates clockwise if the helicity is positive. Thus the horizontal projection of a rising stream tube with positive vertical vorticity also follows a clockwise curve.

Similar effects exist in real and simulated storm flows, as shown by KWR. Figure 10a, taken from that work, shows a series of storm-relative trajectories constructed from the simulated (left) and observed (right) flow fields, all of which start at the 1 km level. These trajectories are actually relative streamlines as a steady state assumption was applied, an assumption which may lead to errors for trajectories moving through slow velocity regions. Trajectory C in the simulated field and B in the observed field (follow the solid lines; the dashed ones are for simulated hydrometeors) are similar to several of those of Fig. 9. Trajectories D in the simulated and C in the observed flows are different, however, in that they have long regions of cyclonic curvature at low levels before entering the updraft and turning anticyclonically. Low-level cyclonic flow is produced in the storms by processes evidently not present in the Beltrami flow model, especially the intense vortex stretching discussed by Klemp and Rotunno (1983). Figure 10b, also from KWR, shows a perspective view of three-dimensional trajectories starting at a level lower than those of Fig. 10a. Their trajectories A–F are mostly similar to C–E′ of Fig. 9. No trajectories similar to F′ are shown, apparently, because the KWR simulation contained almost no southward motion vectors in the principal updraft region at its maximum level. The helicity of the trajectories in Fig. 10b is apparent as they cross each other in similar ways to those of the analytic model. Also, as noted by KWR, the trajectories with maximum altitude gain pass through midlevel along an east–west line and a little to the left of the updraft center. These features suggest that the helical flow model is a good first-order approximation to the storm flow field. In both cases the results appear to compel a qualitative change in Browning’s schematic circulation pattern.
(Fig. 1 of Part I), which envisioned updraft parcels following cyclonic trajectories. In an earlier observational study, Browning and Donaldson (1963) had found some trajectories turning cyclonically and others anticyclonically.

The Beltrami flow model is probably most relevant when the mean flow hodograph has strong curvature, like that of the KWR storm. Because of the apparent high helicity in all rotating storms, however, one might use it to approximate the flow in less curved or even straight hodographs, and thereby to estimate the lateral movement of storms embedded in such flows. Assume that the storm adjusts its propagation rate so as to equalize the Beltrami flow parameter, \( \kappa \), between the storm disturbance and the mean flow. Since \( \kappa \) is the three-dimensional wave number of the disturbance and also the ratio of mean shear to the cross-shear propagation rate, this assumption allows prediction of storm propagation from geometric factors and mean shear alone. The equating relationship is

\[
\kappa^2 = \left( \frac{2\pi}{L_x} \right)^2 + \left( \frac{2\pi}{L_y} \right)^2 + \left( \frac{\pi}{h} \right)^2 = \left( \frac{dU/dz}{-c_y} \right)^2, \tag{28}
\]

where \( L_x \) and \( L_y \) are the horizontal wavelengths of the updraft. Solving for \( c_y \), the \( y \)-propagation rate, yields

\[
c_y = \pm \gamma h dU/dz, \quad \gamma = (1 + 8h^2/L^2)^{-1/2} \pi^{-1} \tag{29}
\]

where \( L = L_x = L_y \) is assumed. For the conditions used in the previous trajectory calculations \( \gamma = 0.2 \).

It is difficult to decide how seriously to take (29) as a prediction algorithm, because of the extreme idealism of the assumptions going into it and the vagueness of real data evaluation of such quantities as \( L \) and \( h \), as well as their complete availability if forecasts are to be made from prestorm environments. If we simply use the nominal value of \( \gamma \) found above and assume \( L \) to be twice a nominal buoyant updraft width of 12 km, the equation seems to give reasonable values. Figure 11 is a plot of the mean hodograph and storm
propagation vectors for nine isolated supercells, composed by Bluestein and Jain (1984). Using the mean shear between 1.6 and 4 km, $5 \times 10^{-3}$ s$^{-1}$, and the previously quoted parameters, a lateral motion of 12 m s$^{-1}$ is predicted, 2 or 3 m s$^{-1}$ larger than found by Bluestein and Jain. Similar results are obtainable from the Bluestein–Jain composites for backbuilding squall line storms, which have many supercell characteristics.

4. Helicity effects on turbulence

Besides its use in illustrating the structure of rotating storms, the concept of helicity suggests the existence of fundamental differences of behavior between storms with and without strong rotation. This is because helicity affects turbulent flow in a way which essentially decreases the nonlinear interactions between scales. The fundamental reason for this is easily seen from inspection of the vector equations of motion and vorticity. A conventional alternate form of expression of the nonlinear momentum advection term is given through a vector identity as

$$ \mathbf{V} \cdot \nabla \mathbf{V} = \nabla (V^2/2) + (\nabla \times \mathbf{V}) \times \mathbf{V}. \quad (30) $$

In the case of purely helical flow the second term on the right vanishes. Upon substituting this into (6) and taking the curl, it is seen that the sum of the nonlinear terms present in the vorticity equation, (7), also vanish. In purely helical flow, stream tubes and vortex tubes are identical. In this situation vortex stretching and tilting, which are crucial to the development of inertial ranges of turbulence, are exactly balanced by advection.

The effects of helicity in turbulent flow have been discussed in several papers in fundamental fluid dynamics. Among the most relevant here is the work by André and LeSieur (1977). They carried out integrations of the Markovian, eddy-damped, quasi-normal closure model for decaying homogeneous and isotropic turbulence at high Reynolds number. The initial state consisted of highly peaked spectra with essentially no dissipation, i.e., flow which would not ordinarily be described as turbulent. For the case without helicity an inertial spectral tail developed rapidly, as shown on Fig. 12. When multiplied by the $5/3$ power of wavenumber and plotted on a linear ordinate scale, as in Fig. 13, the deviations from the inertial range are emphasized. These include, principally, a gradual rise at the large-scale energy-containing end and a small hump at the beginning of the dissipation range.

The results of similar integrations made for the case of large initial helicity are shown on Figs. 14 and 15. On the first, the evolution of the spectrum is superficially similar, but about twice as much time is required to establish an inertial range than for the non-helical case. An inertial range does eventually become established because the total helicity is essentially conserved, but mean squared vorticity is not. Thus relative helicity is confined to the larger scales, with the helicity effect on the inertial range becoming insignificant. The most revealing result shown in Fig. 15 is that the energy spectrum deviates largely from the inertial range for the helicity-containing flow. As scaled by the $2/3$ power of the dissipation rate, the energy of the largest scales is enhanced by a factor of 2 or 3 over that of the non-helical case, and is apparently almost decoupled from the inertial range. Alternatively, one can say that for a given kinetic energy level, a highly helical flow dissipates only about $1/4$ as fast as a turbulent flow without helicity.

A more dramatic example of the apparent importance of helicity in turbulent flow arises from the results of three-dimensional numerical simulations of the Navier-Stokes equations, analyzed by Pelz et al. (1985). Figure 16a, b, from that work, summarizes the critical results from a channel flow simulation with no-slip boundaries at the top and bottom and periodic boundaries down- and cross-stream. These show the condi-

![Fig. 12. Temporal evolution of an isotropic energy spectrum $E(k, t)$, obtained from integrations of the Markovian eddy-damped quasi-normal closure model. No helicity. Initial spectrum $E(k) \sim k^\alpha \exp(-2k^2)$, Reynolds number $5.24 \times 10^9$. (From André and LeSieur, 1977).](image-url)

![Fig. 11. A composite hodograph for nine isolated supercells in Oklahoma, from Bluestein and Jain (1984). The vector indicates the mean storm motion.](image-url)
tional probability densities of relative helicity, labeled here as the cosine of the angle between the vorticity and velocity vectors. In panel 1 the sample data are taken from flow regions where dissipation is less than 5% of its maximum value, while the data for panel b come from regions where the dissipation is greater than 30% of its maximum. Clearly the high dissipation regions have much smaller helicity than those of low dissipation. The probability densities are stated to be “nonnormalized,” so that the larger ordinate values in the low dissipation panel presumably indicate that category to be about 10 times more frequent than the high dissipation category.

The results shown by André and LeSueur (1977) and Pelz et al. (1985), when applied to the convective storm problem, strongly suggest that the effect of helicity is to reduce the diffusing and dissipating effects of turbulence upon rotating storms. That is the central hypothesis of this paper. While it has not yet been subjected to any critical tests using storm simulations or observational data, several qualitatively observed phenomena are consistent with it. First is simply the long life, well-defined organization, and presumably high energy efficiency of these storms. Second is the remarkable and immediate success obtained by numerical models in simulating them. Based on the results of earlier cloud models and of three-dimensional simulations of turbulence by Deardorff, it had been thought that the initial low-resolution simulations, carried out on the CDC 7600 computer by Klemp et al. (1981), would suffer severely from resolution problems, and that only the extra speed attainable on the CRAY-1 computer could lead to reasonably successful results, and even then the model would be straining the computer. Instead the observed low sensitivity to the coarseness of spatial resolution has allowed extensive series of experiments to be carried out by Weisman and Klemp (1982, 1984) using horizontal grid spacings as large as 2 km. Comparable insensitivity to resolution apparently does not exist for less rotationally constrained storms, such as those observed in the hailstorm observing programs in northern Colorado, see e.g., Clark (1979). In addition, sensitivity tests have shown that alterations to the moderately sophisticated subgrid closure schemes used by Klemp et al. (1981) do not significantly alter the simulated evolution. While these pieces of somewhat anecdotal evidence may be unconvincing by themselves, they are uniformly consistent with the theoretical predictions.

A final consequence of the helicity hypothesis is an expectation of increased predictability for helical flow. While this has not been confirmed by, for example, a turbulent closure model, it seems completely consistent with what is known about the predictability of turbulent flow. The partial isolation of the energy-containing eddies from those of the inertial range suggests that errors in observation or analysis at small spatial scales are not likely to infect the larger scales very quickly. Enhanced predictability is also suggested by some of the simulation results. Wilhelmson and Klemp (1981) discuss the evolution of a group of storms observed on 3 April.
1964, before the availability of Doppler radar. As tracked from their reflectivity profiles it appears that most of these storms arise from multiple splitting of a single original storm, together with the products of gust front interactions between the split storm products. Figure 17 shows the environmental sounding for this event and Fig. 18 maps out the evolution of the observed (left) and predicted (right) storms. The strongly sheared wind profile indicates that rotating storms should be expected, and in the simulation the products of the initial split both rotate in the expected directions. The evolution after the split is somewhat complex, but in most respects is faithfully mirrored by the simulation for 4 hours. The model grid square interval is 2 km, so that wave lengths less than 8–12 km, about half the average supercell diameter, cannot be well represented.

Wilhelmson and Klemp (1981) are rather cautious in claiming true predictability, and no other examples have been shown of verifiable simulations over such a long period. I regard the results, however, as strongly indicative of a much greater potential predictability than one would normally anticipate. It is commonly assumed that an e-fold growth in error occurs in about an eddy turnover time, i.e., the time required for fluid to pass through a physical system. For large-scale cyclonic eddies this scaling time is a few days, and correspondingly the practical limit for predictability is probably one to two weeks. The turnover time for thunderstorms is of order 30 minutes, so that the apparent predictability time from the Wilhelmson–Klemp experiment is several times what would ordinarily be expected, especially considering that the initial disturbance configuration was not known at all.

5. Interpretation

In Part I and in this paper, I have identified three processes which may enhance the amplitude or stability of convection storms in a shearing environment, but which are optimized by different storm configurations and movement. In Part I, it was shown that (i) rotating storms can obtain energy from the mean flow, and (ii) they can apparently make use of it to enhance their buoyant updrafts, but the optimal structure and propagation for energy supply from the mean flow is different from that which allows the storm to transfer part of that energy into updraft intensity. This paper has presented evidence that (iii) helicity may reduce the energy losses to dissipation. The structure and propagation for optimizing the helicity effect is similar to that suitable for augmenting the updraft strength, that is, with the vortices coincident with the updrafts and both propagating laterally to the mean flow hodograph. An additional constraint on storm configuration and propagation not mentioned previously is (iv) the maintenance of correlation between vertical velocity and buoyancy. The Beltrami flow solution does not consider buoyancy and does not provide a correct steady state solution to the buoyancy equation. Lateral propagation of a storm implies a ventilating flow at all levels which will tend to carry buoyancy through the updraft and reduce its flux. This ventilation is minimized if the storm is propagating with the mean flow at some level or at some mean point on the hodograph, which is the same condition for maximizing the extraction of kinetic energy from the mean flow. Thus of the above four factors entering into the optimization problem, two, (i) and (iv), favor propagation along the hodograph with separated vorticity and updraft centers, and two, (ii) and (iii), favor lateral propagation with coincident vortex and updraft centers.

Observational and simulation evidence indicates that long-lived isolated storms usually propagate laterally to the hodograph and have nearly coincident updraft and vortex centers. One might expect the effects of fac-
draft without an accompanying rain-loaded and evaporating downdraft. The coincidence of the updraft and vortex centers in this simulation appears to be even more striking than in the supercell-type storms.

A final point relates to the stability of Beltrami flows. Even though net nonlinear interactions are cancelled in these flows, there is no proof to my knowledge that the flows are stable to nonhelical disturbances, and in some cases they appear to be highly unstable. For a mean shear flow with a rotating hodograph, all velocity components have sinusoidal profiles, and each point along the hodograph is a point of inflection for disturbances whose wave fronts lie parallel to the local mean.

tors (i) and (iv) to require a somewhat smaller lateral propagation velocity than that required by the Beltrami flow model. However, recent simulations by Weisman and Klemp (1984), using mean flows with rotating hodographs, show supercell propagation rates approximately at the center of the hodograph, corresponding almost exactly to the assumptions of the Beltrami flow model. In current work, Weisman and Bluestein (1985) have simulated a nonprecipitating storm, apparently similar to some observed by Bluestein and Parks (1983) in western Oklahoma and the Texas panhandle. Strong buoyancy and shear allow generation of a rotating updraft without an accompanying rain-loaded and evaporating downdraft. The coincidence of the updraft and vortex centers in this simulation appears to be even more striking than in the supercell-type storms.

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Fig. 16. Probability densities for the distribution of the angle $\theta$ between vorticity and velocity in a numerical simulation of a turbulent channel flow, conditionally sampled. (a) Points where energy dissipation is less than 5% of its maximum value. (b) Points where the dissipation is greater than 30% of its maximum value. (From Pelz et al., 1985.)

Fig. 17. (a) A skew-$T$ diagram depicting the initial temperature and moisture profiles used for simulating the 3 April 1977 storm. (b) The corresponding environmental wind hodograph, with heights ASL shown in kilometers. The averaged speeds of left-moving observed and modeled storms are indicated by arrows marked L and LM and similarly for right moving storms using R and RM. Wilhelmson and Klemp (1981).
wind vector. Linear instability appears to be present everywhere, even in the absence of buoyancy, and can be relieved most rapidly by nonhelical disturbances with wave fronts normal to the shear. Thus, the evident stability of strongly helical flows remains, in my view, inadequately explained. J. Herring has suggested (personal communication, 1984) that the kind of shearing instability indicated herein is slow compared to the fast and chaotic kind of instability associated with three-dimensional nonhelical flow configurations.

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REFERENCES


