A Minimal Baroclinic Model for the Statistical Properties of Low-Frequency Variability

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ABSTRACT

A recent analysis of atmospheric observations has shown evidence of bimodality in the statistical distribution of wave amplitude in the ultralong (zonal wavenumber group 2–4), low frequency (period > 5 days). Similar analysis of the zonal wind and its average shear shows no clear sign of bimodality. Both are characterized by a very small variance (≈ 1 m s^{-1}) and the associated kinetic energy fluctuations are not sufficient to account for the variations in wave amplitudes. Global energetic analysis confirms this finding: maintenance of the waves is dominated by baroclinic processes. On the other hand, from a theoretical point of view, barotropic models for wave generation and maintenance can be brought into agreement with observed statistics by introducing nonlinear bending of the stationary resonant response to topographic modulation allowing different values of the equilibrium amplitude to correspond to the same value of the zonal flow. However, because of aforementioned energetic difficulties, in a barotropic model, the closure equation (form-drag) for the zonal flow does not select states corresponding to values of the zonal wind within the observed statistics for any realistic value of the external parameters (dissipation and momentum forcing). In this paper we show how the inclusion wave field self-interaction produces resonance bending in a minimal baroclinic model. The two resulting equilibrium stable states can be achieved within a realistic range of the zonal flow. Moreover, the equilibrium states are characterized by stability properties which, on a theoretical level, are much more satisfactory than in the case of linear resonance.

1. Introduction

In a previous paper, Benzi et al. (1986, hereafter BMSS), it was shown that in the Northern Hemisphere winter, ultralong (wavenumber group 2–4) wave amplitude display a bimodal probability distribution (see Sutera, 1986; Hansen, 1985a; Benzi and Speranza, 1986a). On the other hand, the distribution of zonal flow did not show any evident sign of bimodality. Because of the bimodality of the wave amplitude, one would be tempted to connect these observational results with the multiple equilibria theory of Charney and DeVore (1979, hereafter CDV). In fact, this theory predicts bimodal distribution of orographically resonant wave amplitude (see Benzi et al. 1984). On the other hand, the same theory predicts that the zonal flow should also be bimodal distributed, in disagreement with the observation. In BMSS it was suggested that it is possible to restore agreement between theory and observations by including the nonlinear effects of wave self-advection. In fact, in these circumstances it was shown that the resonant response of the wave to the forcing would bend, allowing the possibility of a multiple equilibria for a fixed value of the zonal wind. Unfortunately, when the dynamical equation for the zonal flow (the so-called form-drag equation) was considered, the disagreement with observations remained. In fact the multiple states selected by the stationary zonal momentum balance, corresponded, for any realistic choice of the physical constants (dissipation and momentum forcing), to widely separated zonal flow values. In BMSS, this problem was addressed, but no solution was offered. The physical interpretation of this difficulty can be traced back to the formulation of the barotropic model (see Speranza, 1985a; Benzi and Speranza 1986b). In a nondivergent barotropic atmosphere the only energy source available to the wave growth is the kinetic energy of the zonal flow, so the wave amplitude grows, depleting the zonal flow proportionally to the form-drag; i.e., proportionally to the amplitude of the component out-of-phase with respect to the topography of the perturbation field, which we denote as $A_{\rm out}$. Nevertheless, the resonance bending $A_{\rm out}$ near the resonance has large variations leading to large variation of the zonal flow. Hence, statistically speaking, in this frame we should observe "bimodality" of both zonal flow and perturbation field. However, the resonance bending mechanism would offer an interesting alternative to the previously described process if other energy sources were available to the perturbation field. In particular, if baroclinic sources of energy are considered, the wave could grow by converting available potential energy, leaving the momentum budget approximately unaltered. In particular, at all steady states, the zonal momentum could remain virtually unchanged, achieving the result of having a bimodal distribution of the perturbation field with a unimodal distribution of the zonal flow. This picture appears to be confirmed by observational studies (Hansen, 1986b) which show that the two circulations corresponding to each mode differ mainly in the baroclinic energy content in the ultralong waves. This paper shows how, in a baroclinic model it is possible to derive dynamical statements fitting the statistical properties referred to here—bimodality of the wave amplitude and unimodality of the zonal flow distribution. Moreover, we will show that the stability properties of the equilibrium states derived in the present formulation, which include wave nonlinearity, are, at least theoretically speaking, more satisfactory than the ones encountered in previous work on baroclinic minimal models (Charney and Straus, 1980; Yoden, 1983; Rambaldi and Salustri, 1984; Itho, 1985) where the resonant response to topographic forcing was invariably assumed linear.

In section 2 we set up the model and discuss the physical hypothesis underlying the derivation of the quasi-unidimensional equations that provide the basis of our mathematical procedure. In section 3 we describe the equilibrium solutions, their stability and the statistical implications. Conclusions and reference to problems to be addressed in future work on the subject are contained in section 4.

2. Physical assumptions and basic equations

In BMSS, the theoretical discussion concerning the statistical properties of planetary flow was based on a quasi-unidimensional formulation of the equations of motion. It is worth summarizing here the physical and mathematical assumptions at the basis of such approximation.

The first step in the setting up of the quasi-unidimensional formulation consists in assuming that the streamfunction may be written in the form:

$$\Psi(x,y,t) = -U(t)y + \phi(x,y,t) \qquad (2.1)$$

where $U(t)$ is the total zonal (angular) momentum of the flow in a midlatitude β channel; $\phi(x, y, t)$ represents a zonally asymmetric component with zero projection onto the symmetric flow that dominates the midlatitude circulation. The zonal flow $U(t)$ is uniform in latitude: its only variation is in time and its only coupling with the nonsymmetric component is through the topographic drag. In this respect, our approach differs from that of other authors who rely upon the latitudinal shear of zonal flow in order to introduce nonlinearity into the wave dynamics (see Tung and Rosenthal, 1985, for a recent discussion and a thorough reference list concerning this type of approach). Notice that the distinction is physical and not geometrical, i.e., the same applies on the sphere.

A flow of the form (2.1) would remain on average symmetric in the absence of external modulation of barotropic dynamics.

The equation describing the evolution of (2.1) is deduced from the barotropic vorticity equation:

$$\partial_t q + J(\psi, q) = F \qquad (2.2)$$

where F is any kind of potential vorticity forcing (dissipation, external forcing . . .) and the potential vorticity q is

$$q = \nabla^2\psi + \beta y + h \qquad (2.3)$$

where the usual nomenclature is used (see BMSS) and h is f_0/H times the orography. For flow in a channel of width L, confined by rigid lateral walls,

$$\psi_x = 0,$$

at

$$y = 0, L. \qquad (2.4)$$

The classical approach to the search for stationary solutions of the boundary problem (2.2)–(2.4) consists in the expansion of ψ in eigenfunctions of a Laplace operator. Since at the stationary states the differential operator in (2.2) is not a Laplacian, this choice is arbitrary and, as discussed by Benzi et al. (1984), may lead to inconsistencies. As already stated, in the context of minimal models the choice of one harmonic wave as representation of the nonsymmetric field forces one to invoke the latitudinal shear of the zonal flow as a source of nonlinearity (Jacobian self-interaction vanishes identically for a single harmonic wave). However, observational evidence seems to be against any relevant role of latitudinal shear of the zonal flow (see Hansen, 1986a). We have, therefore, taken the alternative approach of introducing wave–wave interaction as a key source of nonlinearity, using a projection basis different from the set of eigenfunctions of Laplace operator. Our choice, leading to the quasi-unidimensional approximation, consists in the assumption that the nonsymmetric field component can be separated as

$$\phi(x,y,t) = \sum_n g_n(y) A_n(x,t) \qquad (2.5)$$

where the $g_n(y)$ are known functions producing nonempty wave–wave interaction and forming an orthonormal set $\langle g_n | g_m \rangle = \delta_{n,m}$. The consequent development of the quasi-unidimensional approximation is fully documented in BMSS.

Our treatment of the baroclinic problem parallels the barotropic one. In the two-layer formalism, chosen for obvious reasons of mathematical economy, the vorticity equations are

$$\partial_t q_1 + J(\psi_1, q_1) = F_1 \qquad (2.6a)$$

$$\partial_t q_2 + J(\psi_2, q_2) = F_2 \qquad (2.6b)$$

where $F_{1,2}$ again represents forcings and

$$q_1 = \nabla^2\psi_1 + \beta y + F(\psi_2 - \psi_1) \qquad (2.7a)$$

$$q_2 = \nabla^2\psi_2 + \beta y + F(\psi_1 - \psi_2) + h \qquad (2.7b)$$

in standard notation. Separation of symmetric and nonsymmetric flow components is obtained by assuming that

$$\psi_1(x, y, t) = -U_1(t)y + \phi_1(x, y, t) \quad (2.8a)$$

$$\psi_2(x, y, t) = -U_2(t)y + \phi_2(x, y, t) \quad (2.8b)$$

and wave–wave nonlinearity by defining

$$\phi_1(x, y, t) = \sum_{n=1,N} g_n(y)A_{1,n}(x, t) \quad (2.9a)$$

$$\phi_2(x, y, t) = \sum_{n=1,N} g_n(y)A_{2,n}(x, t) \quad (2.9b)$$

where the $g_n(y)$ satisfy the same conditions as in the barotropic case. We will consider one wave approximation ($N = 1$) with topography $h(x, y)$ projected onto the same g function. The forcing on the wave field is pure Laplacian dissipation:

$$F_1 = -\nu_1\nabla^2\psi_1 \quad (2.10a)$$

$$F_2 = -\nu_2\nabla^2\psi_2. \quad (2.10b)$$

Substituting (2.8)–(2.9) into (2.6) and projecting onto $g(y)$, we obtain the quasi-unidimensional wave equation:

$$\begin{aligned}\partial_t[A_{1xx} - \alpha^2 A_1 + F(A_2 - A_1)] + U_1\partial_x(A_{1xx} - \alpha^2 A_1)\\ + \beta\partial_x A_1 + \delta A_1 A_{1x} + F(U_1 A_{2x} - U_2 A_{1x})\\ = -\nu_1(A_{1xx} - \alpha^2 A_1) \quad (2.11a)\end{aligned}$$

$$\begin{aligned}\partial_t[A_{2xx} - \alpha^2 A_2 + F(A_1 - A_2)] + U_2\partial_x(A_{2xx} - \alpha^2 A_2)\\ + \beta\partial_x A_2 + \delta A_2 A_{2x} + F(U_2 A_{1x} - U_1 A_{2x}) + U_2 h_x\\ = -\nu_2(A_{2xx} - \alpha^2 A_2) \quad (2.11b)\end{aligned}$$

where h has only projection on $g(y)$, i.e., $h(x, y) \rightarrow h(x)g(y)$, and

$$\alpha^2 = -\int g g_{yy} dy$$

$$\delta = \int g g_y g_{yy} dy \quad (2.12)$$

are the nonlinearity coefficients of our system. Another coefficient, $\gamma = \int g^2 g_y dy$, originally appears in the projected equations (2.11), but vanishes because of the boundary conditions (2.4). As in the barotropic case, the choice of nontrigonometric $g(y)$ allows nonlinear interaction of waves to take place in the midlatitude channel.

In order to close the wave dynamics described by (2.11), we need to write additional equations for the time evolution of the zonal flow. Since the symmetric circulation defined in (2.8) has no horizontal vorticity, the derivation from the potential vorticity equation is not straightforward, involving a limit to uniform latitudinal structure (Hart, 1979). An alternative is to derive the quasi-unidimensional approximation for the zonal flow directly from the momentum equation. Since neither procedure is particularly illuminating in the context of the problem in question, we do not give an account here, the results being in any case very familiar.

Energetic closure of (2.11) is guaranteed by two equations. The first is the form-drag equation for the average zonal momentum $U = (U_1 + U_2)/2$:

$$\partial_t U = -\overline{A_2 h_x} - \nu_U(U - U^*) \quad (2.13)$$

where the overbar represents zonal average and ν_U is the inverse of a typical relaxation time of the system. As usual, U^* is the external momentum forcing, provided in the real atmosphere mostly by convergence of zonal momentum fluxes associated with small and fast eddies which are not explicitly represented in our minimal model. Such forcing is often omitted in the baroclinic case (e.g., as in Charney and Straus, 1980) because the average baroclinicity associated with planetary latitudinal thermal contrast is available as a primary driving of the circulation, as we shall see in the discussion of the next closure equation. However, this procedure leads to the consequence that, in order to maintain a stationary balance in (2.13), dissipation must be, against any observational evidence, compensated by form-drag acceleration. We, therefore, maintain explicit momentum forcing U^* as in the barotropic case. The second closure equation describes the time evolution of the average baroclinicity $m = (U_1 - U_2)/2$:

$$C\partial_t m = 2F\overline{A_2 A_{1x}} + \overline{A_2 h_x} - \nu_m(m - m^*) \quad (2.14)$$

where, again, ν_m is the inverse of a relaxation time and m^* is external baroclinicity generation associated with global latitudinal thermal contrast; C is a constant representing the inertia of the average baroclinicity field m. This constant is determined by two contributions: the first, associated with the horizontal vorticity of the zonal flow, is kinematic, while the second, associated with vertical shear, is thermodynamic. The thermodynamic inertia dominates in the earth atmosphere.

Equations (2.11), (2.13) and (2.14) form a closed system of partial differential equations in the zonal shape of the wave field $A_{1,2}(x, t)$, the total zonal momentum $U(t)$ and the average baroclinicity $m(t)$. The purpose of the next section is to show that this system of equations admits multiple wave amplitude stationary solution in a limited range of values of U and m, as required by observations.

3. Multiple baroclinic equilibria

In order to set up an approximation suitable for the search of multiple equilibrium solutions of (2.11), (2.13) and (2.14), we observe that, in the presence of orography, the symmetric circulation $U_2 = 0$ produces rather realistic instability patterns (see Buzzi et al., 1984, for a discussion of orographic baroclinic in the unidimensional formulation). Of particular importance is the baroclinic character of the instability, as well as its vertical coherence and its stationary phase. Moreover, we are interested in limited deviations from symmetric circulation. We shall, therefore, consider states with most of the momentum concentrated in the upper level. The rest of our scaling consists in the classical quasi-resonant approximation (see BMSS) which provides the nonlinear resonance folding.

Specifically, having defined a small parameter ϵ (usually the wave detuning with respect to resonance), we assume:

$$\left.\begin{aligned} \delta &= O(\epsilon) \rightarrow \delta\epsilon \\ U_2 &= O(\epsilon^2) \rightarrow \epsilon^2 U_2 \\ \nu &= O(\epsilon^2) \rightarrow \epsilon^2 \nu \\ X &= \epsilon^2 x \\ \tau &= \epsilon^2 t \end{aligned}\right\}, \tag{3.1}$$

where all the dissipations have been assumed equal to ν and slow space–time modulations have been introduced. The arrows indicate substitution of the scaled variables to the original ones. We expand the wave field:

$$A_{1,2}(x,t,X,\tau) = A_{1,2}^{(0)}(x,t,X,\tau) + \epsilon A_{1,2}^{(1)}(x,t,X,\tau) + \epsilon^2 A_{1,2}^{(2)}(x,t,X,\tau) + \cdots \tag{3.2}$$

and consequently expand Eq. (2.11). At the lowest (zero) order we obtain

$$U_1(A_{1xxx}^{(0)} - \alpha^2 A_{1x}^{(0)}) + \beta A_{1x}^{(0)} + F A_{2x}^{(0)} U_1 = 0 \tag{3.3a}$$

$$(\beta - FU_1) A_{2x}^{(0)} = 0 \tag{3.3b}$$

which define a nondissipative, free baroclinic Rossby wave:

$$A_1^{(0)}(X,\tau,x) = a_1(X,\tau)\exp ik_s x + (*) \tag{3.4a}$$

$$A_2^{(0)}(X,\tau,x) = 0. \tag{3.4b}$$

The wave is stationary, with resonant wavenumber $k_s = (\beta/U_1 - \alpha^2)^{1/2}$, except for slow modulation in space and time. The vanishing of $A_2^{(0)}$ reveals that the wave is essentially confined to the upper level as the zonal flow: only at second order in ϵ corrections in the wave field of the lower level will appear.

At the first order in the small parameter ϵ

$$U_1(A_{1xxx}^{(1)} - \alpha^2 A_{1x}^{(1)}) + \beta A_{1x}^{(1)} - \delta A^{(0)} A_{1x}^{(0)} + F A_{2x}^{(1)} U_1 = 0 \tag{3.5a}$$

$$(\beta - FU_1) A_{2x}^{(1)} = 0 \tag{3.5b}$$

which describe a nonlinearly modified, nondissipative, free Rossby wave:

$$A_1^{(1)}(X,\tau,x) = \delta\frac{[a_1(X,\tau)]^2}{2(\beta - \alpha^2 U_1)} + \delta\frac{[a_1(X,\tau)]^2}{2(\beta - \alpha^2 U_1 - 4k_s^2 U_1)} e^{2ik_s x} + (*) \tag{3.6a}$$

$$A_2^{(1)} = 0 \tag{3.6b}$$

modulated on the zonal wavenumbers 0 and $2k_s$. The slow space–time modulation $a_1(X,\tau)$ is determined by the balance with frictional dissipation and topographic action at the second order in ϵ:

$$\partial_\tau[A_{1xx}^{(0)} - (F+\alpha^2)A_1^{(0)}] + U_1(A_{1xxx}^{(2)} - \alpha^2 A_{1x}^{(2)}) + \beta A_{1x}^{(2)} + (\beta - \alpha^2 U_1)A_{1X}^{(0)} + 3U_1 A_{1xxX}^{(0)} - \delta A_1^{(1)} A_{1x}^{(0)} - \delta A^{(0)} A_{1x}^{(1)} + F(A_{2x}^{(2)} U_1 - A_{1x}^{(0)} U_2) = -\nu(A_{1xx}^{(0)} - \alpha^2 A_1^{(0)}) \tag{3.7a}$$

$$\partial_\tau F A_1^{(0)} + \beta A_{2x}^{(2)} + F(U_2 A_{1x}^{(0)} - U_1 A_{2x}^{(2)}) + U_2 h_x = 0. \tag{3.7b}$$

Equation (3.7b) is linear in $A_2^{(2)}$:

$$A_{2x}^{(2)} = -\frac{U_2 h_x + FU_2 A_{1x}^{(0)} + F\dot{A}_1^{(0)}}{(\beta - FU_1)} \tag{3.8}$$

which, substituted into (3.7a), gives

$$-\partial_\tau[(\alpha^2 + k_s^2)a_1 + Fa_1] - 2k_s^2 U_1 a_{1X} - \frac{\delta^2 ik_s a_1|a_1|^2}{3k_s^2 U_1} - FU_2 ik_s a_1 - \nu(\alpha^2 + k_s^2)a_1 - \frac{FU_1}{\beta - FU_1} \times [U_2 ik_m h_0 \exp i\Delta kX + FU_2 ik_s a_1 + F\dot{a}_1] = 0 \tag{3.9}$$

where we have assumed

$$h(x) = 2h_0\cos k_m x = h_0\exp i(k_s + \Delta k)x + (*) \tag{3.10}$$

and $\Delta k = k_m - k_s$ is the detuning with respect to resonance. Choice of $a_1(X,\tau) = a(\tau)\exp i\Delta kX$ finally gives the nonlinear equation for near resonant wave amplitude:

$$-\partial_\tau\left[\left(\frac{F^2 U_1}{\beta - FU_1} + F + \alpha^2 + k_s^2\right)a\right] - 2ik_s^2 U_1 \Delta k a - FU_2 ik_s a - \frac{iF^2 U_1 U_2 k_s}{\beta - FU_1}a - \nu(\alpha^2 + k_s^2)a - \frac{\delta^2 ik_s a|a|^2}{3k_s^2 U_1} = \frac{FU_1 U_2 h_0 ik_m}{\beta - FU_1}. \tag{3.11}$$

Substituting (3.8) and (3.11) into the closure equations (2.13) and (2.14), we obtain:

$$\partial_\tau U = \frac{-1}{\beta - FU_1}[FU_2 h_0 ik_s(a - a^*) + Fh_0(\dot{a} + \dot{a}^*)] - \nu(U - U^*) \tag{3.12a}$$

$$C\partial_\tau m = \frac{1}{\beta - FU_1}[FU_2 h_0 ik_s(a - a^*) + Fh_0(\dot{a} + \dot{a}^*)] \times \frac{+2F}{\beta - FU_1}[F(a\dot{a}^* + a^*\dot{a}) - ik_m U_2 h_0(a - a^*)] - 2\nu(m - m^*). \tag{3.12b}$$

Equations (3.8), (3.11) and (3.12) form a closed dynamical system describing the dynamics of the nonlinear (cubic) folding of resonant response to orographic modulation in an almost zonally symmetric baroclinic flow. Here our major concern is to find the stationary states and their stability properties, because of their impact on the statistics. We find stationary solutions by solving for a, as a function of U and m, the cubic algebraic condition resulting from the stationary version of (3.11), and consequently choosing U^* and m^*, satisfying the stationary version of (3.12) for those values of U and m that allow multiple wave amplitude solutions with a choice of a corresponding to its maximum equilibrium value. Having fixed U^* and m^*, we can then compute the multiple equilibria for all other states by means of a Newton iteration procedure. Convergence is quite fast.

An illustration of our results is given in Table 1. The first set of values (Table 1a) corresponds to multiple equilibria near the symmetric circulation ($U_2 = 0$): the two states of maximum and minimum amplitude are stable and separated by about 100 gpm. The intermediate state is unstable with respect to nontraveling baroclinic disturbances of orographic nature. The difference in zonal flow and average baroclinicity between the large and small wave amplitude stable states is confined in the range of meters per seconds as required by the observed statistics. The maintenance of the stable equilibria is, again in agreement with observations (Hansen, 1986a), essentially baroclinic as shown by the energetics displayed in Table 2.

It is interesting at this point to contrast the preceding results with those obtained by studying a barotropic nonlinear system computed from the same equations, setting $m = 0$. To this purpose we show in Table 1b a second set of equilibrium values computed for the above-defined barotropic dynamics. One can immediately see the large differences in the zonal wind of the two extreme (in wave amplitude) equilibria. This difference is due to the balance between zonal and wave kinetic energy, emphasized before as a necessary consequence of barotropy.

In conclusion, our results support the interpretation of the discrepancy between the prediction of barotropic theories and the observed statistics outlined in the Introduction: different wave amplitude states are maintained by baroclinic conversion, with form-drag exerting only a "catalytic" role. This explains how the wave amplitude can display widely varying values and at the same time bimodal statistics, while the zonal flow has almost coincident equilibrium values and, consequently, essentially unimodal statistics.

As regards the theoretical manipulation of minimal models of atmospheric circulation, it is also worth noting that the asymptotic stability of both extreme amplitude equilibria is here critically dependent on the process of "resonance bending." It is, in fact, this bending which permits the coexistence of different equilibria on the same side of the linear resonance, therefore displaying similar stability properties (Speranza, 1985b). We thus escape the embarrassment of finding globally attractive large amplitude states as other authors (Charney and Straus, 1980; Yoden, 1983; Rambaldi and Salustri, 1984; Itho, 1985) in dealing with the same system, but without resonance bending.

TABLE 1b. Stationary solutions of equations (3.8), (3.11) and (3.12) for the case $m^* = 0$. The values of the parameters are the same as table 1a but with $m = 0$.

	U	M	A1	A2
E1	2.1	0.	$2.08 \pm i0.63$	$4.16 \pm i1.21$
E2	4.72	0.	$-1.53 \pm i0.13$	$-1.83 \pm i0.17$
E3	5.1	0.	$-0.58 \pm i0.02$	$-0.61 \pm i0.02$

4. Conclusions

In this paper we show that the difficulties discussed in BMSS, as regards fitting the observed statistics of Northern Hemisphere circulation (bimodal wave amplitude, unimodal zonal wind) with a barotropic model, can be resolved by assuming that the same process of nonlinear resonance folding operates in a baroclinic model atmosphere. This procedure produces results that are not only in agreement with statistical properties of the general circulation deduced from observations, but also with the corresponding energetics dominated by baroclinic conversion.

Resonance bending also permits the production of stationary states characterized by stability properties suitable for the construction of a useful dynamical system. This resolves some technical difficulties faced by previous investigators of the problem. Therefore, this

TABLE 1a. Stationary solutions for equations (3.8), (3.11) and (3.12) computed for the following set of parameters: $C = 5$, $\alpha = 0.2$, $\delta = 0.5$, $F = 1$, $\beta = 1$, $U^* = 2.38$ and $m^* = 1.84$. The rescaling of variables has been performed using a wind scale of 10 m s^{-1} and a characteristic length of 1000 km. In this way the amplitude of the waves is expressed in units of hundreds of meters.

	U	M	A1	A2
E1	2.1	1.7	$0.73 \pm i0.78$	$0.12 \pm i0.11$
E2	2.15	1.72	$-0.76 \pm i0.61$	$-0.1 \pm i0.1$
E3	2.37	1.83	$-0.15 \pm i0.01$	$-0.01 \pm i0.002$

TABLE 2. Energetics of the three stationary solutions E1, E2 and E3 of Table 1a. $C(Az, Ae)$ is the conversion of zonal available potential energy into eddy available potential energy; $C(Ae, Ke)$ is the conversion from eddy available potential energy into eddy kinetic potential energy; $C(Kz, Ke)$ is the conversion of zonal kinetic energy into eddy kinetic energy.

	$C(Az, Ae)$	$C(Ae, Ke)$	$C(Kz, Ke)$
E1	0.041	0.011	0.002
E2	0.033	0.0093	0.002
E3	0.001	0.0002	0.00005

paper complements BMSS, closing the preliminary stage of construction of a minimal model of general circulation, by removing the traditional obstacles against its further elaboration.

Although we performed preliminary numerical integrations of the system described in section 3, we do not discuss here its time behavior and comparison with observational analysis of transitions between large and small wave amplitude states (Itho, 1985; Hansen, 1986a,b; Dole, 1986). Similarly omitted is a thorough discussion of the physical meaning of the nonlinearities introduced in the construction process of the quasi-unidimensional approximation (see the forthcoming paper by Benzi et al., 1986). All of these aspects of the problems in question require a very laborious analysis and are the subject of future work.

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