The Damping of Potential Enstrophy in the Large-Scale Transient Eddies in the Wintertime Troposphere

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ABSTRACT

On the average, there has to be a balance in the atmosphere between the rate at which the potential enstrophy of the transient eddies (TE) is generated by conversion from the time-mean flow and the rate at which it is damped by diabatic and frictional processes. In the quasi-geostrophic framework this conversion rate (and, hence, also the damping rate) can be estimated from conventional circulation statistics for temperature and horizontal wind. This estimation does not require data on vertical velocity and horizontal wind divergence, which are poorly known and play a crucial role in the TE energetics. Thus, from the data, one can presumably get more reliable information about the damping of the TE potential enstrophy than about the diabatic and frictional damping of the TE energy.

An observational study is made of the amount and maintenance of the (quasi-geostrophic) potential enstrophy of the large-scale transient eddies in the troposphere over the Northern Hemisphere in February 1979 by using two sets of FGGE (level III-b) analyses. Both datasets give approximately the same results. The rate of the damping of the TE potential enstrophy appears to have a maximum around 300 and 400 hPa, as does also the TE potential enstrophy itself. The associated time scale is of the order of ten days. Synoptic and physical interpretation of the results is given.

It is suggested that potential enstrophy might provide more useful a framework than energetics for verification and intercomparison of atmospheric large-scale models.

1. Introduction

Differential heating of the air between different latitudes is known to be the "prime mover" of the general circulation of the atmosphere. On the average, the forcing associated with this heating has to be balanced by damping caused by friction and diabatic processes that operate mainly on spatial scales much smaller than those on which the forcing occurs.

Our knowledge of the damping processes in the atmoshpere is unsatisfactory. For example, estimates of the globally averaged rate of dissipation of kinetic energy in the upper troposphere differ by a factor of five (Kung and Tanaka, 1983). Another unsolved problem in the area of atmospheric energetics is the role of different components of diabatic heating (radiation, release of latent heat, turbulent heat flux divergence) in the budget of available potential energy of the large-scale eddies. The dissipation of kinetic energy and the generation of the available potential energy by the net diabatic heating in the eddies are normally evaluated as residuals in the energetics calculations. The results are sensitive to errors in the estimates of vertical velocity, which is still notoriously difficult to estimate from observations.

It should be possible, at least in principle, to estimate the net effect of diabatic and frictional processes on potential vorticity more accurately than their effect on the energetics. This is because the estimation of the relevant terms in the potential vorticity dynamics does not involve vertical velocity, but can be done by using more accurately known data on temperature and horizontal wind.

A diagnostic study of the potential enstrophy budget of wintertime stationary eddies was reported by Holopainen et al. (1982). It was shown that these time-mean waves lose potential enstrophy to the large-scale transient eddies. As far as the authors know, no corresponding observational analysis of the potential enstrophy budget of the transient eddies (TE) in the troposphere has so far been reported in the literature. Some observational aspects of the potential enstrophy budget in the stratosphere have been discussed by Schoeberl (1982) and Schoeberl and Smith (1986). (An extensive general discussion of the potential vorticity concept, particularly in the isentropic framework, and a review of the relevant literature is contained in Hoskins et al., 1985.)

The purpose of this paper is to present new information on the vertical distribution of the net effect of diabatic and frictional processes on the large-scale transient eddies. This will be done in terms of the quasi-geostrophic potential enstrophy. We use daily data on the large-scale circulation over the Northern Hemisphere to estimate this net effect as a residual in the equation for the TE potential enstrophy. The theoretical background for the calculations is given in section 2. Data and the diagnostic results derived from the data are described in sections 3 and 4, respectively. Further discussion on the results is contained in section 5, which also contains a qualitative comparison of the potential enstrophy approach employed here with the more commonly used energetics approach.

2. Theoretical background

In this chapter we first discuss the quasi-geostrophic potential vorticity equation. We then derive the equation for the budget of the TE potential enstrophy. Finally, we outline the basic features of the atmospheric general circulation in terms of potential enstrophy so as to indicate the broader framework of the diagnostic results derived in later chapters.

a. Quasi-geostrophic potential vorticity equation

In the pressure coordinate system, the quasi-geostrophic potential vorticity (referred to in the following simply as the potential vorticity) is defined as (e.g., Hoskins, 1983)

$$q = \eta + q_s \tag{1a}$$

where

$$\eta = \zeta + f \tag{1b}$$

$$q_s = f\frac{\partial}{\partial p}\left(\frac{\theta''}{\partial\tilde{\bar{\theta}}/\partial p}\right). \tag{1c}$$

In these expressions η is the absolute vorticity, ζ the relative vorticity and θ the potential temperature. $\tilde{X}$ denotes an area-average of an arbitrary quantity X on a pressure surface and X'' a deviation from this average. As usual, $\bar{X}$ denotes the time-average of X.

According to (1a) the potential vorticity q is the sum of absolute vorticity and a quantity which essentially measures the ratio of local, instantaneous static stability to the average static stability at the pressure level in question. The equation for q is written in the form

$$\partial q/\partial t + \mathbf{V}\cdot\nabla q = D \tag{2}$$

where $\mathbf{V}$ is the nondivergent geostrophic wind, ∇ denotes the gradient operator on an isobaric surface and D the source of q due to diabatic and frictional processes:

$$D = -f\frac{\partial}{\partial p}\left(\frac{(c_p)^{-1}(p_0/p)^\kappa Q''}{\tilde{\bar{S}}}\right) - g\frac{\partial}{\partial p}(\mathbf{k}\cdot\mathrm{curl}\boldsymbol{\tau}) + \mathbf{k}\cdot\mathrm{curl}\mathbf{F}_H. \tag{3}$$

In (3) Q is the diabatic heating per unit mass and $S = -\partial\theta/\partial p$. $\boldsymbol{\tau}$ is the turbulent stress due to vertical subgrid-scale momentum fluxes and $\mathbf{F}_H$ the horizontal force due to the corresponding horizontal fluxes. As usual, g denotes the acceleration of gravity, $\kappa = R/c_p$, where R and c_p denote the gas constant and the specific heat at constant pressure, respectively; $p_0 = 1000$ hPa and $\mathbf{k}$ is the unit vector in the vertical.

According to (2), which is the quasi-geostrophic counterpart of the more general conservation law concerning "Ertel's potential vorticity" discussed, e.g., by Hoskins et al. (1985), the potential vorticity q is materially conserved in adiabatic frictionless flow ($D = 0$) on nondivergent isobaric trajectories. One aspect of potential vorticity q is that it is not, in the interior of the atmosphere, directly influenced by the vertical velocity. In particular, q is not influenced by lower boundary vertical velocities caused by mountains and/or Ekman pumping except in the sense that these contribute to the horizontal velocity which advects q (As shown in appendix A, this contribution is most obvious for the vertically-averaged, "barotropic" part of the flow).

b. Equation for the TE potential enstrophy budget

As usual, we will denote the deviation of any quantity X from its time-average $\bar{X}$ by X' and call it the transient eddy (TE) part of that quantity. Taking a time-average of (2) gives an equation for the potential vorticity of the time-mean flow [e.g., see Eq. (6a) in Holopainen et al., 1982]. Then, by subtracting this equation from (2) one obtains the quasi-geostrophic potential vorticity equation for the transient eddies. It can be written as

$$\partial q'/\partial t + \bar{\mathbf{V}}\cdot\nabla q' + \mathbf{V}'\cdot\nabla\bar{q} + (\mathbf{V}'\cdot\nabla q' - \overline{\mathbf{V}'\cdot\nabla q'}) = D' \tag{4}$$

in which

$$\nabla\bar{q} = \nabla\bar{\eta} - f\frac{\partial}{\partial p}(\nabla\bar{\theta}''/\tilde{\bar{S}}). \tag{5}$$

Multiplying (4) with q' and taking a time-average gives the equation for the TE potential enstrophy $\frac{1}{2}\overline{q'^2}$:

$$\partial/\partial t\left(\frac{1}{2}\overline{q'^2}\right) + \bar{\mathbf{V}}\cdot\nabla\left(\frac{1}{2}\overline{q'^2}\right) + \overline{\mathbf{V}'\cdot\nabla\left(\frac{1}{2}q'^2\right)} + \overline{\mathbf{V}'q'}\cdot\nabla\bar{q} = \overline{D'q'}. \tag{6}$$

If averages are now taken over an area, at the periphery of which the normal components of fluxes disappear (as in the case of the whole globe and, approximately, in the case of an hemisphere) the horizontal advection terms on the lhs of (6) disappear because of the nondivergency of $\mathbf{V}$. In the case of time-averages over a period of one month or longer the first lhs term in (6) also becomes small. For such averages we can write

$$\widetilde{\overline{D'q'}} \approx \widetilde{(\overline{q'\mathbf{V}'})_D\cdot\nabla\bar{q}} \tag{7}$$

valid at any pressure level in the interior of the atmosphere where the quasi-geostrophic dynamics apply. Negative values mean that potential enstrophy is converted from the time-mean flow to the transient eddies and dissipated by diabatic and frictional processes (see Fig. 1). In (7) $(\overline{q'\mathbf{V}'})_D$ is the divergent part of the TE flux of potential vorticity. As shown in Appendix B, this quantity (as well as $\nabla\bar{q}$) can be estimated from conventional circulation statistics of temperature and horizontal wind. Hence, (7) can be used to provide an indirect estimate of $\widetilde{\overline{D'q'}}$.

We define a ratio

$$\nu(p) = -\widetilde{\overline{D'q'}} \Big/ \left(\frac{1}{2}\widetilde{\overline{q'^2}}\right), \tag{8}$$

which can be considered as a gross measure of the rate of damping of TE potential enstrophy; $1/\nu$ is the corresponding time scale.

Equations (7) and (8) are the basic expressions used in this study.

c. *General circulation of the atmosphere in terms of potential enstrophy*

A full discussion of the general circulation of the atmosphere in terms of potential enstrophy is beyond the scope of the present paper. The purpose of this subsection is only to indicate the larger framework within which our results belong.

Figure 1 shows the basic features of the time-mean budget of atmospheric (quasi-geostrophic) potential enstrophy, averaged over the whole isobaric surface. Mean-flow potential enstrophy is generated by diabatic heating on the planetary scale. (The vertical distribution of the horizontally-averaged diabatic heating tends to destabilize the atmosphere (e.g., see Fig. 6 in Holopainen and Fortelius, 1986). Generation of mean-flow potential enstrophy means that, compared with this average destabilization, the high potential-vorticity air in high latitudes is stabilized and the low potential-vorticity air in lower latitudes is destabilized by the distribution of the diabatic heating.) Due to the inherent instabilities, mean-flow potential enstrophy is converted to that of the transient eddies. Damping of the potential enstrophy by diabatic and frictional processes is likely to be strongest at small horizontal and vertical scales and thus to affect mainly the transient eddies. For an average over the whole isobaric surface and over a long time, the generation, conversion and damping terms in Fig. 1 must be equal. (Fig. 1 is essentially the same as Fig. 2 in the study of stratospheric dynamics by Schoeberl and Smith, 1986, except that their study concerns the interaction between the zonally-averaged flow and the corresponding eddies).

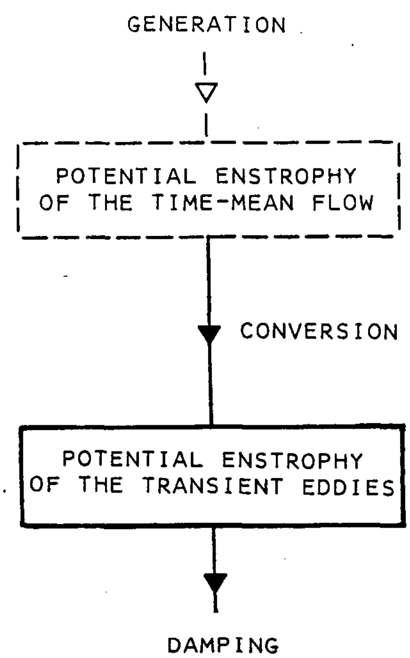

FIG. 1. Schematic diagram for the horizontally-averaged time-mean budget of the atmospheric potential enstrophy at an arbitrary pressure level. The part of the potential enstrophy budget studied in this paper is indicated by heavy continuous lines and arrows.

Figure 1 is for an arbitrary pressure level. In the potential enstrophy framework there is no coupling of different levels in the form of vertical flux divergence terms, which in most budget studies give contributions of different sign at different levels, and the vertical average of which vanishes. In this respect, the potential enstrophy budget is simpler than, e.g., the energy budget, in the horizontally-averaged form of which $\partial(\Phi\omega)/\partial p$ is one of the dominating terms (e.g., see Fig. 18 in Arpe et al., 1986). Accordingly, residual estimates of the net diabatic and frictional effects in the atmospheric energetics at any particular level have much larger relative error than in the case of vertically integrated energetics. Because irreversible processes occur at all levels, one might expect the potential enstrophy cascade at all levels to be in the direction shown in Fig. 1.

Some general comments on the potential usefulness of the description of the atmospheric general circulation in terms of potential enstrophy will also be presented in section 5. Otherwise, the rest of this paper will deal only with those processes delineated in Fig. 1 with heavy continuous lines and arrows. Actually we estimate from data only the conversion term, $\widetilde{\overline{q'V'} \cdot \nabla\bar{q}}$. However, according to (7) this is also an indirect estimate of the damping term $\widetilde{\overline{D'q'}}$.

3. Data and procedures used

Estimates of the amounts of potential enstrophy and its gross damping rate were calculated for the Northern Hemisphere for February 1979, the main full month of the Special Observing Period I of the Global Weather Experiment or FGGE. The temperature and horizontal

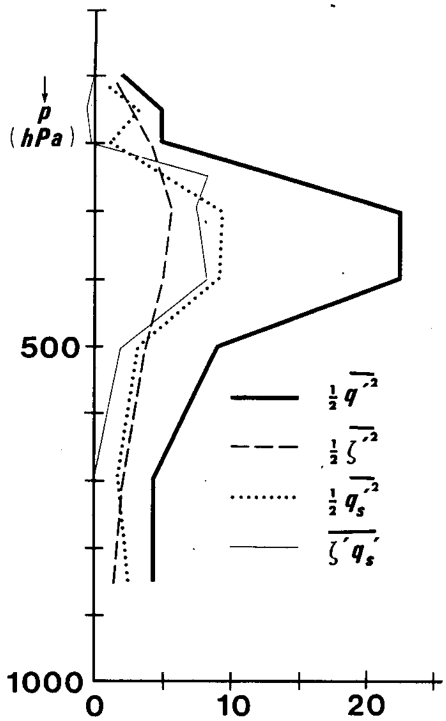

FIG. 2. Northern Hemisphere averages for February 1979 of the potential enstrophy in the transient eddies, $\frac{1}{2}\overline{q'^2}$ (solid line). The dashed, dotted and light continuous lines show the distribution of $\frac{1}{2}\overline{\zeta'^2}$, $\frac{1}{2}\overline{q_s'^2}$ and $\overline{\zeta' q_s'}$, respectively. Unit: $10^{-10}\ s^{-2}$.

wind data used here consist of the (FGGE level III-b Main) analyses prepared by ECMWF (European Centre for Medium Range Weather Forecasts) and GLA (Goddard Laboratory for Atmospheres). Estimates of $\widetilde{\overline{D'q'}}$ were calculated from both datasets, those of $\frac{1}{2}\widetilde{\overline{q'^2}}$ from the ECMWF data only.

The TE potential vorticity flux divergences and the associated divergent fluxes were estimated from the horizontal TE fluxes of heat and momentum (see appendix B).

The vertical derivatives were evaluated using the cubic spline technique.

4. Results and their interpretation

In this section, Northern Hemisphere averages of $\frac{1}{2}\overline{q'^2}$, $\overline{D'q'}$ and $\nu(p)$ are presented for pressure levels below 100 hPa. No values are given for levels below 850 hPa, however, because quasi-geostrophic equations cannot be expected to apply in this layer of large frictional influence.

Figure 2 shows, the hemispherically averaged values of the TE potential enstrophy and its ingredients:

$$\frac{1}{2}\overline{q'^2} = \frac{1}{2}\overline{\zeta'^2} + \frac{1}{2}\overline{q_s'^2} + \overline{\zeta' q_s'}. \qquad (9)$$

The first term on the rhs of (9), shown by the dashed line in Fig. 2, has a broad maximum around 300 hPa. This is an expected result, because the amplitude of the TE velocity fluctuations is known to be largest in the upper troposphere. The second term, shown by the dotted line, also has its maximum in the upper troposphere. Large north–south excursions of air particles and large differences in the mean static stability between high latitudes and the subtropics are reasons for this maximum. The third term, shown by the light continuous line, is large and positive between 250 and 400 hPa, and practically zero in the lower troposphere and above the 200 hPa level. The synoptic interpretation of the positive covariance $\overline{\zeta' q_s'}$ in the upper troposphere is clear: the upper troughs (where $\zeta' > 0$) have relatively large static stability and the upper ridges have (where $\zeta' < 0$) relatively small static stability (e.g., see Figs. 7.7c and 7.9 in Palmén and Newton, 1969).

The residual estimates of $\widetilde{\overline{D'q'}}$ are shown in Fig. 3. They are seen to be negative at nearly all levels. This means that the diabatic and frictional processes damp the potential enstrophy of the transient eddies. The largest damping is seen to take place in the layer 300–400 hPa. Although some differences exist, the two datasets give very much the same result.

The sign of $\widetilde{\overline{D'q'}}$ depends on whether the divergent TE potential vorticity flux is on the average up or down the gradient of the time-mean potential vorticity. One part of the scalar product in (7) is determined by the

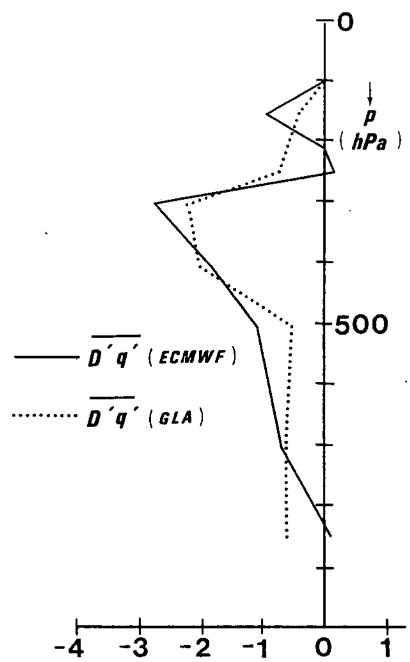

FIG. 3. Northern Hemisphere averages for February 1979 of the estimates of the potential enstrophy damping in the transient eddies. Solid (dotted) line shows the estimate obtained from the ECMWF (GLA) analyses. Unit: $10^{-15}\ s^{-3}$.

mean TE potential vorticity flux in the meridional direction, $[\overline{q'v'}]\partial[\bar{q}]/\partial y$; the brackets denote an average with respect to longitude. The distribution of $\partial[\bar{q}]/\partial y$ in February 1979 (not shown) has the same basic features as the corresponding pattern for the long-term wintertime conditions (e.g., Pfeffer, 1981; Fullmer, 1982) in that it has a maximum in the upper troposphere, where its magnitude exceeds β $(=df/dy)$ by a factor of 2–5. In the atmosphere, the potential vorticity flux by the transient eddies is predominantly southwards, as was first shown by Wiin-Nielsen and Sela (1971), who also found that on the average this flux is dominated by $[\overline{q'_s v'}]$. This equatorward flux in combination with a positive $\partial[\bar{q}]/\partial y$ contributes to the negative values arising from (7) for $\widetilde{\overline{D'q'}}$. However, the remaining part of $\widetilde{\overline{D'q'}}$, proportional to the conversion of potential enstrophy from the stationary eddies to the transient eddies, also appears to be important and contributes in the same direction. This is in agreement with the finding of Holopainen et al. (1982) that the large-scale transient eddies extract potential enstrophy from the stationary eddies.

Synoptic examples of the net southward flux of potential vorticity are, as discussed by Hoskins et al. (1985), the irreversible southward intrusions of high potential vorticity air in connection with the formation of cutoff lows, and the northward intrusions of low potential vorticity air in connection with blocking highs. (A schematic longitude-pressure structure of atmospheric eddies giving $[\overline{q'_s v'}] < 0$ in the upper troposphere, is given, e.g., in Fig. 5 of Townsend and Johnson, 1985).

We do not know the error bars of our estimates of $\widetilde{\overline{D'q'}}$. The difference between the estimates obtained from the two datasets is, of course, of some relevance in this connection. In contrast to many studies in which the residual term is calculated as a difference between two large terms (and therefore the relative accuracy of its estimate is very much smaller than that of these two large terms), the residual in the potential enstrophy budget (i.e., $\widetilde{\overline{D'q'}}$) is not such a difference.

Irreversible processes could be expected to act in a diffusive sense on quasi-conservative quantities such as potential vorticity. For potential enstrophy, such a diffusion means damping. Qualitatively, then, the negative values shown in Fig. 3 are as anticipated. The cascade of potential enstrophy from resolved to unresolved scales obviously gives a negative contribution. In the expression (3) for D, the diabatic heating per unit mass (Q'') can be written as the sum of contributions from the net radiative heating, the net release of latent heat and the heat flux convergence due to subgrid-scale processes. Qualitatively, it seems likely that radiation, too, must be an important contributor to the negative values of $\widetilde{\overline{D'q'}}$ in the upper troposphere. In blocking highs, for example, radiation must be the dominant heating process and thus $Q'' < 0$. The quantity $(p_0/p)^\kappa Q''/\tilde{\tilde{S}}$ must be negligible in the lower stratosphere and hence, the contribution of radiation to D' is positive in a blocking high. Because $q' < 0$ (both ζ' and q'_s are negative) a negative radiative contribution to $\widetilde{\overline{D'q'}}$ is likely to arise in the upper troposphere in blocking cases.

At present it is not possible to say very much quantitatively about the contributions of different components of D to $\widetilde{\overline{D'q'}}$. The only aspect that can be commented on concerns the role of $\mathbf{F}_H$. If the horizontal resolution scale in our data falls in the inertial subrange characterized by constant enstrophy flux toward smaller scales, this flux should be given by the contribution of $\mathbf{F}_H$ to $\widetilde{\overline{D'q'}}$. In our units the latest estimate of this flux, reported by Boer and Shepherd (1983), is $-0.3 \times 10^{-15}\ \mathrm{s}^{-3}$ and thus about one third of the vertical average of $\widetilde{\overline{D'q'}}$ seen in Fig. 4.

In principle, models which provide the first-guess fields to large-scale analyses can also be used to provide, via their parameterization schemes, quantitative values of the different components of D and their contribution to $\widetilde{\overline{D'q'}}$. For the time being, such estimates are probably

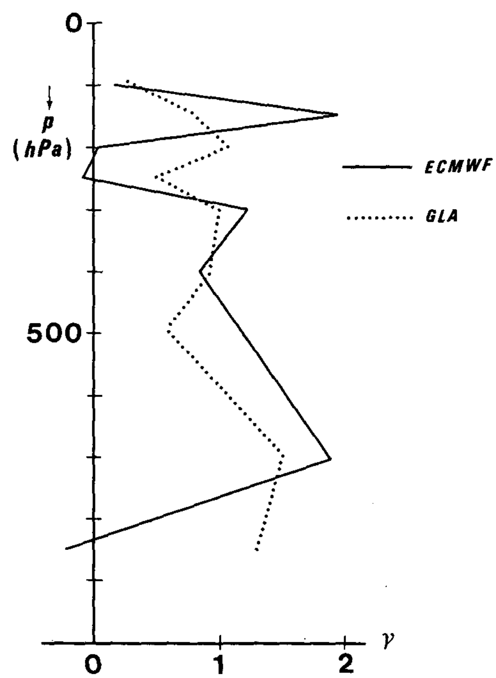

FIG. 4. Damping coefficient $\nu(p) = -\widetilde{\overline{D'q'}}/(\frac{1}{2}\widetilde{\overline{q'^2}})$, calculated using the values of $\widetilde{\overline{D'q'}}$ in Fig. 3 and values of $\frac{1}{2}\widetilde{\overline{q'^2}}$ in Fig. 4. Solid and dotted lines refer to the ECMWF and GLA data, respectively. (Potential enstrophy values have been calculated from the ECMWF data only). Unit: $10^{-6}\ \mathrm{s}^{-1}$.

not very reliable for many reasons. For example, clouds must be important in determining the radiative heating of the air. Yet, very little is known about the space-time variability of cloudiness, and its simulation in the models is at present unsatisfactory.

Figure 4 shows the damping coefficient, defined in (8). It is seen that in most of the troposphere, ν is of the order of $10^{-6}\ s^{-1}$, which corresponds to a damping time scale ($1/\nu$) of about 10 days. This is a gross measure of the damping time scale of the TE potential enstrophy during the whole month. In an individual situation, as, for example, during "breaking" of large-scale waves and the subsequent formation/disappearance of upper-level shear lines, the e-folding dissipation time for the associated concentrations of high potential vorticity can be as small as 1 day (e.g., Holopainen and Rontu, 1981). In some other situations, however, the diabatic processes may even temporarily produce potential enstrophy (i.e., give a positive contribution to the local values of $\overline{D'q'}$).

The vertical distribution $\nu(p)$ is of considerable theoretical interest in connection with the instability of stationary and quasi-stationary planetary waves. Held et al. (1986) showed that external Rossby waves in vertical shear can be destabilized by the TE thermal damping at the lower boundary and also by the TE damping of the potential vorticity if this latter damping is larger in the lower than in the upper troposphere. Our profile of $\nu(p)$ in Fig. 4 does not give any convincing evidence about vertical variation. One might, however, tentatively infer an average decrease of $\nu(p)$ from 700 hPa to 250 hPa.

The results presented in Figs. 2–4 are hemispheric averages. The quasi-geostrophic theory is, however, not valid in the deep tropics, which deserve special analysis. The results shown in this paper reflect mainly what happens in the extratropics. Vertical profiles corresponding to those shown in Figs. 2–4 were also calculated for the region 20°–90°N. The resulting profiles (not shown) for $\widetilde{\tfrac{1}{2}\overline{q'^2}}$ and $\widetilde{\overline{D'q'}}$ were very much the same as those in Fig. 2 and Fig. 3, respectively, except that the magnitude was (as one can expect) much larger; the profile of $\nu(p)$, however, changed hardly at all from that seen in Fig. 4.

5. Discussion

The conservation law for potential vorticity is the backbone of dynamic meteorology. Documentation of atmospheric large-scale behavior in terms of potential enstrophy should then be at least as important as in terms of energy, particularly when the latter has considerable ambiguities (e.g., Plumb, 1983). It appears appropriate here to discuss briefly the advantages and shortcomings of the potential enstrophy approach and the energetics approach in the investigation of the effects of diabatic and frictional processes in the atmosphere.

When the whole mass of the atmosphere is considered, the by now classic picture of the atmospheric energy cycle (originating from Lorenz, 1955) is that available potential energy of the time-mean flow is generated by the latitudinal differences in the net diabatic heating. This is converted to TE available potential energy, and that in turn to TE kinetic energy. A small amount of the TE kinetic energy is fed back to the time-mean flow and maintains its kinetic energy against friction. The net result of the interplay between the time-mean flow and the TE is an energy conversion of mean-flow energy to TE energy. On the average this net energy input into the eddies is compensated by damping, which occurs in the form of frictional dissipation of the TE kinetic energy and thermal damping of the TE available potential energy. As mentioned in subsection 2c, residual estimates of the net diabatic and frictional effects in the atmospheric energetics at any particular level have much larger error bars than in the vertically-integrated case.

The description of the atmospheric general circulation in terms of potential enstrophy (Fig. 1) is qualitatively the same as in terms of energy: the time-mean flow that is generated by latitudinal differences in diabatic heating breaks down to transient eddies, which in turn are damped by diabatic and frictional processes.

Potential enstrophy considerations have at least two advantages over those of energetics. First, estimates of terms in the potential enstrophy budget involve only the relatively accurately known fields of temperature and horizontal wind, whereas the energetics depends also on poorly known fields of vertical velocity and horizontal wind divergence. (The vertical TE heat fluxes, which are of paramount importance in the TE energetics, do not enter the first-order budget of TE potential enstrophy; they have to be considered, however, when the potential enstrophy budget of the time-mean flow is studied). Second, potential enstrophy calculations provide information for individual pressure levels, whereas estimates of the corresponding energetic processes can probably be worked out with comparable accuracy only for the vertically-integrated case. An additional remark, already made earlier in section 2, is that the potential enstrophy budget does not directly depend on the presence of mountains. Mountains have, however, a direct effect on the energy conversion from the mean flow to eddies and this effect is difficult to estimate accurately.

The obvious conclusion from the above discussion is that the potential enstrophy budget could possibly be at least as useful a tool as the energy budget in diagnosing the net effect of diabatic and frictional processes in the atmosphere, and in the verification and intercomparison of large-scale models.

In addition to the TE potential enstrophy considered here, there are several other quadratic quantities whose budgets are of interest when the behavior of the transient eddies is considered. One such concept is "eddy activity", which was introduced by Edmon et al. (1980) (who actually called it "Eliassen-Palm wave activity") and extended to transient eddies by Plumb (1986). Another example is "pseudoenergy" by Andrews (1983). The concept of "vertically-integrated pseudomomentum" by Held et al. (1986) combines the budget of "eddy activity" in the interior of the atmosphere with the budget of the eddy temperature variance at the lower boundary. These quantities and their budgets are difficult to diagnose from data, one of the reasons being that the relevant expressions become infinite at those points where the horizontal gradient of the basic state potential vorticity vanishes.

An alternative to the quasi-geostrophic framework used in this study is the use of "Ertel's potential vorticity" and the isentropic coordinate system (e.g., Townsend and Johnson, 1985; Hoskins et al., 1985). No systematic study of the atmospheric damping processes has been done so far using this approach.

A lot of diagnostic studies have been made in the past concerning the conventional enstrophy in the horizontal wavenumber domain (e.g., Boer and Shepherd, 1983). Similar studies should also be made for the potential enstrophy; in this case, spectral transfer in three-dimensional wavenumber space is involved. In addition, detailed mesoscale observational and modeling studies along the lines of Shapiro (1976, 1980) and Gidel and Shapiro (1979) are needed.

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APPENDIX A

The Effect of the Lower Boundary Vertical Velocities on the Vertically Averaged (Barotropic) Part of the Flow

In order to illustrate the effects of the lower boundary vertical velocities we derive here the vorticity equation for the vertically-averaged flow from the quasi-geostrophic potential vorticity equation (2). The essential point in the derivation is that one also has to include in the total amount of potential vorticity in the atmospheric air column the contribution of the temperature anomaly at the lower boundary (for more detailed discussion, see section 4b in Hoskins et al., 1985).

The vertical boundary condition which must be used in connection with (2) is provided by the thermodynamic energy equation. In our notation this equation, when applied at the lowest pressure level p_B, can be written as

$$\partial\theta''/\partial t + \mathbf{V}\cdot\nabla\theta'' = \tilde{\tilde{S}}\omega_M + \tilde{\tilde{S}}\omega_F + (c_p)^{-1}(p_0/p)^{\kappa}Q'' \quad (\text{at } p = p_B). \qquad \text{(A1)}$$

The pressure level p_B is assumed to coincide with the top of the Ekman layer. In (A1), ω_M and ω_F denote the lower boundary vertical velocity due to mountains and friction, respectively. Using the Ekman–Taylor theory (which, among other things, assumes that the stress at the top of the boundary layer is small compared with the surface stress) we can write

$$\omega_F \approx -f^{-1}g\mathbf{k}\cdot\text{curl}\boldsymbol{\tau}_s, \qquad \text{(A2)}$$

where $\boldsymbol{\tau}_s$ is the surface stress.

The vorticity equation for the vertically-averaged (barotropic) part of the flow is obtained by first averaging (2) vertically over the mass of the whole air column between $p = 0$ and $p = p_B$. When (A1) is then multiplied by $-f/(\tilde{\tilde{S}}p_B)$ and added to the vertical average of (2), and (A2) is also used, one obtains

$$\partial\hat{\zeta}/\partial t + \widehat{\mathbf{V}\cdot\nabla\eta} = \frac{f}{p_B}\omega_M - \frac{1}{p_B}g\mathbf{k}\cdot\text{curl}\boldsymbol{\tau}_s + \mathbf{k}\cdot\text{curl}\hat{\mathbf{F}}_H. \qquad \text{(A3)}$$

Here $\hat{X} = (1/p_B)\int_0^{p_B} X dp$ denotes the vertical average of X.

Equation (A3) (which can be derived more directly by taking the vertical mass average of the normal vorticity equation) shows that even if q is not directly influenced by the forced vertical velocities at the lower boundary, $\hat{\zeta}$ (and therefore also $\hat{\mathbf{V}}$) are.

APPENDIX B

Estimation of Divergent Fluxes of Quasi-geostrophic Potential Vorticity from the Horizontal Fluxes of Momentum and Heat

We derive here the expression used for the TE potential vorticity flux divergence. For convenience we drop ( )″ and let θ denote the deviation from its isobaric average. By using the expression

$$q' = \zeta' + q_s'$$

we first write

$$\nabla\cdot\overline{q'\mathbf{V}'} = \nabla\cdot\overline{\zeta'\mathbf{V}'} + \nabla\cdot\overline{q_s'\mathbf{V}'}. \qquad \text{(B1)}$$

The first term on the rhs of (B1) can be written as a curl of a horizontal nondivergent force, which can be estimated if the horizontal TE momentum fluxes $\overline{u'u'}$, $\overline{u'v'}$ and $\overline{v'v'}$ known [see Eq. (4) in Holopainen et al., 1982]. The second rhs term can be written as the sum of three terms

$$\nabla\cdot\overline{q_s'\mathbf{V}'} = -\frac{\partial}{\partial p}\left\{\frac{\nabla\cdot f\overline{\theta'\mathbf{V}'}}{\tilde{\tilde{S}}}\right\} + f\frac{\overline{\nabla\theta'}}{\tilde{\tilde{S}}}\cdot\frac{\partial\mathbf{V}'}{\partial p} + \beta\frac{\overline{\theta'}}{\tilde{\tilde{S}}}\frac{\partial v'}{\partial p}. \qquad \text{(B2)}$$

For $\mathbf{V}'$ in a thermal wind balance with the temperature field the second rhs term of (B2) disappears. The third rhs term can be written as

$$-(\tilde{\bar{S}})^{-1}\frac{\beta}{f}\frac{R}{p}\left(\frac{p}{p_0}\right)^{\kappa}\frac{\partial}{\partial x}\left(\frac{1}{2}\overline{\theta'^2}\right).$$

This term also vanishes in the case of zonal averages and when θ'^2 does not depend on longitude. Locally, it is not generally exactly zero. However, its typical magnitude appears to be small in comparison with that of the first rhs term in (B2). Therefore, we can write

$$\nabla\cdot\overline{q_s'\mathbf{V}'}=-\frac{\partial}{\partial p}\left\{\frac{\nabla\cdot f\overline{\theta'\mathbf{V}'}}{\tilde{\bar{S}}}\right\}, \tag{B3}$$

which can be evaluated if the horizontal TE fluxes of heat are known. [Note that in Holopainen et al. (1982) the expression (7a) for D^{HEAT}, corresponding to (A3) here, is different from (B3) in that the Coriolis parameter occurs in D^{HEAT} outside the del operator.] Equation (B3) must be considered more correct in the sense that its global average at each pressure level disappears, whereas that of D^{HEAT} does not. In the middle latitudes the difference between the two expressions is insignificant.

The divergent part of the TE potential vorticity flux was calculated by solving at each pressure level the Poisson equation for the potential function χ with the convergence of the TE potential vorticity flux as the source function:

$$\nabla^2\chi=-\nabla\cdot\overline{q'\mathbf{V}'}. \tag{B4}$$

This was done by expanding the rhs of (B4) in spherical harmonics and truncating the series at the total wavenumber 30 and zonal wavenumber 20. (Dealing with a single hemisphere, we used only the even functions). The divergent part of the TE flux of potential vorticity was then determined as

$$(\overline{q'\mathbf{V}'})_D=-\nabla\chi.$$

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