# Evolution of a Rossby Wave Packet in Barotropic Flows with Asymmetric Basic Current, Topography and $\delta$-Effect*

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ABSTRACT

Using the Rossby wave packet approximation and the WKB method, the evolution of a single geostrophic synoptic disturbance has been studied. The structural changes of the wave packet due to the variation of $\beta$ with latitude, the asymmetric basic currents and the variety of topography are thoroughly discussed. However, there are different effects on the wave packet in the various asymmetric basic currents, depending on the different positions of the wave packet relative to the basic current and the variety of topography. The $\delta$-effect always lengthens the Rossby wave packet's longitudinal scale and causes the westward-tilting trough line to lean toward the $Y$-axis, i.e., to the north. When the Rossby wave packet is located to the left (right) side of a southwesterly jet its longitudinal scale (latitudinal scale) will lengthen and its latitudinal scale (longitudinal scale) will shrink, while the westward-tilting trough line will become more westward (toward the $Y$-axis). Linearly sloping topographies will not affect the structure of the Rossby wave packet, but nonlinear topographies do affect the structure of the packet. The results suggest that the mountains, especially the Rocky Mountains, may decrease (increase) the $X$-propagating disturbance system when it is westward- (eastward) tilted. The effects of topography on distributions of east–west oriented, north–south oriented, convex and concave models have been discussed in detail. Two examples of the entire evolution of the Rossby wave packet are also presented.

## 1. Introduction

There have been many studies on the subject of Rossby waves (e.g., Phillips, 1965; Platzman, 1968; Pedlosky, 1979; Held, 1983; Karoly and Hoskins, 1982). As is well known, the propagation properties of the Rossby wave are related to the linear change of the Coriolis parameter with respect to latitude, called the $\beta$-effect. So far, few have considered the effect of variation with latitude of nonlinear change (Hoskins and Karoly, 1981). In what follows, we will introduce the parameter $\delta = -d\beta/dy$. We propose to study the effect of $\delta$ on the evolution of the disturbance systems.

There is growing evidence of the importance of the role of the earth's topography in both the atmosphere and ocean. Since Petterssen (1956) pointed out the importance of the topographic effect in the atmosphere, numerous studies have been done on this subject (e.g., recently, Tung and Lindzen, 1979; Hart, 1979). A good summary of the topographic effects is found in Smagorinsky (1980). After Charney initiated the theory of multiple equilibria, the role of topography has been given more attention (Charney and DeVore, 1979; Charney and Straus, 1980; Yoden and Mukougawa, 1983; Yoden, 1985; Cessi and Speranza, 1985). For instance, Hayashi and Golder (1983) computed the general circulations of the atmosphere with and without mountains, by using GFDL spectral general circulation models. The results show that, in the absence of mountains, the eastward-moving wave (wavenumber 4–6) components are markedly increased in the Northern Hemisphere, which suggests that the mountains reduce the components of the eastward-moving synoptic wave. However, there is still a lack of understanding of the structural changes of the disturbance due to influences of differing topographies. In the present paper, we will study these structural changes and show that the main results relative to the effects of topography are consistent with those found by Hayashi and Golder (1983).

The purpose of this study is to investigate the effects of (i) the variation of $\beta$ with latitude (which we call the $\delta$-effect), (ii) asymmetric basic currents, for instance, the asymmetric jets, and (iii) the topography upon the evolution of the Rossby wave packet. There are few theoretical studies of the structural change of the Rossby wave packet. Zeng (1982, 1983a,b) considered the effect of the symmetric zonal flow upon the evolution of the Rossby wave packet. He found that the development of a Rossby wave packet in the upper level of the atmosphere depends on the packet's structure and location with respect to the zonal flow, regardless of the stability of the zonal flow. The spatial scale or wavelength of developing (decaying) disturbances increases (decreases). Meridional tilt of barotropic decaying (developing) trough lines increases (decreases). However, Zeng did not consider (i) the to-

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pography, which plays an important role in the evolution of the flow, especially in low levels, and (ii) the variation of $\beta$ with respect to latitude. Therefore, it is worthwhile including in the investigation asymmetric basic currents, and the variation of $\beta$ with latitude and topography.

The model equations will be given in the following section. More discussions will focus on the structural changes of a synoptic disturbance due to these factors. The results of some model computations will be presented in section 3. In section 4, we will compute two concrete examples of the evolution of the Rossby wave packet and give some discussions for the evolution. The final section will be conclusions with remarks.

## 2. Equations governing the evolution of the barotropic Rossby wave packet

The linearized potential vorticity equation for synoptic scales can be written in the following nondimensional form:

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}+V\frac{\partial}{\partial y}\right)(\nabla^2\psi-F\psi)+(B_1+\beta_1)\frac{\partial\psi}{\partial x}-(B_2+\beta_2)\frac{\partial\psi}{\partial y}=0, \quad (2.1)$$

where

$$B_1=FU+\beta_0-\epsilon\delta_0 y-\frac{\partial^2 U}{\partial y^2}, \quad (2.2a)$$

$$B_2=\frac{\partial^2 V}{\partial x^2}-FV, \quad (2.2b)$$

where $U$ is the $x$-component of basic current, which is a function of latitude and time; $V$ is the $y$-component of basic current, which is a function of longitude and time; and

$$\beta_1=\frac{\partial\eta_B}{\partial y}, \quad \beta_2=\frac{\partial\eta_B}{\partial x}, \quad (2.2c)$$

$$F=\frac{f_0^2L^2}{gD}, \quad \beta_0=\left.\frac{\partial f}{\partial y}\right|_{y=y_0},$$

$$\delta_0=-\left.\frac{\partial\beta}{\partial y}\right|_{y=y_0}=-\left.\frac{\partial^2 f}{\partial y^2}\right|_{y=y_0}, \quad (2.2d)$$

where $\eta_B$ is the relative height of the topography; $\beta_2$ and $\beta_1$ describe the east–west and the north–south slopes of the topography, respectively; $F$ is the Froude number; $L$ and $D$ are horizontal and vertical characteristic scales of the disturbance, respectively. Also, $\delta_0$ is the variation of $\beta$ with latitude and $\epsilon$ is a small parameter, which is defined as

$$\epsilon=O\left(\frac{L}{R}\right), \quad (2.2e)$$

where $R$ is the average radius of the earth.

We simplify an individual single synoptic disturbance system as a Rossby wave packet. Then the geostrophic stream function for the synoptic disturbance can be considered as a Rossby wave packet. Since $\beta$ changes very slowly with latitude and in most cases the synoptic basic flows vary slowly with spatial variables and time in geosphysical fluid, then the functions in our model (2.1) will be varying very slowly with spatial variables and time as far as only large-scale topography with smooth shape has been considered. On the other hand, the processes of the evolution of the generated disturbance systems, such as the slowly varying trough systems and cyclones after being generated in the real atmosphere, are carried on more slowly than their generating processes. In such cases, the WKB method, which has been used successfully in many studies of geophysical flows (McWilliams, 1976; Pedlosky, 1979; Lindzen and Rosenthal, 1981; Zeng, 1982, 1983a,b), will be a natural choice to investigate the present problem since it has such characteristics. We restrict our discussion to the case in which the basic flows are varying slowly in spatial variables and time and the topographies are large-scale with smooth shape, i.e., they change slowly in spatial variables.

The time and space variables are as follows:

$$T=\epsilon t, \quad X=\epsilon x, \quad Y=\epsilon y, \quad (2.3)$$

$$\psi=\Psi(X,Y,T)e^{i\theta(X,Y,T)/\epsilon}, \quad (2.4)$$

where

$$\Psi(X,Y,T)=\Psi_0(X,Y,T)+\epsilon\Psi_1(X,Y,T)+\cdots, \quad (2.5)$$

$$\sigma=-\frac{\partial\theta}{\partial T}, \quad m=\frac{\partial\theta}{\partial X}, \quad n=\frac{\partial\theta}{\partial Y}, \quad (2.6)$$

and where $\sigma$, $m$ and $n$ are called, respectively, the local frequency, the local wavenumber along the $X$-direction and the local wavenumber along the $Y$-direction of the Rossby wave packet.

Substituting (2.5) into (2.1), using (2.3), (2.4) and (2.6), we obtain the zeroth approximation, or the dispersion relation:

$$(\sigma-Um-Vn)K^2+(B_1+\beta_1)m-(B_2+\beta_2)n=0, \quad (2.7)$$

where

$$K^2=m^2+n^2+F. \quad (2.8)$$

From the first approximation, the amplitude equation is

$$\left(\frac{\partial}{\partial T}+U\frac{\partial}{\partial X}+V\frac{\partial}{\partial Y}\right)K^2\Psi_0-(\sigma-Um-Vn)\times\left\{\left(\frac{\partial m}{\partial X}+\frac{\partial n}{\partial Y}\right)\Psi_0+2\left(m\frac{\partial}{\partial X}+n\frac{\partial}{\partial Y}\right)\Psi_0\right\}-\left\{(B_1+\beta_1)\frac{\partial}{\partial X}-(B_2+\beta_2)\frac{\partial}{\partial Y}\right\}\Psi_0=0. \quad (2.9)$$

From the dispersion relation (2.7), the frequency equation is

$$\sigma = Um + Vn - \frac{m}{K^2}(B_1+\beta_1) + \frac{n}{K^2}(B_2+\beta_2). \quad (2.10)$$

Therefore, the phase velocities are

$$C_X = \frac{\sigma}{m} = U + V\frac{n}{m} - K^{-2}\left\{(B_1+\beta_1) - \frac{n}{m}(B_2+\beta_2)\right\}, \quad (2.11)$$

$$C_Y = \frac{\sigma}{n} = \frac{m}{n}U + V - K^{-2}\left\{(B_1+\beta_1)\frac{m}{n} - (B_1+\beta_2)\right\}, \quad (2.12)$$

and the group velocities are

$$C_{gX} = \frac{\partial\sigma}{\partial m} = U - K^{-4}\{(B_1+\beta_1)K^2 - 2m[(B_1+\beta_1)m - (B_2+\beta_2)n]\}, \quad (2.13)$$

$$C_{gY} = \frac{\partial\sigma}{\partial n} = V + K^{-4}\{(B_2+\beta_2)K^2 + 2n[(B_1+\beta_1)m - (B_2+\beta_2)n]\}. \quad (2.14)$$

Using (2.6), the definitions of $\sigma$, $m$ and $n$, (2.10), (2.13) and (2.14) yield

$$\frac{D_g\sigma}{DT} = m\left(\frac{\partial U}{\partial T} - K^{-2}\frac{\partial B_1}{\partial T}\right) + n\left(K^{-2}\frac{\partial B_2}{\partial T} + \frac{\partial V}{\partial T}\right), \quad (2.15)$$

$$\frac{D_gm}{DT} = -\left[n\frac{\partial V}{\partial X} - \frac{m}{K^2}\frac{\partial\beta_1}{\partial X} + \frac{n}{K^2}\left(\frac{\partial B_2}{\partial X} + \frac{\partial\beta_2}{\partial X}\right)\right], \quad (2.16)$$

$$\frac{D_gn}{DT} = -\left[m\frac{\partial U}{\partial Y} - \frac{m}{K^2}\left(\frac{\partial B_1}{\partial Y} + \frac{\partial\beta_1}{\partial Y}\right) + \frac{n}{K^2}\frac{\partial\beta_2}{\partial Y}\right]. \quad (2.17)$$

In addition, the equations governing the whole structural change of the Rossby wave packet are

$$\frac{D_g(m^2+n^2)}{DT} = -2mn\left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right) + \frac{2m^2}{K^2}\frac{\partial\beta_1}{\partial X} - \frac{2mn}{K^2}\left(\frac{\partial B_2}{\partial X} + \frac{\partial\beta_2}{\partial X}\right) + \frac{2mn}{K^2}\left(\frac{\partial B_1}{\partial Y} + \frac{\partial\beta_1}{\partial Y}\right) - \frac{2n^2}{K^2}\frac{\partial\beta_2}{\partial Y}, \quad (2.18)$$

$$\frac{D_g}{DT}\left(-\frac{n}{m}\right) = \frac{\partial U}{\partial Y} - \frac{n^2}{m^2}\frac{\partial V}{\partial X} - \frac{1}{K^2}\left(\frac{\partial B_1}{\partial Y} + \frac{\partial\beta_1}{\partial Y}\right) + \frac{n}{mK^2}\frac{\partial\beta_2}{\partial Y} + \frac{n}{mK^2}\frac{\partial\beta_1}{\partial X} - \frac{n^2}{m^2K^2}\left(\frac{\partial B_2}{\partial X} + \frac{\partial\beta_2}{\partial X}\right), \quad (2.19)$$

where

$$\frac{D_g}{DT} = \frac{\partial}{\partial T} + C_{gX}\frac{\partial}{\partial X} + C_{gY}\frac{\partial}{\partial Y}. \quad (2.20)$$

Therefore, all the equations governing the evolution of the Rossby wave packet (2.15)–(2.19) have been obtained.

Let

$$\Psi_0 = |\Psi_0|e^{i\alpha(X,Y,T)}, \quad (2.21)$$

where

$$|\Psi_0| = |\Psi_0(X,Y,T)|. \quad (2.21)$$

Using the above, we rewrite Eq. (2.9) as

$$\frac{D_g\alpha}{DT} = 0, \quad (2.22)$$

$$\left(\frac{\partial}{\partial T} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right)K^2|\Psi_0| + K^{-2}[(B_1+\beta_1)m - (B_2+\beta_2)n]\left[\left(\frac{\partial m}{\partial X} + \frac{\partial n}{\partial Y}\right)|\Psi_0| + 2\left(m\frac{\partial}{\partial X} + n\frac{\partial}{\partial Y}\right)|\Psi_0| - \left\{(B_1+\beta_1)\frac{\partial}{\partial X} - (B_2+\beta_2)\frac{\partial}{\partial Y}\right\}|\Psi_0| = 0. \quad (2.23)$$

## 3. Structural change of the Rossby wave packet

It can be shown that, in the present model, energy is not conserved but rather the "wave action" (Whitham, 1974), which is the ratio of the energy density and the frequency. Equation (2.22) states that the energy of the Rossby wave packet will propagate with the group velocity. On the other hand, from (2.3), (2.4) and (2.21), one has

$$\psi \approx \Psi_0 e^{i\theta/\epsilon} = |\Psi_0|e^{i(\alpha+\theta/\epsilon)}; \quad (3.1)$$

since $\epsilon$ is a small parameter, $\theta/\epsilon$ will be much greater than $\alpha$ and the Rossby wave packet will mainly move with the wave vector determined by the phase function. Previous studies primarily focused on the effects of basic flow, topography and the variation of the Coriolis parameter with latitude on the properties of propagation of the Rossby waves.

The propagation properties cannot completely describe the evolution of the packet. Does the Rossby wave packet change its structure with time due to these factors? If yes, then how do these factors alter the structure?

### a. *The earth's rotation: the δ-effect*

The so-called $\delta$-effect is caused by the effect of the second derivative of the Coriolis parameter with respect to latitude on the structural change of the Rossby wave packet. If we only consider the $\delta$-effect, modeling conditions near the polar regions, then (2.16)–(2.19) will become

$$\frac{D_gm}{DT} = 0, \quad (3.2)$$

$$\frac{D_gn}{DT} = -\frac{m}{K^2}\delta_0, \quad (3.3)$$

$$\frac{D_g}{DT}(m^2+n^2) = -\frac{2mn}{K^2}\delta_0, \quad (3.4)$$

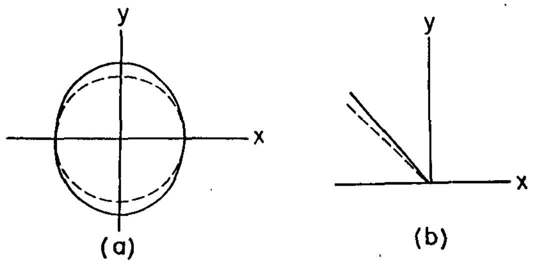


FIG. 1. The $\delta$-effect. (a) change of the spatial scale and (b) change of the trough (or ridge) line of the Rossby wave packet due to the variation of with latitude. Dashed line at $T = 0$, solid line at $T = 200$.

$$\frac{D_g}{DT}\left(-\frac{n}{m}\right)=\frac{1}{K^2}\delta_0, \tag{3.5}$$

where $\delta_0$ is always greater than zero.

From the definition of the local wavenumber, when the phase function is constant, the slope of the trough (or ridge) line is derived as

$$\left(\frac{\partial X}{\partial Y}\right)_{\theta=\mathrm{const}}=-\frac{\partial\theta/\partial Y}{\partial\theta/\partial X}=-\frac{n}{m}. \tag{3.6}$$

Therefore, $-n/m < 0$ means that the slope of the disturbance system is tilted to the west with respect to the $Y$-axis and $-n/m > 0$ means that the disturbance system is tilted to the east. Without loss of generality, we suppose $m > 0$. Then, the disturbance system has a westward (eastward) tilt with respect to the $Y$-axis for $n > 0$ ($n < 0$). In the following, we will use the terminology trough (ridge) line instead of the slope of the system, which is more commonly used in the atmospheric sciences.

From (3.2)–(3.5) one finds that $\delta$ will not affect the packet's longitudinal scale, but it does affect its latitudinal scale. This enlarges the packet's latitudinal scale by decreasing the latitudinal wavenumber.[1] This means that the whole spatial scale will become enlarged. Equation (3.4) indicates that the $\delta$-effect makes the westward-tilting trough move toward the $Y$-axis while it makes the eastward-tilting trough line move eastward. For example, taking $\delta_0 = 0.5$, which corresponds to the case in which the wavelength is about 4000 km in the middle latitude regions, we compute the $\delta$-effect, shown in Fig. 1. In Fig. 1a, the broken line is an ideal streamline at $T = 0$, and the solid line is the streamline after 200 dimensionless time steps, which is about five days in dimensionless time scale, a typical time scale for synoptic systems. Hereafter we will always take $F = 1$ and $\epsilon = 0.1$, and choose $m = n = 6$ at $T = 0$, in all our model computations without declaration. For convenience of comparison, we take an ideal streamline at $T = 0$ as the isolated symmetric eddy. In Fig. 1b, the broken line is the trough line at $T = 0$, while the solid line is the trough line after 200 dimensionless time steps. The results show that the latitudinal scale of the wave packet increases by 20 percent, so that the whole spatial scale has been increased by about eight percent and the trough line tilts toward the $Y$-axis by 5 degrees.

[1] Hereafter, the spatial scale change will always be considered in a westward-tilting system, i.e., $n > 0$, unless stated otherwise. Obviously, the eastward-tilting disturbances can also be discussed using the same method.

*b. Asymmetric basic currents*

If only the main part of the asymmetric basic current effect on the structural change of the Rossby wave packet is considered, then (2.16)–(2.19) become

$$D_g m/DT=-\frac{n}{K^2}(m^2+n^2)\frac{\partial V}{\partial X}, \tag{3.7}$$

$$D_g n/DT=-\frac{m}{K^2}(m^2+n^2)\frac{\partial U}{\partial Y}, \tag{3.8}$$

$$D_g(m^2+n^2)/DT=-2mn\left(\frac{\partial U}{\partial Y}+\frac{\partial V}{\partial X}\right)\frac{m^2+n^2}{K^2}, \tag{3.9}$$

$$D_g\left(-\frac{n}{m}\right)\Big/DT=\left(\frac{\partial U}{\partial Y}-\frac{n^2}{m^2}\frac{\partial V}{\partial X}\right)\frac{m^2+n^2}{K^2}. \tag{3.10}$$

As examples, we consider two typical asymmetric basic currents, which often occur in southeast Asia, i.e., the southwesterly jet and the southeasterly jet. Figure 2 shows an ideal southwesterly jet and an ideal southeasterly jet, respectively.

On the left side of a southwesterly jet, we have

$$\frac{\partial U}{\partial Y}<0,\quad \frac{\partial V}{\partial X}>0. \tag{3.11}$$

Thus, from (3.7)–(3.10) one could know that at such a location the longitudinal scale of the packet will increase while its latitudinal scale will decrease. Also, the westward-tilting trough line will become more westwardly while the eastward-tilting trough line will tend toward the $Y$-axis.

On the right side of the southwesterly jet,

$$\frac{\partial U}{\partial Y}>0,\quad \frac{\partial V}{\partial X}<0. \tag{3.12}$$

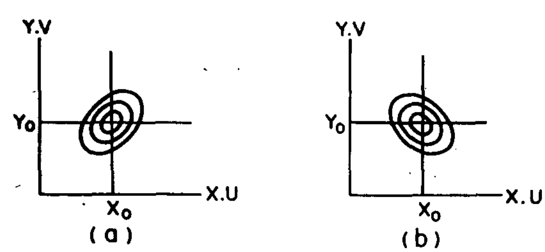


FIG. 2. The ideal jet profiles. (a) the southwesterly jet and (b) the southeasterly jet. Centers are located at $(X_0, Y_0)$ and basic isotachs are given by solid lines.

Therefore, the packet's latitudinal scale will increase and its longitudinal scale will decrease with time. The westward-tilting trough line will tend toward the $Y$-axis while the eastward-tilting trough line will become more eastwardly tilted. The southeasterly jet would act similarly but in reverse.

Figure 3 illustrates the main results of spatial structural changes of the Rossby wave packet in a southwesterly jet. In Fig. 3, we take $\partial U/\partial Y = -0.02$ and $\partial V/\partial X = 0.01$ on the left side of the jet, which corresponds to the basic current changes of about 2 m $s^{-1}$ in the $U$ component and 1 m $s^{-1}$ in the $V$ component 100 km out from the jet center. The broken line is an ideal streamline of the packet at $T = 0$ and the solid line is the streamline after 200 dimensionless time steps. The results show that in such a southwesterly jet the packet's longitudinal scale enlarges 40 percent while its latitudinal scale shrinks 20 percent. In addition, the whole spatial scale has been decreased by about 10 percent and its westward-tilting trough line tends toward the west by about 16 degrees on the left side of the jet. However, on the right side of such a southwesterly jet, where we chose $\partial U/\partial Y = 0.02$, $\partial V/\partial X = -0.01$, the results show that the packet's longitudinal scale decreases 10 percent and its latitudinal scale increases by 80 percent. Hence, the whole spatial scale has been increased by about twelve percent and the westward-tilting trough line tends towards the $Y$-axis about 18 degrees.

Figure 4 shows the results of an ideal southeasterly jet. Here again, we have computed the results after 200 dimensionless time steps, considering such a southeasterly jet, where $\partial U/\partial Y = 0.02$ and $\partial V/\partial X = 0.01$. The results show that on the left side of the jet, the whole scale will increase while the westward-tilting trough line will tend toward the $Y$-axis. The longitudinal scale increases about 30 percent, while the latitudinal scale increases seventy percent and the trough line tilts about 8 degrees. However, on the right side of the jet, the whole scale of the packet will decrease while the westward-tilting trough line will tilt toward the west. The longitudinal scale shrinks about 20 percent, the latitudinal scale decreases about 30 percent. Therefore, the whole spatial scale has been increased about 35 percent and the trough line tilts westward about 4 degrees after 200 dimensionless time steps. Figure 5 shows the results of a trough line, where the broken line is the trough line at $T = 0$, solid lines A, B, C and D are the westward tilting trough line after 200 dimensionless time steps, corresponding to the cases in which the wave packet is on the left side and right side of a southwesterly jet and on the left side and right side of a southeasterly jet, respectively.

It can be seen from these figures that there is an obvious difference between the effects of these two jets upon the packet. In the southwesterly jet, the change of the packet's whole spatial scale depends on the magnitudes of $\partial U/\partial Y$ and $\partial V/\partial X$, since its latitudinal scale decreases as its longitudinal scale increases. However, in a southeasterly jet both the longitudinal scale and the latitudinal scale increase or decrease (depending on the location relative to the jet) simultaneously.

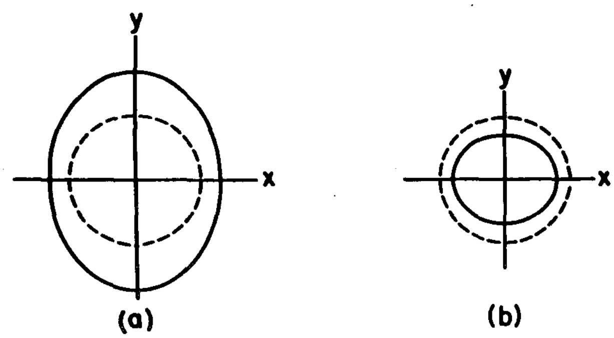


FIG. 4. The spatial scale change of the Rossby wave packet in southeasterly jet. (a) on the left side and (b) on the right side of the jet. (Dashed line) an ideal streamline at $T = 0$, (solid line) the ideal streamline at $T = 200$.

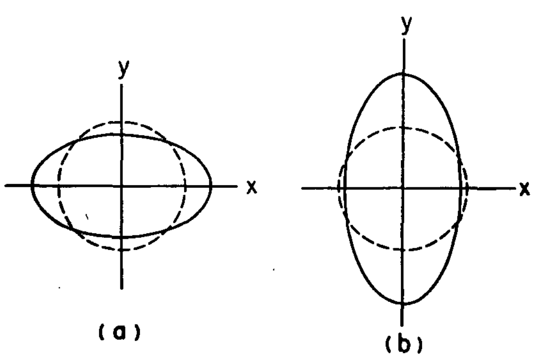


FIG. 3. The spatial scale change of the Rossby wave packet in southwesterly jet. (a) on the left side and (b) on the right side of the jet. (Dashed line) an ideal streamline at $T = 0$, (solid line) the ideal streamline at $T = 200$.

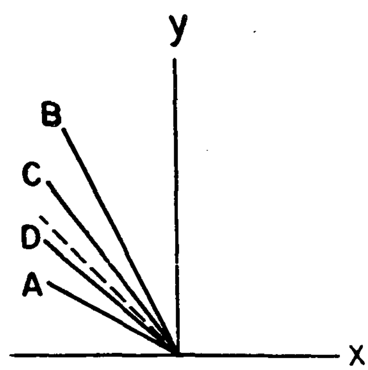


FIG. 5. The trough (or ridge) lines. The broken line is at $T = 0$, solid lines A, B, C and D are trough lines at $T = 200$, corresponding to the cases on the left side and the right side of the southwesterly jet and on the left side and on the right side of the southeasterly jet, respectively.

Therefore, the change of the spatial scale in the southeasterly jet will be significant in some cases, i.e., the jet will largely affect the spatial scale of the packet. Furthermore, from Fig. 3 one can find that the Rossby wave packet, when located on the left side of the southwesterly jet, will lengthen its longitudinal scale and shrink its latitudinal scale so that finally the wave packet can easily be absorbed by the zonal current. This phenomenon was called "rotational adaptation" by Zeng (1979). However, here the "rotational adaptation" is caused not only by the rotation, but rather by the combination of the rotation (and/or the topography) and the jet. Yet, when located at the right side of southwesterly jet, the "rotational adaptation" will not happen. If the increase of longitudinal scale is simply considered as developing the disturbance system, or taking the latitudinal scale to be infinite, i.e., $n = 0$, the result suggests that the disturbance develops more easily on the right side of the southwesterly jet than on the left.

### c. *Topography*

Considering the effect of topography, from (2.16)–(2.19) one has

$$D_g m/DT = K^{-2}\left(m\frac{\partial\beta_1}{\partial X} - n\frac{\partial\beta_2}{\partial X}\right), \qquad (3.13)$$

$$D_g n/DT = K^{-2}\left(m\frac{\partial\beta_1}{\partial Y} - n\frac{\partial B_2}{\partial Y}\right), \qquad (3.14)$$

$$D_g(m^2+n^2)/DT$$
$$= K^{-2}\left[2m^2\frac{\partial\beta_1}{\partial X} - 2n^2\frac{\partial\beta_2}{\partial Y} + 2mn\left(\frac{\partial\beta_1}{\partial Y} - \frac{\partial\beta_2}{\partial X}\right)\right], \qquad (3.15)$$

$$D_g\left(-\frac{m}{n}\right)\Big/ DT$$
$$= K^{-2}\left[-\frac{\partial\beta_1}{\partial Y} + \frac{n}{m}\frac{\partial\beta_1}{\partial X} - \frac{n^2}{m^2}\frac{\partial\beta_2}{\partial X} + \frac{n}{m}\frac{\partial\beta_2}{\partial Y}\right]. \qquad (3.16)$$

#### 1) LINEARLY DISTRIBUTED TOPOGRAPHY

Equations (3.13)–(3.16) show that linearly distributed topography, i.e., the height of the topography is a linear function of $X$ and $Y$, has no effect on the structural change of the Rossby wave packet. In other words, the packet above the linearly sloping topography can have the same structure as that above a flat surface. From previous studies (e.g., Pedlosky, 1979) or Eq. (2.11)–(2.14) in section 2, it is known that the linearly sloping topography does affect the properties of the propagations in both phase and energy. It may be interpreted that when the wave packet is propagating in the nearly sloping topography, it can not acquire or lose its potential vorticity from or into the environmental vorticity field with higher or lower speed. This is due to the linearly sloping topography which only produces a uniform environmental vorticity gradient field. Therefore, the linearly sloping topography can not change the structure of the wave packet. This is reminiscent of the $\beta$-plane approximation. In that case the Rossby wave can be found with its phase velocity and energy propagation. However, we did not find a change in the structure of the Rossby wave. The reason is that in the potential vorticity equation, the gradient of planetary vorticity was artificially taken to be constant. So, after arriving at its new position the Rossby wave could not gain or lose its relative vorticity with higher or lower speed from or into planetary vorticity.

#### 2) NONLINEARLY DISTRIBUTED TOPOGRAPHY

Nonlinear topography, i.e., topography described by a nonlinear function of $X$ and $Y$, has a substantially different effect on the structure of the Rossby wave packet than linear topography. Nonlinear topography will affect not only the phase velocity and the energy propagation properties, but also the packet's structure. For simplicity, four types of topographies will be considered. Other more complex types can be discussed by using the same method.

*North–south oriented topography.* The Rocky Mountains may be considered a typical example. In this case,

$$\beta_1 = 0, \quad \frac{\partial\beta_2}{\partial Y} = 0, \quad \frac{\partial\beta_2}{\partial X} < 0. \qquad (3.17)$$

Then, the equations (3.13)–(3.16) can be written as

$$D_g m/DT = nK^{-2}\left|\frac{\partial\beta_2}{\partial X}\right|, \qquad (3.18)$$

$$D_g n/DT = 0, \qquad (3.19)$$

$$D_g(m^2+n^2)/DT = 2mnK^{-2}\left|\frac{\partial\beta_2}{\partial X}\right|, \qquad (3.20)$$

$$D_g\left(-\frac{n}{m}\right)\Big/ DT = \frac{n^2}{m^2K^2}\left|\frac{\partial\beta_2}{\partial X}\right|. \qquad (3.21)$$

Obviously, in this case, the Rossby wave packet will shrink its longitudinal scale while its latitudinal scale will remain unaltered. The whole spatial scale will shrink with time. The westward-tilting trough will tilt more toward the $Y$-axis while the eastward-tilting trough line will tilt further eastward. Figure 6a and Figure 8 show the results of such a kind of topography, considering $\partial\beta_2/\partial X = -0.8$, where the broken line is the ideal streamline at $T = 0$ while the solid line is the streamline after 200 dimensionless time steps. Here, if we consider the topography to be a hyperbolic surface, then $\partial\beta_2/\partial X = -0.8$ corresponds to $\eta_B = H - 0.8xX$. One can find from the figure that its latitudinal scale remains the same while its longitudinal scale decreases ten percent, which implies that the whole spatial scale has been decreased about ten percent and its westward-tilting trough line tends to the $Y$-axis about five degrees,

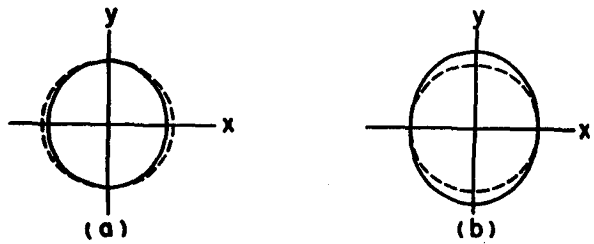


FIG. 6. The spatial scale change of the Rossby wave packet on (a) the north–south and (b) the east–west oriented topography. (Dashed line) an ideal streamline at $T = 0$, (solid line) the ideal streamline at $T = 200$.

after 200 dimensionless time steps, as shown in Fig. 6a and in Fig. 8 by solid line A.

*East–west oriented topography.* Here,

$$\frac{\partial\beta_1}{\partial Y}<0, \quad \frac{\partial\beta_1}{\partial X}=0, \quad \beta_2=0 \tag{3.22}$$

Thus, the Eqs. (3.13)–(3.16) will take the following forms:

$$D_g m/DT=0, \tag{3.23}$$

$$D_g n/DT=-mK^{-2}\left|\frac{\partial\beta_1}{\partial Y}\right|, \tag{3.24}$$

$$D_g(m^2+n^2)/DT=-2mnK^{-2}\left|\frac{\partial\beta_1}{\partial Y}\right|, \tag{3.25}$$

$$\frac{D_g}{DT}\left(-\frac{n}{m}\right)=K^{-2}\left|\frac{\partial\beta_1}{\partial Y}\right|. \tag{3.26}$$

Therefore, an east–west-oriented topography will not alter the Rossby wave packet's longitudinal scale but will increase its latitudinal scale, enlarging the entire spatial scale. The westward-tilting trough line will tend toward the $Y$-axis while the eastward-tilting trough line will tilt further east. Figure 6b shows the result of this kind of topography, taking $\partial\beta_1/\partial Y = -0.5$, corresponding to $\eta_B = H - 0.5yY$, where again the broken line is an ideal streamline at $T = 0$ and the solid line is the streamline after 200 dimensionless time steps. It is shown that after 200 dimensionless time steps the packet's longitudinal scale will remain the same as before, while its latitudinal scale will increase about 20 percent. In this case, the whole spatial scale has been increased about 8 percent, and the packet's westward-tilting trough line will tend toward the $Y$-axis by about 5 degrees, which is the solid line B in Fig. 8.

*Convex topography.* The Tibet Plateau may be considered an example of this type of topography. Then,

$$\frac{\partial\beta_1}{\partial Y}=-b<0, \quad \frac{\partial\beta_2}{\partial X}=-a<0, \tag{3.27}$$

where $a$ and $b$ are taken as positive constants. The governing equations are

$$D_g m/DT=anK^{-2}, \tag{3.28}$$

$$D_g n/DT=-bmK^{-2}, \tag{3.29}$$

$$D_g(m^2+n^2)/DT=2mnK^{-2}(a-b) \tag{3.30}$$

$$D_g\left(-\frac{n}{m}\right)\Big/DT=m^{-2}K^{-2}(m^2b+n^2a). \tag{3.31}$$

From these equations, it can be seen that the convex topography will shrink the longitudinal scale. If we consider an $X$-propagating disturbance system, the result strongly suggests that the mountains, especially the Rocky Mountains (where $b = 0$), will decrease the longitudinal component of the system. This result is consistent with that of the spectral general circulation models of Hayashi and Golder (1983). They found that in the absence of mountains, the length-scales of eastward-moving wave (wavenumbers 4–6) are markedly increased in the Northern Hemisphere. The mountains did decrease the eastward-moving wave components. The change of the whole spatial scale depends on the magnitudes of $a$ and $b$. When $a = b$, which describes symmetric topography about the vertical axis, the whole scale will not change. The westward-tilting trough line will tend toward the $Y$-axis while the eastward-tilting trough line will be tilted more toward the east. If we consider the Tibet Plateau to be topography typical of the case $b > a$, the topography lengthens the packet's whole spatial scale and moves its westward-tilting trough line toward $Y$-axis while its eastward-tilting trough line will tilt more to the east. These results are consistent with the observations and the results of Hayashi and Golder (1983). This kind of topography has the same qualitative effect on the structural change of the packet as that of the southwesterly jet located northwest of the packet. Therefore, when there is a southwesterly jet over the plateau, the change of spatial scale will be much more obvious. From this discussion, we may conclude that when the Rossby wave packet passes over the plateau, its latitudinal scale will lengthen and the weather system will tend to develop more easily. This agrees with observational data (Zhu et al., 1982).

Taking $a = b = 0.5$, we have computed the structural changes of the wave packet. The results, after integrating for 200 dimensionless time steps, are given in Fig. 7a and the solid line C in Fig. 8. It can be found from Fig. 7a that in such a case the longitudinal scale will decrease about ten percent and the latitudinal scale will increase forty percent after 200 dimensionless time steps. The trough line will tilt toward the $Y$-axis about 12 degrees after 200 dimensionless time steps, as shown in Fig. 8 by the solid line C.

*Concave topography.* This kind of topography can be more easily found in oceans than in the atmosphere. However, the basins in both oceans and the atmosphere may be considered as this type of topography. The effect of this type of topography can be easily discussed with

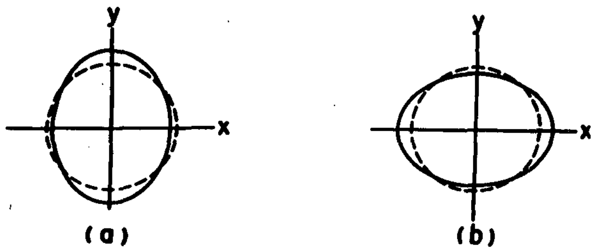


FIG. 7. The spatial scale change of the Rossby wave packet on (a) convex and (b) concave topography. (Dashed line) an ideal streamline at $T = 0$, (solid line) the ideal streamline at $T = 200$.

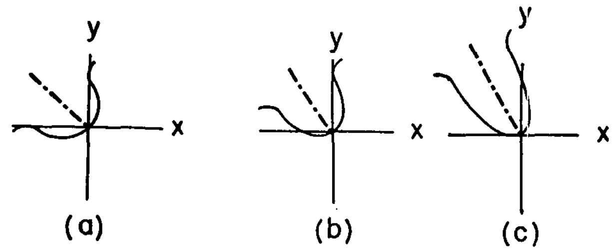


FIG. 9. The evolution of the Rossby wave packet on the right side of a southwesterly jet and convex topography. (a) at $T = 0$, (*b*) at $T = 100$ and (c) at $T = 200$. The solid line is the ideal streamline and heavy broken line is the trough line.

the same method as for the convex topography. Figure 7b and solid line D in Fig. 8 illustrate this effect upon the structural changes of the wave packet, where $\partial\beta_2/\partial X = 0.5$ and $\partial\beta_1/\partial Y = 0.5$ have been taken. The computations show that after 200 dimensionless time steps the wave packet's longitudinal scale has lengthened about 20 percent while its latitudinal scale has shrunk about 10 percent and the trough (or ridge) line will tilt westward about 7 degrees.

In Fig. 8, the broken line is the trough (or ridge) line at the beginning, i.e., at $T = 0$, the solid lines A, B, C and D are trough (or ridge) lines after 200 dimensionless time steps, corresponding to the north–south, east–west, convex and concave topographies, respectively.

## 4. Two examples and discussions

We now consider two examples of the entire evolution of the Rossby wave packet. In the first example, we look at a case in which the evolution of the Rossby wave packet takes place on the right side of a southwesterly jet using convex topography. According to our results, on the right side of a southwesterly jet using convex topography, the Rossby wave packet will evolve in such a manner that its longitudinal scale will become small and its latitudinal scale will become large, while the westward-tilting trough (or ridge) line will tend to tilt toward the $Y$-axis. Figure 9 illustrates the entire evolution of the Rossby wave packet from model computations, taking $\delta_0 = 0.2$, $\partial U/\partial Y = 0.02$, $\partial V/\partial X = -0.01$, $\partial\beta_2/\partial X = -0.2$ and $\partial\beta_1/\partial Y = -0.2$. We assume that the Rossby wave packet is westward-tilting and its streamline is always symmetric about its trough line. Initially, the wave packet is supposed to be located to the west of the topography, the trough line is tilted westward 45 degrees and one of its streamlines has an ideal profile, as shown in Fig. 9a, where the solid line is an ideal streamline and the heavy broken line is its trough line. After computing 100 dimensionless time steps, two and a half days of the dimensional time scale, Fig. 9b has been obtained. If we assume that at this point the wave packet has just arrived at the middle of the topography, then according to our arguments and computations in section 2, the phase speed is accelerated during these 100 dimensionless time steps. It has been found from the computations that the longitudinal scale has decreased about 10 percent and the latitudinal scale has increased about 40 percent. However, the whole spatial scale has increased by about 12 percent and the trough line has tilted 12 degrees more toward the $Y$-axis. The pattern of computing for another 100 dimensionless time steps is shown in Fig. 9c. During this time interval the phase speed has been decreased. The longitudinal scale remains almost the same while the latitudinal scale has increased dramatically, to almost two and a half times as large as that at $T = 0$. Yet, the whole spatial scale of the wave packet has increased only about another 6 percent during these 100 dimensionless time steps and the trough line has now been tilted a total of 13 degrees toward the $Y$-axis since $T = 0$.

Figure 10 shows another example of the evolution of the Rossby wave packet when located on the left side of a southwesterly jet with the concave topography, taking $\delta_0 = 0.2$, $\partial U/\partial Y = -0.02$, $\partial V/\partial X = 0.01$, $\partial\beta_2/\partial X = 0.2$ and $\partial\beta_1/\partial Y = 0.2$. Figures 10b and 10c are the results after computing from $T = 0$ for 100 and 200 dimensionless time steps, respectively. It can be seen in the figures that the longitudinal scale has increased 20 percent during the first 100 dimensionless time steps and fifty percent for 200 dimensionless time steps, while the latitudinal scale has decreased about 10 percent for each 100 dimensionless time steps. On

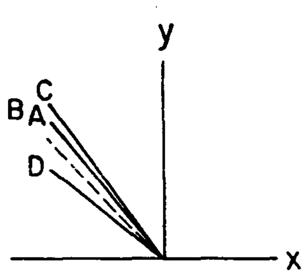


FIG. 8. The trough (ridge) lines. The broken line is at $T = 0$, solid lines A, B, C and D are at $T = 200$, corresponding to on the north–south-oriented, the east–west-oriented, convex and concave topography, respectively.

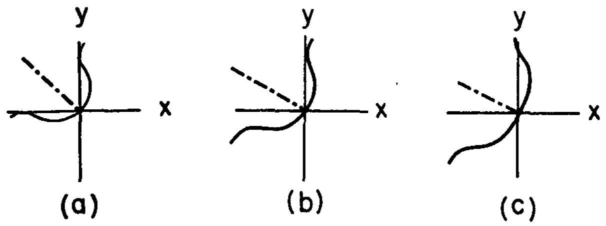


FIG. 10. As in Fig. 9, except on the left side of a southwesterly jet and concave topography.

the other hand, the whole spatial scale of the wave packet has decreased only about 3 percent for each 100 time steps while its trough line has further tilted 9 degrees and 17 degrees with 100 and 200 dimensionless time steps, respectively. Compared with the results in the first example, it is interesting to notice that the evolution speed in this case is different from that in the first case, although we used the same order of magnitude for each parameter in both cases.

Suppose that there is a disturbance on a north–south oriented type topography, shown in Fig. 11. It could be proven from the Eqs. (2.11)–(2.14) in section 2 that on the west side of this topography the wave packet will move toward the northeast while the energy will propagate toward the southwest. On the other hand, the Rossby wave packet will move toward the southwest while its energy will propagate toward the northeast on the east side of topography. Therefore, in this case, the topography will result in the Rossby wave packet moving up the topography and its energy propagating down the topography, shown in Fig. 11 where the double arrow line points in the direction of the energy propagation and the solid arrow line beside the topography gives the direction of the propagation of the wave packet. Thus, it can be seen from the figure that the latitudinal scale of the wave packet may shrink because the energy moves away from the packet, i.e., the packet loses its energy into the basic current, which is the only place for the wave packet to give its energy in the present model. If the system is westward-tilted, then the trough line will tend to the $Y$-axis because of the difference in phase velocity of the wave packet between the two sides of the topography. Similarly, we can consider the case of the evolution of the Rossby wave packet on an east–west-oriented topography, shown in Fig. 12. Since in the present case the energy moves toward the packet, i.e., the wave packet gains energy from the basic current, which is the only available energy source here, the wave packet's longitudinal scale will stretch, making the whole spatial scale of the wave packet lengthen. The difference in the wave packet vectors between the two sides of the topography will result in the westward-tilting trough line tending toward the $Y$-axis.

## 5. Main results and remarks

From the above discussions and some model computations, we draw the following main conclusions:

1) The $\delta$-effect results in a structural change in the packet. The Rossby wave packet's latitudinal scale increases and its westward-tilting trough line tends toward the $Y$-axis while the eastward-tilting trough line tends toward the east. This is called the $\delta$-effect. The computations show that the whole spatial scale has increased by about 8 percent and the westward-tilting trough (or ridge) line has tilted 5 degrees toward the $Y$-axis after 200 dimensionless time steps.

2) The Rossby wave packet, when located on the left (right) side of the southwesterly jet, will lengthen (shrink) its longitudinal scale and shrink (lengthen) its latitudinal scale. The westward (eastward)-tilting trough line will tilt further westward (toward the $Y$-axis) when the packet is located on the left side of the jet. The westward (eastward)-tilting trough line will tilt toward the $Y$-axis (the east) when located on the right side of the jet. The Rossby wave packet, when located on the left (right) side of a southeasterly jet will enlarge (shrink) its longitudinal scale and latitudinal scale. The numerical computations show that the whole spatial scale of the wave packet could shrink (lengthen) by about 10 percent (12 percent) on the left side (right side) of a southwesterly jet after 200 dimensionless time steps. The westward-tilting trough (ridge) line could tilt up to 16 degrees (18 degrees) toward the west (the $Y$-axis) on the left side (right side) of a southwesterly jet. The

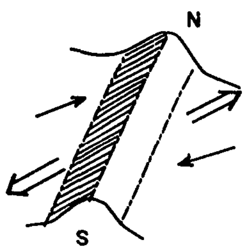


FIG. 11. A simple interpretation of the effect of a north–south-oriented topography upon the evolution of the packet. The packet's latitudinal scale shrinks since the energy propagates away from the packet. The solid arrow points in the wave vector direction, the double arrow is the direction of the energy propagation.

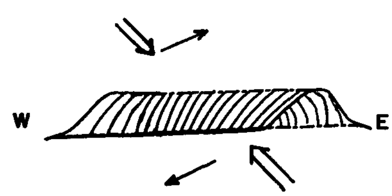


FIG. 12. A simple interpretation of the effect of an east–west-oriented topography upon the evolution of the packet. The packet's longitudinal scale stretches since the energy propagates toward the packet.

whole spatial scale could increase (decrease) by 35 percent (25 percent) on the left side (right side) of a southeasterly jet after 200 dimensionless time steps. The westward-tilting trough line could tilt about 8 degrees (5 degrees) toward the $Y$-axis (the west) on the left side (right side) of a southeasterly jet.

3) A linear topography will not alter the structure of the Rossby wave packet. However, the nonlinear topography does affect the structure of the packet. The results suggest that mountains, especially the Rocky Mountains, may decrease (increase) the longitudinal scale of the westward (eastward)-tilting disturbance system. The north–south-oriented topography will only decrease the Rossby wave packet's longitudinal scale and the westward (eastward)-tilting trough line will tilt toward the $Y$-axis (further eastward). The east–west-oriented topography will only increase the packet's latitudinal scale. The convex topography will shrink its longitudinal scale and lengthen its latitudinal scale. The east–west-oriented topography will result in the westward (eastward)-tilting trough line tilting toward the $Y$-axis (further eastward). The concave topography will lengthen the Rossby wave packet's longitudinal scale and shrink its latitudinal scale, and the westward (eastward)-tilting trough line will tilt further westward (toward the $Y$-axis). The model computations have proven the results. For instance, the computations show that the north–south (east–west)-oriented topography could result in the whole spatial scale decreasing (increasing) by about 10 percent (8 percent) after 200 dimensionless time steps.

The WKB method is only valid in the case in which the functions in the model vary slowly with spatial variables and time. Obviously, the narrow jets and sharp topographies do not satisfy this condition. Note that the present model makes it impossible to investigate three-dimensional problems, which are of importance in geophysical fluid dynamics. Therefore, further studies could be focused on the three-dimensional problem in the presence of asymmetric basic current, topography and the variation of $\beta$ with respect to latitude. In the present discussion we only take one wave packet as a disturbance. Actually, the real geophysical fluid is not so simple as one single wave packet, therefore using the concept of multiple wave packets to investigate some nonlinear or seminonlinear phenomena in the geophysical fluid will be interesting.

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